Wind power statistics in Britain, and their consequences for the integration of wind generation into electricity grids

Thesis

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Wind Power Statistics in Britain, and their Consequences for the Integration of Wind Generation into Electricity Grids.

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ABSTRACT.

This thesis describes work on the statistics of the temporal and spatial variation of wind power in the UK, and the consequences of these variations for the use of wind generation of electricity both by the Central Electricity Generating Board (CEGB) and more generally. The statistical work is based on the use of spectral analysis, and cross correlation analysis of wind power time series estimated from Meteorological Office records of hourly mean wind speeds at 30 UK recording sites over a period of 9 years. Estimates of wind power capacity credit and wind operating reserve requirements as functions of wind power penetration into the conventional grid, geographical separation of wind power plant and specific power of wind turbines are made. An important result from this work, is that cross correlation coefficients of changes in wind power for lead times of a few hours are dominated by diurnal fluctuations in wind speed. These in turn are highly variable in both magnitude and sign, depending on among other things measurement height. The dominance of diurnal fluctuations can be seen more clearly in the coherence function plots for wind power at pairs of sites. The most important consequence of the resultant uncertainty in cross correlation coefficients will be on estimates of wind operating reserve requirements, which to a first approximation are linearly dependent on these coefficients.

CONTENTS

FIGURES AND EQUATIONS.

The conventions that I have used in labelling and referring to figures and equations are as follows. Figures and equations are referred to in the order of appearance in each chapter section. So the first equation which appears in chapter section 4.2 will be referred to as equation 4.2.1. The first figure that appears in chapter section 2.4 will be referred to as fig 2.4.1. For the sake of neatness, equations are labelled simply by the last of the three numbers by which they are referred to. If the reader wishes to locate a particular equation referred to in the text, they should first find the appropriate chapter section, and then find the equation within it. Figures are labelled fully.

Figures will be found at the end of each chapter section. This convention enables the most efficient use of space to be made.

LIST OF FIGURES.

Acknowledgements.

I would like to thank my supervisor Dr. Gary Alexander and my collaborator Dr. John Dixon for the help and encouragement they have given me in doing the work described in this thesis. I would also like to thank my colleagues in the Open University Energy Research Group for their subliminal input to the work. Finally I would like to thank Chiu Lai-fong, without whose love and help I would have given up long ago.

1.0 INTRODUCTION

This thesis describes a wide range of work carried out over a period of about three years, in the general area of wind energy systems. This line of research was prompted by a number of factors. The first of these was a general concern with the problems of energy supply, which I became aware of in the final year of my undergraduate course. It is worth remembering how far the study of energy has come since 1975. At the time the energy problem was seen almost exclusively as one of energy supply. Given that energy demand was expected to increase indefinitely in a rather simple exponential fashion, following the trend established in the post-war years, and given that oil and gas production were near their peaks, how was the gap between demand and fossil fuel production to be filled? The answer, in the case of the industrialised countries, was widely thought to be nuclear power, and the result in the case of the UK was a series of proposals for nuclear power construction programmes of quite staggering size (Department of Energy 1975, UKAEA 1974 and 1976, Flowers et al 1976). It has been put to me that the French nuclear power programme since the oil crisis of 1973 makes the programmes mentioned above look rather more realistic than I give them credit for. By 1982 the French had commissioned some 23 GW of nuclear capacity (Syrota 1983) and by 1990 this is expected to have increased to over 50 GW (Nature, 23 Feb 1984). However the rate of ordering of nuclear power stations in France has been uncertain since 1983, and it seems likely that the rate in the future will be substantially lower than in the past (by perhaps a factor of 3, see Electrical Review, 19/26 August 1983). Falling predictions of energy demand for the years to the end of the century (eg. Financial Times European Energy

Report, 27 May 1983) suggest possible overcapacity in the French electricity industry, which may be offset to some extent by attempts to market electricity for uses that are arguably unsuitable, such as boilers (Nature, op cit). Developments in electricity-using technologies (see for example Nørgaard 1983) and more general studies of national energy systems (for example Leach et al 1979), suggest that no growth in electricity demand is necessary in the forseeable future and indeed that demand may advantageously be cut. In the light of these observations, it could be said that the French nuclear programme to which I have referred was somewhat overambitious as originally conceived. The British proposals to which I earlier referred were considerably larger. For example the UK Atomic Energy Authority's 1975 reference programme (UKAEA, 1976) envisaged 104 GW of nuclear capacity by 2000, and 426 GW by 2030. A document published the previous year (UKAEA, 1974) envisaged 278 GW by the year 2000.

The technical feasibility and social desirability of these programmes appeared doubtful. Indeed with the energy demands posited, this would have been true almost regardless of the supply technologies assumed (see for instance Chapman 1975). While it was difficult to see how nuclear power production could be built up to the level or at the rate which would have been needed, it was also difficult to see how renewable energy sources could have supplied more than a small part of the total energy demand. What was necessary for a constructive criticism of the orthodox energy paradigm of the time was analysis which included the question of the origin of energy demand. The first attempt of which I became aware to construct a renewable energy scenario for an industrial country, Denmark, was made by Sørensen (Sørensen

1975). This started from an analysis of Danish energy demand, which was handled in a purely physical way with no reference to level of economic activity. From this starting point Sørensen was able to demonstrate the technical feasibility of a renewable energy supply system for an industrialised country.

In the following years, a series of studies appeared which questioned the basis of the establishment energy demand forecasts, on the grounds of the implausibility and undesirability of their economic growth assumptions (Chapman op cit, see also Mishan 1969), and on the question of the link between economic growth and energy demand growth (Foley et al 1976, Chapman op cit, Lovins 1977, Leach et al 1978, Leach et al 1979). Of these studies, only Lovins' attempted detailed analysis of renewable energy supply systems. Nevertheless, these studies were of great importance in that they established the plausibility of bounded energy demand in the industrialised countries, the condition which is necessary if renewable energy technologies are to satisfy a large part of energy demand.

Given a) a disatisfaction with orthodox proposals for solving the "energy problem", in particular with the proposals for the expansion of nuclear power and b) the demonstration that the problem was soluble, in principle at least, using renewable energy technologies, the factor which drew me to the study of wind energy was the advanced state of the technology at the time. On the basis of work done over the whole of the 20th century, wind generation of electricity was clearly feasible, and had for example been in widespread use Denmark (thought this did not represent a large fraction of total electricity produced) (Golding 1955, Putnam 1948). The key question in the short term appeared to be the capital cost of producing wind turbines and it seemed a reasonable expectation at the time that costs would fall (see for example Ryle, 1977). The extent to which wind power costs are still in fact highly uncertain can be judged from chapter 2 of this thesis.

I joined the Open University Energy Research Group in 1977, with the preliminary objective of working on wind energy. Something like the next two years were spent gaining a general background in energy technology, and in economics, a subject I had not studied previously. In the subsequent study of wind energy I have attempted to apply both technical and economic analysis. In doing so I became aware of the discrepancy between the claims which are often naively made for economics and its limitations. While noting the quite fundamental objections which have been raised to economic analysis, I have come to the conclusion that in certain circumstances it may yield valid insights into the structure of a problem. I have no objection to its use provided that the limitations to any particular piece of analysis are clearly recognised and stated. Economic analysis is limited for a variety of reasons, of which the following appear particularly important:

1) uncertainty in the numerical values of input data. These data may be prices or physical data. The classic statement of the case is contained in Morgenstern 1965.

2) systematic distortion of numerical values may arise at the system boundaries of any analysis. Tariffs are a frequent symptom of this problem, a tariff being used to model in economic terms the world outside a given system boundary, or a major sub-system which the analyst does not wish to or is unable to model in more

detail. Distortion may also arise from the total absence of important sub-systems in the analysis.

3) the problem of distribution. The assumption behind welfare economics, is that society should strive to improve the welfare of its members. The problem of the estimation of welfare or changes in welfare is intractable in any analytic sense (see for instance Little 1957). Certain schools, for instance the Paretian, try to avoid the problem by avoiding inter-personal comparisons of welfare. Cash flows are frequently taken as surrogates for "flows" of welfare. To the extent that economic analysis attempts to tackle the problem of inter-personal comparison, it cannot claim to objective, and to the extent that it avoids it, it runs the risk of being irrelevant.

The second chapter of this thesis consists of a review of a commonly used economic model for planning the mix of generating plant in a large inter connected electricity supply system. The assumption of size allows fairly free use of the techniques of calculus. This economic model provides a background against which to assess the value of wind generators operating as fuel savers in such a system (in the short term). Considerable analysis of the value of wind generators based on costs expected or quoted for modern prototypes has been done (Sørensen 1978, Sørensen 1979, Lowe 1980, Dixon and Lowe 1981a, Musgrove 1981, Dixon and Lowe 1981c). Much of this work has taken the fuel saving value of wind based on a simple linearised model of an electricity system, as a first approximation to the value of wind generation, and has considered factors such as capacity credit as higher order corrections. In my view this approach has much to commend it, since:

1) the complexities and uncertainties surrounding the evaluation of the first order approximation are at least as great as those surrounding several of the higher order terms, and

2) the simple fuel saving value accounts, in the case of the UK, for of the order of 70% of the total value of wind generation (see Dixon and Lowe 1982).[1]

In the UK the first order approximation to the value of wind electricity appears to exceed the cost, but with considerable uncertainties surrounding conventional plant fuel costs, capital and running costs of wind systems, output of wind systems (Bossanyi 1981). These uncertainties are probably larger than the expectation value of the sum of the higher order terms.[2] Arguably therefore, the correct step to take is to start a construction programme of the types of wind generators for which the above statements look like being true, in order:

1) to refine the first order estimates

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[1] Clearly, this statement depends on many factors, the chief two being the discount rate and fuel costs in conventional stations. In the case of Australia, cheap coal and a higher discount rate give rise to reversed fractions of capacity credit and fuel saving value (see Diesendorf et al 1981).

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[2] Although this statement is framed in the language of statistics, the key word "probably" should be understood in its subjective sense. The statement could be justified by numerical argument, but the status of many of the numbers would still be that of subjective guesses.

2) if the refined first order estimates suggest that wind power on a large scale would be economic, to refine estimates of the higher order quantities.

In the absence in the UK of a wind generator construction programme of the type suggested above, it is necessary to make and refine estimates of the higher order quantities by theoretical analysis and numerical modelling techniques (see for examples Martin and Diesendorf 1980, Diesendorf et al op cit, Wittle 1981, Rockingham 1980, Johanson and Goldenblatt 1979, Dixon and Lowe 1982).

Economic analysis of wind turbines in the context of a large interconnected grid can conveniently be broken into two areas. These are, the iterative refinement of the value of wind energy at low penetrations [1] and in the short term, and the estimation of the value of wind energy at high penetrations, in the long term and possibly in energy systems that differ radically from the present ones. The grounds for this divison are:

1) the estimation of the value of wind in the latter case is subject to far larger uncertainties than the former. The effort which it is worth devoting to detailed economic analysis in the latter case is therefore limited. The range of plausible assumptions

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[1] Penetration may be defined in several ways, eg. installed wind power normalised by installed grid power, mean annual wind energy output normalised by mean annual grid energy output. The precise definition to be chosen will depend on the purpose of the analysis.

in the latter case is large or unlimited, leaving little place for the economic optimisation of the type that I indulge in in chapter 2 of this thesis. Effort is better directed at the technical elaboration of possible systems - in other words scenario building.

2) in an ideal world, it would be the estimates of the value of wind generation at low penetrations in the short term which would provide the justification or otherwise for an initial construction programme. The wisdom of extending such an initial programme would depend on the results of the initial programme, and on other relevant factors at that time, since much of what one would like to know to determine the ultimate contribution of wind energy to a grid is unknown, or indeed unknowable, beforehand.

In the interests of clarity it is desirable to separate statements concerning these two different areas.

The third chapter of this thesis presents a study of the spatial and temporal fluctuations in the power output from wind turbines in the UK. The approach taken is in quite sharp contrast to the "economic" approach of the first half of the thesis, for reasons which have been alluded to above. Summarised, these are:

the details of spatial and temporal fluctuations in wind turbine output are not strongly relevant to the case for the initial installation of wind turbines - this depends primarily on the first order analysis presented in chapter 2 below. The uncertainties involved in deciding whether very large scale installation of wind generators is a good thing are great, and any attempt to apply to apply the techniques of optimisation used freely in chapter 2 would be potentially misleading.

The questions which I have tried to answer in this work are:

1) what is the change in wind power output from a single site which can be expected over a given (arbitrary) period? (changes in climate have not been dealt with here).

2) how does this statistic vary with wind turbine characteristics (rated wind speed, cut-in wind speed etc.) ?

3) how does this statistic vary if the power output is assumed to come from several turbines separated by distances of tens to hundreds of kilometres?

4) what size of storage will suffice to allow a wind turbine to supply a constant power demand at a given level of failure to meet the demand solely with the wind turbine/storage system?

The answers to these question are of interest in the following areas:

1) the study of the capacity credit for wind generators operating in a conventional electricity supply system.

2) the estimation of second order effects in the economic evaluation of wind generators, caused by finite time constants for changes in power ouput of conventional power stations (in particular, time constants for starting up from zero output).

3) the size of storage which would be required for "renewable" electricity supply systems based, for instance, on wind and conventional dammed hydro (see for instance Sørensen 1980).

4) the size of storage which would desirable for wind powered district heating systems (see for instance Margen 1979).

In this chapter I have not attempted to answer the above questions directly, but have tried to add to the basis of information which is necessary for them to be answered.

The power output of wind turbines is characterised by fluctuations with a wide range of periods. An effective starting point for much of the analysis is the Fourier transform, which yields directly estimates of the variation due to each frequency over the range analysed. Further statistics can be constructed from the spectra thus obtained, by relatively simple transformations. The value of spectra as a means of presenting aspects of a random process in a visually digestible way, is enhanced by the speed of the algorithms which are available for producing spectra from digital input data. These algorithms are known collectively as fast Fourier transforms (FFT's) and without them the work described in this thesis would have been considerably more difficult. Like many workers, I owe a debt to Cooley and Tukey.

Spectra of wind speeds have frequently been used in an intuitive manner to illustrate points regarding the variability of wind power output (eg. Sørensen 1978, Lowe and Alexander 1981). I have attempted to put these intuitive statements onto a somewhat firmer footing. Much of the resultant mathematics is rather elementary, but is applied to wind power for the first time, as far as I am aware. The approach taken here is clearly a general one, which could be applied usefully to any energy source characterised by large, wide band fluctuations in intensity - the one that comes immediately to mind is wave power.

The fourth and fifth chapters of this thesis concern the questions of the effects of variations in wind power on its capacity credit, and the effect of changes in wind power over short periods on the fast response plant that is required in a system containing wind plant.

Chapter 4 uses a framework for analysis which was first developed for wind power by Rockingham. This framework is developed and reformulated here, and results from chapter 3 are incorporated.

Given a definition of capacity credit which can be applied consistently to all types of plant, the evaluation of capacity credit for wind (or any other form of generation) may be a purely physical question. However changing the definition can lead, in theory, to changes in capacity credit. At the end of the third chapter I have paid some attention to an alternative to the "loss of load probability" (LOLP) definition of capacity credit, namely one based on spot pricing of electricity. It appears to me that technical questions can rarely, if ever, be separated from value judgements, and my purpose in presenting this brief review of spot pricing theory, is to illustrate this point. I do not wish to suggest that spot pricing is necessarily better in practice than systems currently in use, but it is sobering to realise the extent to which present systems are "second best" even when viewed from within classical economics.

Chapter 5 is heavily based on work done in collaboration with Dr. John Dixon of the Open University Technology Faculty, whose guidance and encouragement are warmly acknowledged here. The inspiration for this work again comes from Rockingham. The chapter incorporates many of the results from chapter 3, to give estimates of the spinning reserve cost of wind in the UK, under a range of assumptions.

There are two appendices to this thesis. The first contains graphical output from the analysis described in chapter 3. These graphs may give some intuitive insight into the nature of spatial and temporal fluctuations of wind power, in particular into the problem posed by diurnal variations in wind power output estimated from wind speed data measured at 10 to 20 metres height (it is advisable to read chapter 3 of this thesis before spending much time on this appendix). The second appendix contains listings of the most important of the computer programs used in the work described in this thesis.

2. AN ECONOMIC MODEL OF UTILITY PLANNING

2.1 INTRODUCTION

The major problem in determining the objectives of an electricity utility is to differentiate clearly between the normative and the actual. The major problem in specifying or approximating utility behaviour, is to arrive at objectives which are clearly stateable and sufficiently simple to allow of quantitative speculation, while not departing too far from an inevitably complex reality.

In what follows, I have attempted to consider utility behaviour, as it affects planning of the mix of different types of capacity, from within the framework of classical economics. In doing this I do not imply that this framework is without problems, or that it is in practice used in a consistent manner throughout the electricity system. This is not because I think that these problems are unimportant, but rather because I think that other people are better equipped than I am to do this type of work.

The basic approach in this chapter, is examine a simple linearised model of utility costs and to examine the conditions which will result in a minimisation of these costs. At each stage I have attempted to sketch the limits to any simple analysis of utility behaviour, which I feel are very important. The last part of this chapter is a summary of estimates of the "first order" costs and benefits which might be derived from a wind program in the UK. While these estimates are subject to considerable uncertainty, they form the background and indeed the justification for the work presented in later chapters.

2.2 FIRST ORDER APPROXIMATION - LINEARISATION OF COSTS.

The purpose of this section is to present a first order approximation to the operation of a non-hydro utility which enables estimates to be made of the costs of various generation options (in the light of forgoing comments on utility aims). The analysis in this section will basically exclude all effects due to rates of change of demand and available generating capacity, and capacity credit, which will be discussed in chapters 4 and 5.

The first major simplification to be made at this stage is to assume that the costs of operating generating plant of any type can be described by a linear function of the total time for which the plant is required to operate. If we take the year as the basic unit of time (in keeping with financial accounting practice) then the specific cost of operating plant of type i for a fraction x of the year is:

1. $\phi_i + x\gamma_i$ (£/kW)

(notation after Rockingham, 1980). In this formulation, ϕ_i are chiefly charges on capital (but including maintenance and manning costs which are not proportional to total number of hours of operation, and other costs which are fixed, such as research and development), and γ_i in the case of conventional plant, are chiefly fuel costs. Having determined ϕ_i and γ_i for a range of possible generating options, it is possible to graph the annual costs of each type of plant against its annual load factor (the fraction of the year for which the plant is required to operate). The result of this is shown schematically in fig 2.2.1. This type of curve is known as a screening curve and its use is quite general to problems in which a varying demand for some good can be met technically by plant with a variety of fixed and running costs. If I assume that the objective of the utility is to install plant which will result in the lowest annual running costs, then fig 2.2.1 illustrates that the choice of plant depends on the load factor of the demand to be met. At very high load factors it is economical to install plant with high capital costs, where these are offset by low running costs. At low load factors, the running cost of plant becomes decreasingly important, and the plant with the lowest capital costs gives the lowest annual running costs. Certain types of plant (eg. type 4 in fig 2.2.1) may not give lowest annual costs at any load factor. The load factors at which the annual costs of two types of plant are equal, are called "break-even load factors" (BELF's). Clearly, to be physically significant, BELF's must lie in the range $0<x<1.0$. In fact the upper bound may be considerably less than unity, since plant cannot be operated continuously. A typical achieved load factor for pressurised water reactors (probably the most relevant to any future UK programme) is 0.65 (Surrey and Thomas 1980) . The screening curve tells us which types of plant it is economic to install in a system, and between what load factors it should be operated to achieve lowest annual costs for the whole system.

Having linearised costs of operating different types of plant one can calculate total system costs for any given mix of plant in the system and establish the optimum mix of generating plant for a given pattern of demand. At this level of approximation, it is sufficient to describe demand by a load duration curve. The load duration curve can be defined in a number of ways, but is basically a graph of the level of demand versus the fraction of time for which that level of demand is exceeded. If the load duration curve is $f(x)$, then:

2. dx=fraction of time demand lies in the range $f<y<f+df$

Suppose there are n types of plant, and that the installed capacity of each type is c_i ($i = 1,2,\ldots n$). If the types are run in order of decreasing running costs (type $i=1$ has the lowest running costs), then the installed capacity of each type of plant can be written:

3. $c_i = f(x_i) - f(x_{i-1})$

The above equation defines the x_i. This is illustrated in fig 2.2.2. At this stage the values of the c_i and x_i are quite arbitrary. All I am assuming is that as the level of demand increases, the plant with the lowest running costs out of all the available non-running plant is turned on. From equation 2.2.1 the running cost of meeting demand dc_i at a load factor x with plant of type i is given by $\gamma_i x dc_i$. The quantity xdc_i is equal to the area of the shaded element in fig 2.2.3. Therefore the running cost of meeting all demand between load factors a and b with this type of plant is proportional to the sum of all such elements, and is equal to the area ABCD in fig 2.2.3 multiplied by γ_i. This cost can be re-expressed as:

$$\gamma_i \{ \int_a^b (f(x)-f(a))dx \quad + \quad b(f(b)-f(a)) \}$$

The fixed cost of providing plant to meet the demand between the load factors a and b is simply ϕ_i (f(b)-f(a)) (the fixed charge per kW multiplied by the number of kilowatts).

Given the above strategy, the total annual cost of operating the electricity system is given by:

$$4. \quad a = \Sigma\{ \gamma_i \int_{x_i}^{x_{i-1}} (f(x)-f(x_{i-1}) \, dx \quad + \quad x_i(f(x_i)-f(x_{i-1}))$$
$$+ \phi_i(f(x_i)-f(x_{i-1})) \}$$

A number of useful results can be derived from this simple analysis. The total annual cost can be minimised with respect to the installed capacities of the different types of plant by partial differentiation of the expression for a with respect to the x_i:

$$5. \quad \partial a/\partial x_i = -\gamma_i[f(x_i)-f(x_{i-1})] \quad + \quad \gamma_i[f(x_i)-f(x_{i-1})]$$
$$+ \gamma_i x_i \; f(x_i)/ \; x_i \quad + \quad \phi_i \; \partial f(x_i)/\partial x_i$$
$$+ \gamma_{i+1}(f(x_i)-f(x_i)) \; - \; \gamma_{i+1}x_{i+1}\partial f(x_i)/\partial x_i$$
$$- \phi_{i+1}\partial f(x_i)/\partial x_i$$

and simplifying:

$$6. \quad \partial a/\partial x_i = \partial f(x_i)/\partial x_i \{ x_i(\gamma_i - \gamma_{i+1}) + (\emptyset_i - \emptyset_{i+1}) \}$$

If I now set $\partial a/\partial x_i = 0$, I get a set of equations for the x_i which minimise the total annual running cost of the electricity system. Note that $\partial f(x_i)/\partial(x_i)$ does not go to zero for real load duration curves. I will call these values $\hat{x}_i$:

$$7. \quad \hat{x}_i = (\emptyset_{i+1} - \emptyset_i) / (\gamma_i - \gamma_{i+1})$$

The $\hat{x}_i$ as determined above are the break-even load factors which could be calculated from the screening curve (fig 2.2.1) with the assumption that the lowest system costs could be achieved by operating the plant with the lowest annual costs at each load factor.

Vimukta (1977) derived the result that, for a system in which in which the mix of plant has been optimised (as defined by the BELF's), the total annual system operating cost is given by:

$$8. \quad a = \emptyset_1 f(0) + \sum_{i=1} \gamma_i \int_{\hat{x}_{i-1}}^{\hat{x}_i} f(x)dx$$

From this equation, together with the fact that first order derivatives of a quantity are zero at its maxima or minima, it can be seen that for an optimised system, the additional cost of installing capacity to meet an increase of peak demand of d is $\emptyset_1 d$, ie. the cost of providing the increased capacity with peaking plant. This is true regardless of the type of plant actually built (provided d is small). This conclusion is perhaps clearer from an analysis presented by Rockingham (1980), for a system containing two types of plant. Rockingham's analysis can easily be generalised as follows.

Let us assume that a small additional amount dc_j of plant of type j is added to an electricity system. The increase in annual running costs which will result from this increase in capacity is (to first order in dc_j):

$$9. \qquad da = \emptyset_j dc_j - dc_j \sum_{i=1}^{j-1} x_i(\gamma_i - \gamma_{i+1})$$

In this equation, the first term is the intuitively obvious cost of the additional plant. The following summation is running cost reductions which arise as addition of non-peaking plant reduces the total annual output of all plant higher in the merit order. This is illustrated for a system comprising three types of plant in fig 2.2.4. Note that equation 2.2.9 applies generally, not just to systems in which the mix of plant has been optimised.

If I assume that the plant mix is optimised, I can substitute for x_i using equation 2.2.7 above, giving the following:

$$10. \qquad da = \emptyset_j dc_j - dc_j \sum_{i=1}^{j-1} (\emptyset_{i+1} - \emptyset_i)$$

Equation 2.2.10 can be simplified by noting that the $\emptyset_i$ can be cancelled in pairs, leaving:

$$11. \qquad da = \emptyset_1 dc_j$$

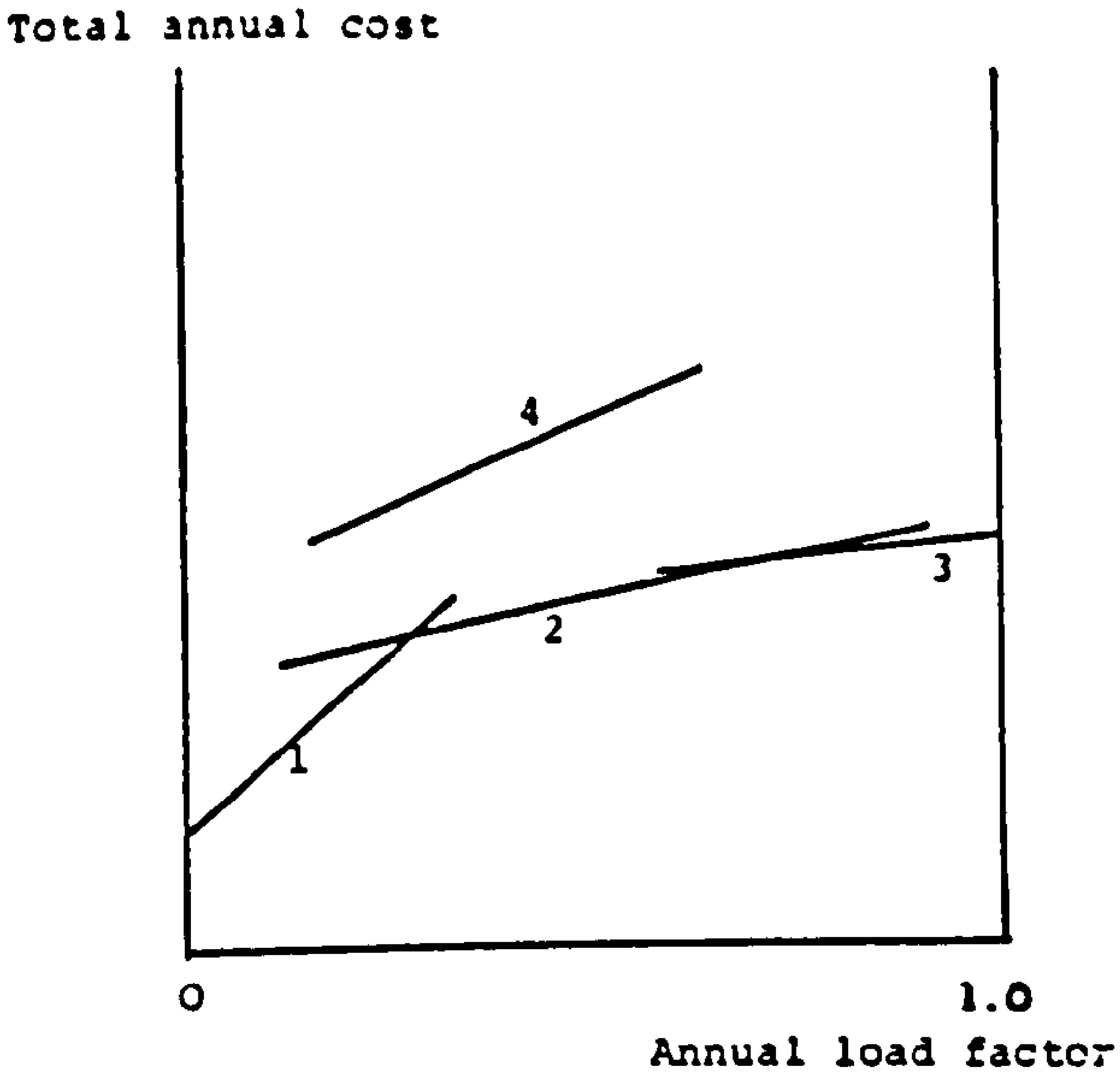

Fig 2.2.1 Total annual costs vs. annual load factor for different types of plant (sketch).

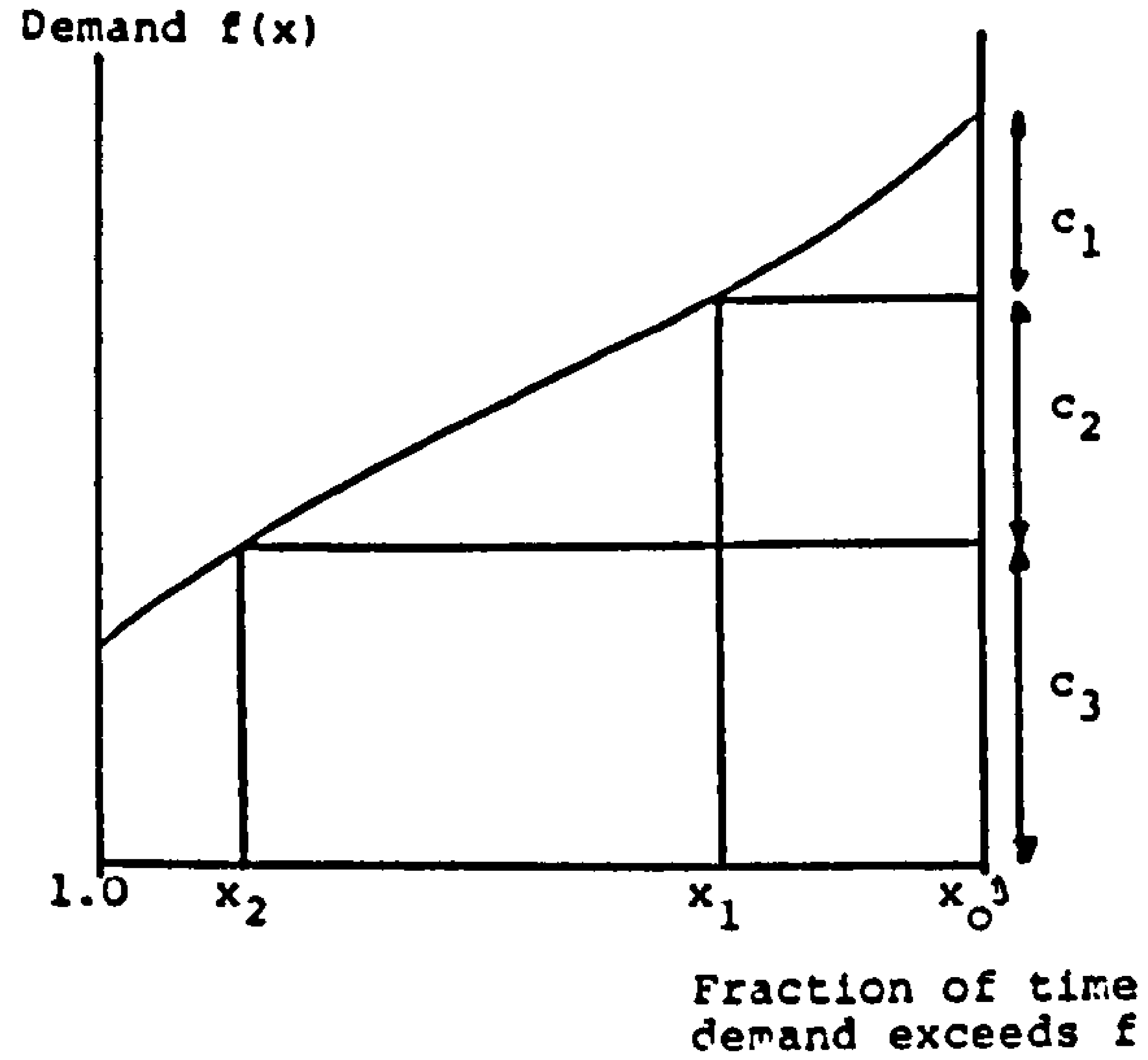

Fig 2.2.2 Relationships of installed capacities c_i to load factors x_i.

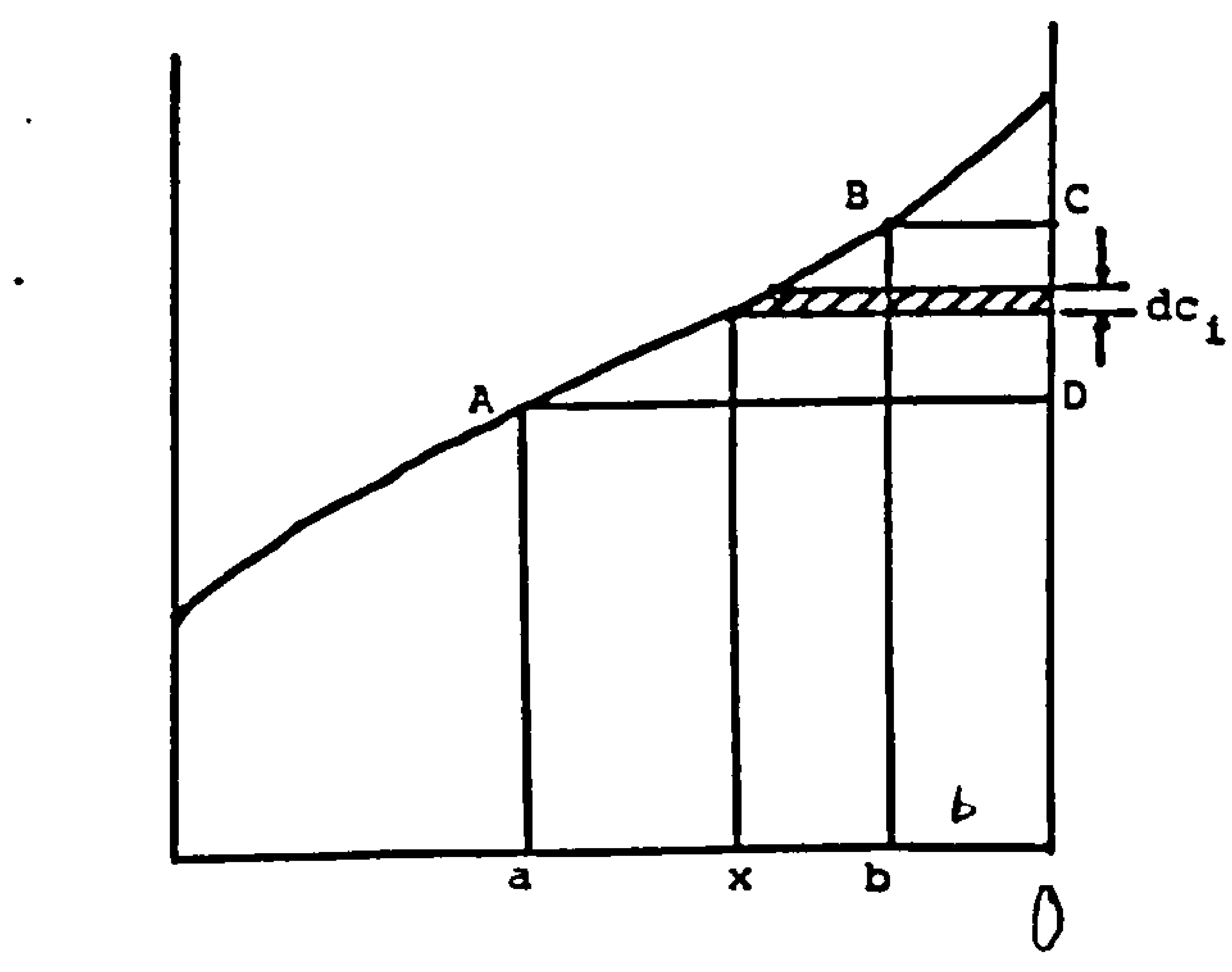

Fig 2.2.3 Interpretation of areas on load duration curve as annual runnig costs.

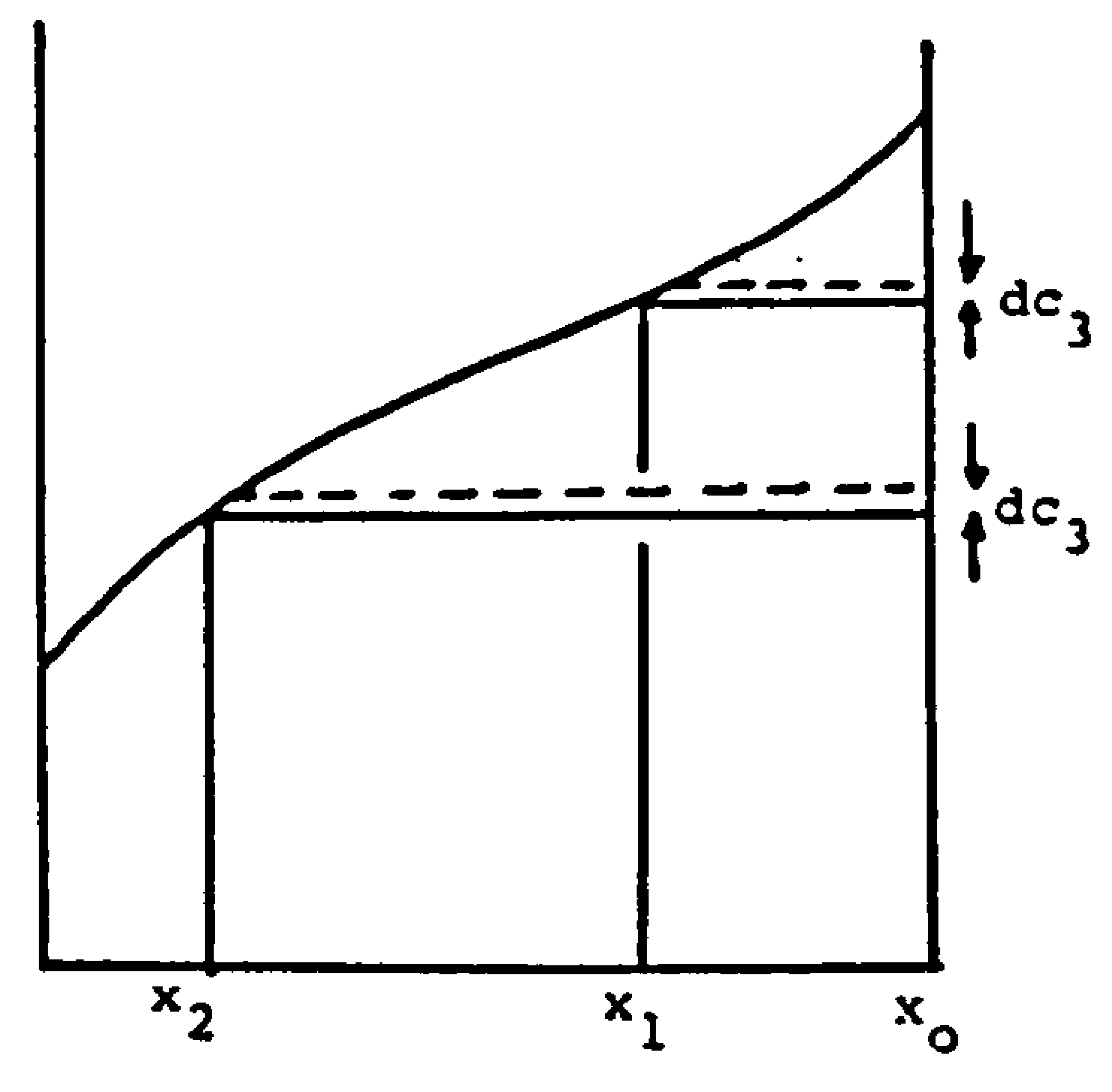

Fig 2.2.4 Running cost savings due to introduction of small increment of base load plant into a system comprising 3 types of plant.

Running cost saving $\quad dc_3\left\{x_1(\gamma_1 - \gamma_2) + x_2(\gamma_2 - \gamma_3)\right\}$

Note this result is equivalent to taking the derivative of a with respect to c_j. This result leads naturally to the concept of net effective cost of capacity (NEC). In general the net effective cost of adding plant of type j to an existing electricity generating system is:

12. nec_j = initial capital cost - running cost savings which result due to the addition of extra capacity

In the more specific case of a linearised but unoptimised system, the net effective cost of capacity is given by equation 2.2.9 above. It is worth noting that NEC's may be either positive or negative depending on the values of the x_i. Inspection of equation 2.2.9 will show that plant with less than optimum installed capacity will have a net effective cost less than $\emptyset_1$, while if there is too much of a given plant type it will have a net effective cost greater than $\emptyset_1$.

2.3 INCORPORATION OF TIME INTO THE SIMPLE MODEL

The analysis presented above does not include the dimension of time. In general utilities do not try to minimise their costs in any particular year, but a weighted sum of annual costs over many years. Taking a very simple view of utility economics (see eg. Berrie, 1968), the objective is not to minimise a, but:

1. $$\text{MINIMISE} \left(A = \sum_{n=1} a_n \, (1/1+r)^n \right)$$

where a_n is the cost in year n of operating the system, and r is the discount rate. Analytical solutions to this minimisation problem may be obtained in very limited cases by calculus of variations. In general however, numerical methods need to be used. In reality utilities are faced with the fact that running costs and certain fixed costs may vary from year to year. The discount rate may also fluctuate, either because it is directly tied to prevailing market interest rates (as for instance in the USA) or because the government decides to change the required rate of return for a nationalised industry (as in the case of the CEGB).

The concept of net effective capacity can be extended to cope with the additional variable, as follows:

$$2. \qquad NEC_j = \Phi_j - \sum_{n=1}^{N} \sum_{i=1}^{j-1} x_{i,n} (\gamma_{i,n} - \gamma_{i+1,n}) (1/1+r)^n$$

where Φ_j is the present value of the fixed costs of plant type j over its life (I have expressed these costs as a present value in year 0, as they would normally be composed mostly of capital costs which would be incurred on or before year zero), $x_{i,n}$ and $\gamma_{i,n}$ are the obvious extensions of load factor and running costs for plant type i, and N is the physical life of the plant in question. One particular case is of interest here. This is where $NEC_j \leq 0$ for some j. Equation 2.3.2 can be rewritten:

$$3. \quad NEC_j = \Phi_j - \sum_{n=1}^{N} b_n \; (1/1+r)^n$$

Here b_n is the value of the system running cost savings in year n due to the installation of a small quantity of plant of type j. The condition that $NEC_j \leq 0$ is sometimes said to justify the immediate construction of plant of this type, up to the point where the b_n are too low to satisfy the equality. It is not immediately apparent (it was not apparent to me for a long time) that the process of considering marginal changes to plant mix (or indeed to any variable that affects system costs) may also be extended to the consideration of the time at which an investment programme is started. Consider an investment programme which involves the indefinite operation of a particular type of generating plant (with replacement as necessary). I will assume that the annual system running cost savings of this programme are monotonically increasing. The costs and savings of such a programme are illustrated in fig 2.3.1. The marginal change in the present value of the costs of operating the system due to the programme are:

$$4. \quad dA/dc_j = (1/1+r)^{t-1} \sum_{n=t-1} \Phi_j \; (1/1+r)^{nN} - \sum_{n=t} b_n \; (1/1+r)^n$$

This change in costs may be maximised by differentiating the above expression with respect to t and setting the result equal to zero:

5. $$d/dt\ (dA/dc_j) = (1/1+r)^{t-1} \sum_{n=t-1} \Phi_j\ (1/1+r)^{nN}\ \{1-(1/1+r)\}$$

$$-b_t(1/1+r)^t \qquad = \qquad 0$$

The above expression may be simplified by evaluating the sum and rearranging:

6. $$\Phi_j\ \{1 - (1/1+r)\}\ \{1 - (1/1+r)^N\}^{-1}\ (1/1+r)^{t-1} = b_t(1/1+r)^t$$

Simplifying again I get:

7. $$\Phi_j = b_t\ (1/1+r)\{1 - (1/1+r)^N\}/\{1 - (1/1+r)\}$$

This result demonstrates that, in the case where system savings due to a particular type of plant are increasing, the "best" way to time the installation programme is so that the first units come on stream in the first year in which the additional fixed charges are equal to the resulting system running cost savings in that year. Thus it is not sufficient simply to determine that a particular class of plant has a negative net effective cost to justify immediate construction.

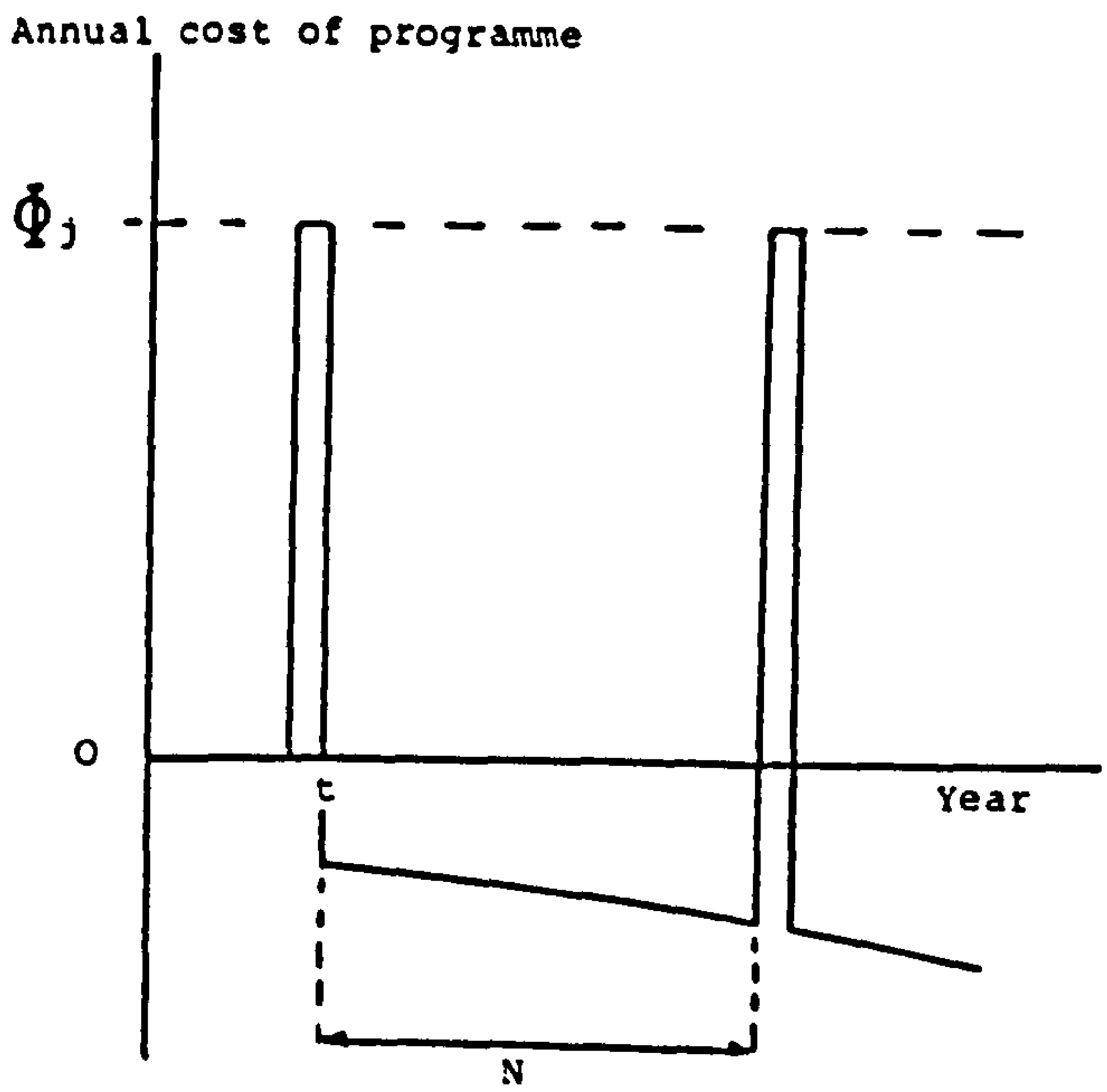

Fig 2.3.1 Cash flow due to installation and operation of new plant with physical life N years.
Note monotonically increasing system savings.

2.4 COSTS AND ECONOMIC VALUE OF WIND TURBINES IN THE UK.

Since 1980 there have been a number of papers which have attempted to survey the likely costs and benefits of a program of wind generators in the UK, and it is necessary to review these here. It is not intended to extend the work already done in this area, because the uncertainties which surround the most basic variables in the calculations make the task rather unrewarding.

2.4.1 ECONOMIC VALUE OF WIND TURBINES IN THE UK.

Numerous recent papers have touched on this subject (Rockingham 1980, Rockingham and Taylor 1981, Whittle 1981, Lowe 1980, Dixon and Lowe 1981b, Dixon and Lowe 1981c, Musgrove 1981) but have not dealt in detail with the assumptions regarding fossil fuel costs which underly any estimates of the economic value of wind. Estimates of the value of wind over the lifetimes of turbines are particularly problematical, since these require both estimates of fossil fuel prices, and a comprehensive modelling of the operation of the CEGB system into the first decade of the next century. All of the factors involved are the subject of political debate, as well as technical uncertainty, which perhaps explains the reluctance of academics to get involved.

The most detailed estimates of the long term marginal value of wind generation in the UK are naturally enough those provided by the CEGB, in the guise of net effective cost calculations for nuclear power. The very similar way that marginal nuclear power and small amounts of wind power behave in a large electricity grid allow us to assume that net system savings for the former are applicable with no adjustment to the latter (this point is developed in chapter 4). The CEGB's Annual Report and Accounts 1979/80 estimated the net fuel saving value for nuclear capacity commissioned in 1986, as 2.68 p/kWh, based on a coal price escalation of 6% between 1980 and 1986, and about 2% thereafter. This estimate is based on a computer model of the CEGB, the details of which are as yet unpublished.

The long term net fuel saving value of low running cost base load electricity generating systems (low penetration wind or nuclear) is quite heavily dependent on fuel cost escalation assumptions. The CEGB have been criticised (Report of the Select Committee on Energy, 1982) for the estimates contained in the 1979/80 AR&R, on the grounds that they have an agreement with the National Coal Board that there will be no coal cost escalation until 1986, if the CEGB maintains its coal consumption at 70 Mte. With a discount rate of 5% the very high initial rate of escalation of coal price assumed in the 1979 AR&R has a large effect on the levelised cost of coal over the life of typical generating plant. A possible justification for the CEGB approach is that coal prices in the UK do not adequately reflect the cost of production, due to subsidies to the NCB. The price rises therefore represent a movement of coal prices to a level reflecting the "true" production cost.

The public inquiry into the proposed construction of a Pressurised Water Reactor at Sizewell has led to the publishing by the CEGB of a set of energy scenarios including a range of possible coal price "trajectories" (see fig 2.4.1). This figure emphasises that the long term movement of energy prices is fraught with uncertainty, even if the effects of inflation are removed. Estimates of possible energy consumption by industrialised countries for the end of the century have declined by a factor of 2 or more over the last 10 years (see for example D.En 1978, Leach et al 1979, Olivier et al 1983). Even on the basis of the very high gnp growth rates assumed by Leach et al (op cit) it is difficult to see how a rational energy policy (ie. one in which investment decisions in the energy consuming sectors were made on the same economic basis as in the energy supply industries) would lead to energy demand growth. The reason for this is that investment decisions on the demand side are currently made on the basis of rather short expected pay-back times. Beijdorf (1979) suggests 3 to 5 years is typical. The nationalised energy supply industries in the UK effectively use a 5% discount rate, equivalent to a 15 or 16 year pay-back time [1]. If one assumes that low or zero gnp growth is now a permanent feature of highly industrialised countries, then the pressure on world oil reserves which was to have caused a doubling or more of oil prices by the end of the century (D.En 1979) may not materialise until the middle of the next century.

If we can assume that coal price rises will be monotonic for the next 40 years or so, and that wind will displace coal at the margin in electricity generation over that period, then

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[1] There are some minor exceptions to this in the case of the CEGB. Investments defined as optional (certain spares, and at least until recently investment in combined heat and power) are assessed at a 15% discount rate.

the argument set out in section 2.3 of this chapter suggests that the correct coal price to use in analysis of the timing of a wind generation (or nuclear) programme is the current coal price. The economic test of the programme is considerably simplified in this case, since one is required to estimate coal prices over perhaps a decade (assuming that wind is deployed on a large scale in the UK by the end of the 1980's), rather than four. The fuel cost of operating CEGB base load coal plant in 1979/80 was 1.29 p/kWh, equivalent to roughly 1.42 p/kWh at mid 1980 prices. As can be seen from fig 2.4.1, CEGB estimates of possible coal prices by the end of the current decade span a range of a factor of 1.45. It has been suggested (eg. Johnson 1983), that the bottom of the range is more likely to be realised, suggesting a first order system fuel saving value for wind within a few percent of 1.42 p/kWh. The top end of the range is about 2.15 p/kWh. The significance of this range will be discussed shortly. It should be noted that these estimates based on base load coal plant fuel costs are conservative since the system fuel savings from wind plant will include savings from lower merit plant. The factor by which the former underestimates the latter is likely to be of the order of 0.80 ($\simeq$2.15/2.68 which is the ratio of the effective fuel cost of baseload coal generation to the nuclear system fuel saving contained in appendix 3 of the CEGB Annual Report and Accounts 1979/80). It must be remembered that any estimates of system fuel savings over periods of several decades are highly sensitive to policy decisions, both inside and outside the electricity supply industry. The estimates quoted above would change perhaps by a factor of 2 in the event of heavy investment either in combined heat and power, or nuclear power.

2.4.2 ESTIMATES OF WIND TURBINE COSTS.

This section deals only with the costs of large wind turbines, on the grounds that these have at least until recently been thought to offer the expectation of lowest electricity costs. It is possible that the consensus on this point is changing in favour of so-called medium sized machines (50-300 kW, see eg. Musgrove 1983a and 1983b).

Estimates of costs for large modern land based wind turbines have to be based on estimates published for turbines designed outside the UK, since as far as the author is aware, no UK designs fall into this category. This inevitably poses problems of financial accounting which are in principle unlikely to be satisfactorilly resolved. Some of these problems have been outlined by Dixon and Lowe 1981c, and will only be discussed briefly here.

One of the first estimates for the cost of a compliant multimegawatt wind turbine in the UK was published by myself (Lowe R.J. 1980). This was based on energy cost and accounting framework information for the Boeing Mod2, presented by Robbins and Thomas (1979). The estimates of "mature product cost" in this latter paper are in disagreement with information given in other papers on the Mod2 (Lowe J. E. and Engle 1979, Douglas 1979) and possibly represent targets rather than actual cost estimates. The second unit cost figures are rather more consistent. My 1980 paper (Lowe R.J. 1980) ignored a large number of factors in the calculation, eg array effects, turbine availability, transmission efficiency on the debit side, and reoptimisation for lower UK wind speeds on the credit side. This paper nevertheless emphasised the importance that the economic assessment framework may have on the economics of wind (or indeed any low running cost technology). In 1979, wind projects in the US were being assessed at an 18% capital charge rate, while public sector energy projects in the UK were being assessed at a 5% real discount rate. The difference (other things being equal) is a factor of over 2 in the cost of electricity.

A considerable extension of my 1980 paper (op cit) was presented by Dixon and Lowe (1981b). This paper assessed the Karlskronavarvet/Hamilton Standard WTS 3 and the Boeing Mod 5 proposal, as well as the Mod 2. The paper made allowances for changes in survival wind speed, lower wind speeds at UK on land sites, and an adjustment of the rated power of the turbines for the lower wind speeds, and concluded that the cost of electricity for the three machines was of the order of 1 p/kWh (1980 prices, calculated for the 100 th machine in all cases). The conclusion was that turbines currently being developed in the US and Sweden showed promise of being economic in the UK. Dixon and Lowe presented a third paper on this subject (1981c) which made a number of refinements to the calculations (eg. including the reduction in energy output due to turbine control strategies), but left the conclusions basically unaltered. An extension made in this paper was a reference to energy and mass analysis (Dixon and Lowe 1981a) to lend plausibility to capital cost estimates for large wind turbines.

Another paper which has dealt in a similar if less detailed way with the assessment of non UK wind turbine designs in UK conditions, was presented by Musgrove (1981). This paper took the same basic cost data for the Mod 2 that was used by

Dixon and Lowe (op cit) and applied a slightly more conservative analysis. A 20 year life was assumed instead of the manufacturers estimates of 30 years, which increases the cost of electricity (including O&M) by a factor of 1.25. A lower (more realistic ?) exchange rate of $2 to £1 was used as opposed to the then current exchange rates used by Dixon and Lowe. A reduction in energy yield due to a lower assumed mean wind speed for lowland UK sites of a factor of 0.72 was estimated, and no allowances for survival wind speed reduction (Dixon and Lowe used a factor of 0.85) or rated power reoptimisation (Dixon and Lowe used a factor of 0.95 based on Kilar et al 1979) were made. Musgrove's final figure for the cost of electricity from the 100 th unit Mod 2 was 1.82 p/kWh (1980 prices).

It is important to stress that there is no disagreement here. It may well be that the most sensible wind planting strategy to adopt in the UK is one of installing unmodified US or Swedish designs, in which case corrections for rated power and survival wind speed are inappropriate.

Perhaps it needs to be said that an economic analysis does not need to be conservative. A conservative analysis is normally biassed. Where risks attached to failure of a particular project are large, a biassed estimate which attempts to produce eg. a cost of energy which is not likely to be exceeded (without normally defining "likely") may be useful. Where costs and credible risks are small in absolute terms, the case for conservative analysis is less strong and "best guess" estimates may be more useful. Pace Lipman, one should not be surprised when "best guess" and "conservative" estimates differ.

Perhaps the most important variable in determining wind turbine costs in the UK is the $ to £ exchange rate. At the time of writing this stands at about $1.4 to £1. Perhaps Lipman in his raporteür's summing up (session 2, Int. coll. on wind energy, Brighton, 1981) was correct when he said that anecdotal evidence suggested an exchange rate of $1 to £1 to be more "reasonable" than the then current rate of over 2. Investigations by Swift (Dixon and Lowe 1981c) suggest that rather obvious consistency properties that one might expect exchange and inflation rates to possess, do not exist, at least over periods as short as ten years.

Summary.

The range of cost of electricity for the Mod 2 based on Musgrove (op cit) and Dixon and Lowe (1981c) is between 1.04 p/kWh and 1.82 p/kWh (1980 prices). If this cost range is compared with the coal prices sketched in fig 2.4.1, one can conclude that if actual costs are near the bottom of the range then this turbine in mass production will already be economic, while if the cost is near the top end of the range, installation may be justified by the end of the decade on all but the lowest of the CEGB coal cost scenarios. This conclusion is sufficiently favourable to wind to justify the rather detailed work on certain system aspects of wind turbine economics which is presented in chapters 4 and 5.

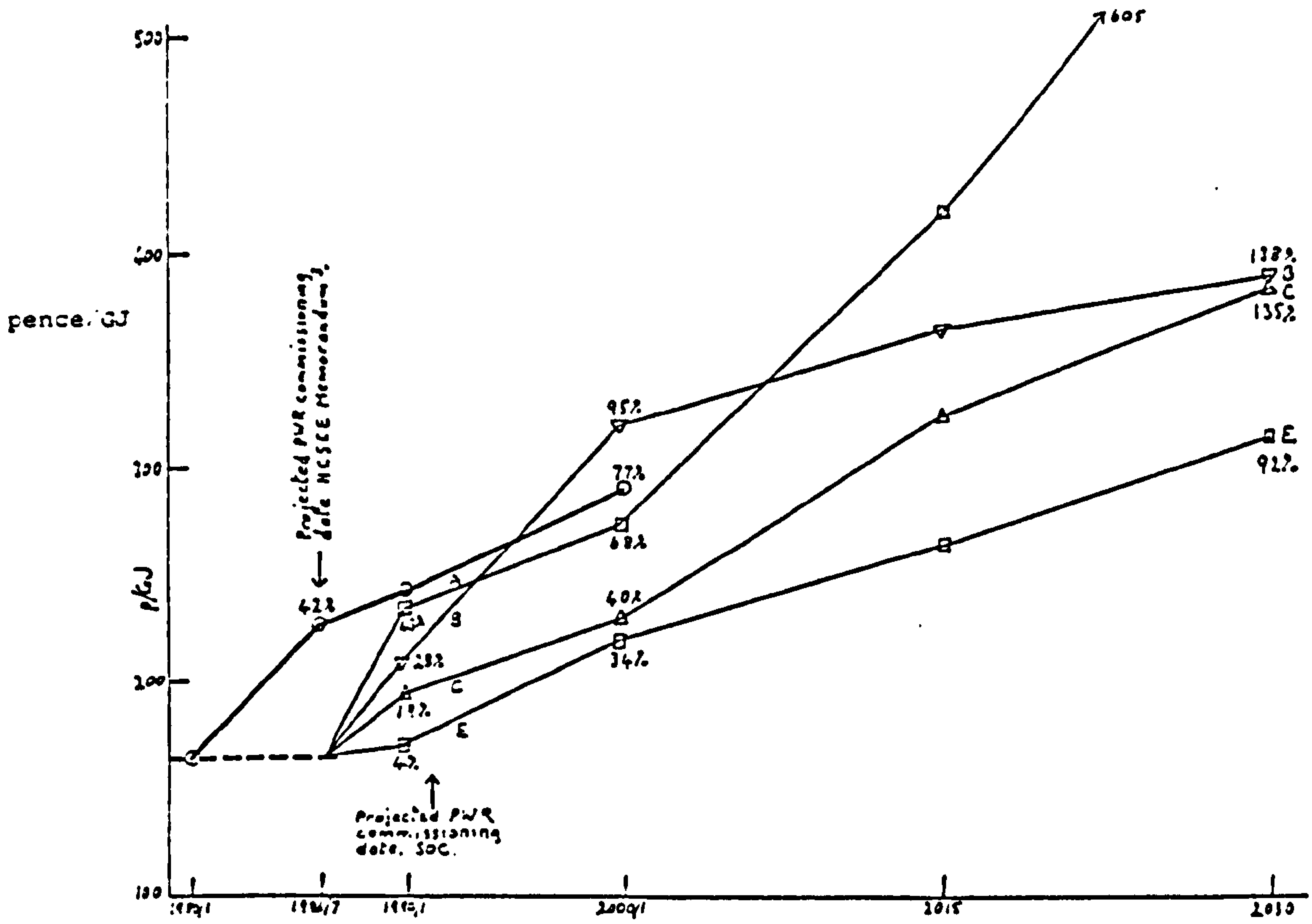


Fig 2.4.1 Future NCB coal prices assumed by CEGB for coal delivered to a central coal-fired power station. March 1982 price levels. From Jeffery, 1983.

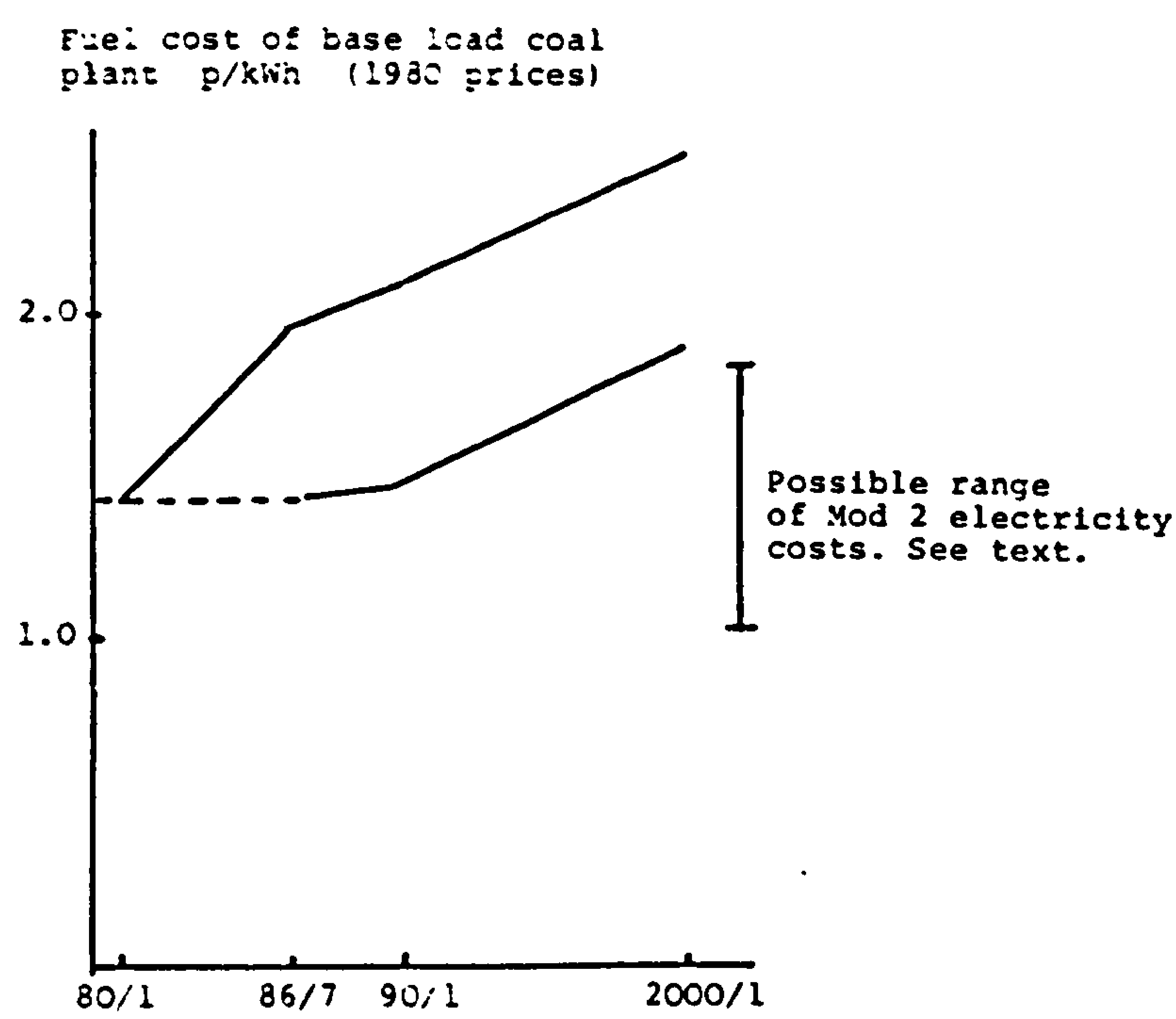


Fig 2.4.2 Comparison of extreme CEGB coal price scenarios (see above) with possible range of Mod 2 electricity costs.

3.0 STATISTICAL ANALYSIS OF WINDPOWER

3.1 INTRODUCTION

The purpose of this chapter has been described briefly in the general introduction to this thesis. The areas to which this chapter is relevant are as follows:

1) the study of capacity credit for wind generators operating in a conventional electricity supply system.
2) the estimation of "second order effects in the economic evaluation of wind generators, caused by finite time constants for changes in power output from conventional power stations (in particular time constants for starting up from zero output).
3) the size of storage which would be necessary for renewable energy supply systems based on wind, or on a combination of wind with other renewables such as hydro.

The rest of this chapter will be devoted to

1) a summary of the wind data that I have used.
2) an exposition of the analysis techniques that I have used.
3) a summary of the results of this preliminary analysis.

The analysis techniques used in the work described in the rest of this chapter are based on spectral analysis. The main reason for this apparent single mindedness is that very efficient algorithms exist for the calculation of Fourier spectra based on the fast Fourier transform. A variety of statistics can be estimated from the spectra thus derived, by appropriate filtering. The main limitation of this type of analysis is that it only makes accessible the second moments of the probability density functions of interest. Where it is possible to assume that these probability density functions are Gaussian, then the fact that we only know the second moments is not important - a Gaussian is completely defined by its mean and variance. Similarly where it can be shown that a quantity of interest depends only on the second moment of a probability density function in a certain limit (eg. in the limit of small penetrations, capacity credit is a function of the variance of wind power output) this limitation is not a problem. Caution is however necessary. Many of the variables pertinent to wind power are not Gaussian, or even approximately Gaussian, and one should beware of implicit assumptions that they are.

3.2 DESCRIPTION OF WIND DATA

The data that I have used to investigate the statistics of wind power, consist of time series of nine years' total duration, of wind speeds averaged and sampled at an interval of 1 hour, for 30 sites in the UK. It is necessary to say something about the quality of this data and the methods that I have used for estimating the power output from hypothetical wind turbines at these 30 sites.

The basic data was bought from the Meteorological Office in the form of records on magnetic tape. In addition to wind speeds in knots, the magnetic tape also contains data on wind direction and gust speeds. The magnetic tapes were transferred to the IBM 370 machine in Cambridge for analysis. Examination of the data on wind direction and gust speeds showed that:

1) about 95% of the wind speed data was present.
2) the wind direction and gust speed data was unuseable owing to the number of missing records.

In order to make the processing of the wind data easier, I made copies of the wind speed data only, from the Met. Office supplied tapes to tapes initialised for the Cambridge tape library service (TLS) system. In transferring the data, I repaired some of the missing values. This was done by simple linear interpolation between the last wind speed record before a block of missing data and the first record after it. Continuous blocks of 13 hours or more of missing data were not interpolated. The data is recorded both in the original Met Office tapes, and in the TLS copies in two byte words, and in both cases missing data is marked by -max (-32768).

The Met Office data was supplied in files with a logical record length of 806 words. A month's worth of data occupies a single logical record. At the beginning of each file is a header block containing details of the anemograph station at which the data was measured. At the beginning of each logical record is a preliminary array containing the month and year, and a summary of the months data. This is followed by the main block which contains the hourly data, and a final daily data block which also contains quality control data.

The TLS copies of the files contain simply 2 logical records of 8808 bytes per site per year. The end of each pair of logical records is padded by end of year markers (99). The site at which the data was measured is identified only by the file name. File names are of the form .WIK7179R , the first three letters being a site acronym, 7179 denoting the period over which the data was collected, and the 'R' denoting that some missing values have been interpolated. In retrospect, this was a mistake. The end of record padding should have contained the site acronym and the year, to enable simple checks to be carried out during the analysis. For some analysis, the effective height of the anemometer would also have been useful.

Most of the data was recorded at effective heights of roughly 10 m above the ground. Two sites in particular depart from this. The first is Bell Rock (BLR) where wind speeds are measured at the top of a light house, at an effective height of about 40 m. The second is Cardington

(CAR), an inland station, where the effective height of the anemometer is 41 m. The siting of the anemometers has not been done with the purpose of prospecting for wind power. Anemometers at airports (the two large airfields in the database are Gatwick (GWK) and Leuchars (LCH)) may give data which is misleading for on-land sites because of the low surface roughness of the land surrounding the anemometer. A table of the sites in the database, the effective and actual heights above ground level of the anemometers, and the quality of the data is presented below. Fig 3.2.1 shows a map of the UK with the locations of the anemograph stations marked.

In order to convert wind speed data into wind turbine output powers, I have used a turbine power/wind speed characteristic for the Boeing Mod 2, taken from Douglas 1979. (This paper deals with a wind tunnel model of the Mod 2). This characteristic is given in terms of the hub height wind speed. Douglas (op cit) states that a variable V/H exponent relationship was used for the Mod 2 design study. The V/H exponent at the design average wind speed of 14 knots (30 feet) implicit in this paper is 0.19, and I have assumed a constant V/H exponent with this value to express the Mod 2 hub height characteristic in terms of the standard wind speed at 30 feet. The resulting turbine power/wind speed characteristic is shown in fig 3.2.2. This characteristic has been converted into a histogram and finally into the subroutine IMOD2 shown in appendix 2.

It should be emphasised that the same turbine power/wind speed characteristic has been used regardless of the details of the individual sites. The assumption is that all the anemograph measurements were made at a standard height of

10 m and that the same V/H exponent applies to all sites. The assumptions are quite crude, but (certainly at the beginning of the analysis) I was mainly interested in the patterns of spatial and temporal fluctuations rather than the absolute values of these statistics. Clearly an important next step in the analysis of wind power output will be to to make better estimates of the effective hub height wind speeds from available data such as the Met Office anemograph stations.

A crude check on the wind speed data can be made by estimating the capacity factor of a hypothetical wind turbine at each site. The capacity factors at each site can then be plotted against the specific power of the wind turbine at each site (the specific power is a measure of the rated power of the wind turbine compared with the flux of kinetic energy through its swept area. This concept will be discussed fully in chapter 4.). Fig 3.2.3 shows a scatter plot of capacity factor F, versus specific power p, for the 30 sites listed in table 1. Superimposed on this is the design F p curve for a Mod 2 calculated from data in Lowe and Engle 1979. The data in the original paper is expressed in terms of the 30 foot wind speed, and I have converted to hub height wind speed by use of a V/H exponent of 0.19, consistent with the above.

Table 1. Summary of wind sites.

	Site	Fraction of good data after interpolation	Effective height m	Actual height m
1.	Stornoway (STY)	1.00	10	10
2.	Wick (WIK)	0.99	10	10
3.	Tiree (TIR)	1.00	12	12
4.	Fort Augustus (FAU	-	16	16
5.	Fraserburgh (FGH)	0.97	11	13
6.	Leuchars (LCH)	1.00	13	13
7.	Bell Rock (BLR)	0.98	38	40
8.	Ballypatrick (BLY)	0.97	13	13
9.	Prestwick (PWK)	1.00	10	10
10.	West Freugh (WFR)	0.98	10	10
11.	Great Dun Fell (GTD)	0.99	10	10
12.	Durham (DRH)	0.93	10	16
13.	Ronaldsway (RON)	1.00	10	10
14.	Squires Gate (SQG)	1.00	11	12
15.	Silpho Moor (SMR)	0.97	9	15
16.	Valley (VAL)	1.00	12	16
17.	Coningsby (CON)	1.00	10	10
18.	Aberporth (ABP)	1.00	10	10
19.	Cardington (CAR)	1.00	41	41
20.	Wattisham (WAT)	1.00	10	17
21.	Gorleston (GOR)	0.91	13	13
22.	Millford Haven (MHN)	1.00	10	10
23.	Porth Talbot (PRT)	0.85	11	12
24.	St. Mawgan (STM)	1.00	13	13
25.	Boscombe Down (BDN)	1.00	17	17
26.	Gatwick (GWK)	1.00	10	10
27.	Dungeness (DNG)	0.99	10	10
28.	Scilly (SCI)	1.00	17	20
29.	Lizard (LZA)	0.99	18	23
30.	Mount Batten (MNT)	1.00	13	13

Note that the effective height of the anemometer is the estimated height of an unobstructed anemometer which would have given the same readings as actually measured by an anemometer which suffers some local obstruction.

Interpretation of wind turbine output based on hour averages

The database described above consists of records of wind speeds averaged over hourly intervals. The effect of averaging and sampling will be discussed in section 3.3. The effect of averaging wind speed over periods of 1 hour and then applying a single machine turbine power/wind speed characteristic is likely to approximate crudely the output which would be obtained from an array of turbines installed at the wind site.

The high frequency part of the spectrum of the power output from an array of wind turbines will be related to the turbulence spectrum of wind speed at the wind site in question. There is a great deal of work on the shape of the turbulence spectrum of wind speed fluctuations (Taylor 1938, Panofsky and McCormick 1954, Van der Hoven 1957, Davenport 1961, Lumley and Panofsky 1964, Busch and Panofsky 1968). Most of the work refers to the frequency spectrum, since most of the raw data which is available is time series data from single points. The frequency spectra can be related to the spatial spectra by use of an assumption originally due to Taylor (see Taylor op cit). Essentially this assumption is expressed by the equation

1. $$S_k(k) = S_n(n)\ dn/dk$$

where the space and frequency spectra have been denoted by S_k and S_n, and

$$k = n/V$$

where V is the average wind speed over some suitable period (in practice usually 1 hour).

The most general non-dimensionalisation of the turbulence spectrum involves the non-dimensional spectral density $nS(n)/u_*^2$ and the reduced frequency $f=nz/V$ (where z is the height above the ground and V is the wind speed averaged over a period of about 1 hour). The quantity u_* is the friction velocity defined as $\sqrt{\langle uw \rangle}$ where u and w are the horizontal and vertical components respectively of the wind speed fluctuations about the respective means. The friction velocity can be measured by drag plate experiments, calculated by measurement and correlation of u and w, or can be estimated from the variation of wind speeds with height. Plots of turbulence spectra in these coordinates are given by Busch and Panofsky (1968), fig 3.2.4.

An approximation to the auto-correlogram of wind speed fluctuations is a simple exponential. The corresponding spectrum has the form

2. $nS(n) = (1/\pi)\ \{\ 2\pi nJ \ + \ (1/2\pi nJ)\ \}$

with a peak at $n=1/J$ (J is the time constant of the exponential correlogram). Actual turbulence spectra may be approximated by functions of the form of equation 3.2.2, although the fit is not a good one (Pasquill 1974). The length scale in the exponential auto-correlogram is equal to the frequency at which the spectral density nS(n) peaks. To a first approximation the length scale of the horizontal wind speed is

3. $L_x = z/f_m$

where f_m is the reduced frequency of the peak of the spectrum. What determines L_x ? Work by Berman (1965) and by Busch and Panofsky (op cit) show that f_m is not constant over a range of heights, as would be expected from simple similarity theory, but increases with height. In addition there is a large amount of scatter in plots of L_x or f_m versus height z. Fig 3.2.5 is taken from data in Berman (op cit). Berman fitted a power law

4. $L_x = 167\ z^{0.29}$

to the data in fig 3.2.5, an effort which must be described as brave. The values of L_x which may be obtained from the plot, for hub heights of the order of 100 m are perhaps in the range 500 to 900 m. This estimate is rather longer than the estimate of Lipman et al (1980) , based on work by Shiotani and Iwatani (1976), of 200 m.

We have still to consider the cross wind fluctuations in wind speed. Pasquill (op cit) states that the spectrum of cross wind fluctuations is imperfectly known. The length constant L_y does not appear to scale with height, and values of 150 to 300 m are suggested for heights in the range 15 to 92 m. Lipman et al (op cit) suggest a value of 50 m, again based on Shiotani et al (1976).

The effect of wind speed fluctuation on the output from an array of wind turbines has to be estimated as a function of wind speed, as the sensitivity of turbine output to wind speed variations is a strong function of wind speed. For any given wind speed we can define a diversity factor D for wind speed fluctuations. This factor represents the reduction in the rms wind speed fluctuation averaged over an array of points, compared with the wind speed fluctuation at a single point. For an exponential auto correlogram, with one space dimension, Farmer et al derived the following equation for D

5. $D^2 = \{N \tanh (x/2L)\}^{-1}$

A similiar expression for the two dimensional case was derived by Bossanyi et al 1980a.

6. $D^2 = \{\coth(x/2L_x) \coth(y/2L_y)\} / N$

It should be noted that the above two equations apply strictly only to infinite arrays. However, Bossanyi has shown (Lipman et al op cit) that arrays as small as 4x4 were effectively infinite for the purposes of these equations.

If we use the two dimensional equation for diversity factor to estimate the effect of correlation, assuming a 1 km machine spacing, D is in the range $1.0/N < D < 2.0/N$, depending on the choice of L_x and L_y. For a 100 machine array, the diversity factor will lie in the range $0.1 > D > 0.14$.

The total turbulent fluctuation of wind speed can be expressed as rms fluctuation divided by mean wind speed. Turbulence levels in arrays may be of the order of 20%. I have estimated the variance of power of a single turbine, due to turbulent fluctuations of wind speed as a function of hourly mean wind speed, for the turbine characteristic shown in fig 3.2.2 above and assuming 20% turbulence. The distribution of turbulent wind speed fluctuations was assumed to be Gaussian for the purpose of this exercise. The result is shown in fig 3.2.6. This graph is dominated by fluctuations generated at the cut out wind speed. It must be noted however that an instantaneous cut out strategy was assumed for these calculations. Bossanyi (Bossanyi 1981) has shown that the frequency of wind turbine shut downs can be greatly reduced by a variety of strategies, including introducing hysteresis into the algorithm controlling shut downs. I have not attempted to evaluate in detail the effects of more sophisticated control strategies on the variance of turbine power output due to turbulence, because the diversity factor of fluctuations at cut out will almost certainly be increased by strategies aimed at reducing the frequency of shut downs and therefore the variance of power output near the cut out frequency.

The average variance due to turbulence may be obtained by the integration of the product of the function shown in fig 3.2.6 with a suitable probability distribution function of mean wind speeds. This has been done assuming a Rayleigh distribution with two characteristic wind speeds, corresponding to specific powers of approximately 1.0 and 1.5 respectively. The results are shown below:

characteristic speed (m/s)	fractional std.dev of power
4.8	0.52
4.2	0.58

In the above the fractional standard deviation is the root mean variance of power variations, non-dimensioanlised by the mean power output of the turbine. A lower limit on the average variance of turbine power may be obtained by simply assuming that the variance due to cut outs is supressed completely. If this is done the graph of fig 3.2.6 changes from the function shown by the full line to the one shown by the dotted line. The change in the fractional standard deviation is much less dramatic than might be expected from fig 3.2.6, because wind speeds near cut out are relatively infrequent. The complete supression of variance at cut out, at a characteristic wind speed of 4.8 m/s (corresponding to a specific power of about 1.0), reduces the fractional standard deviation to 0.44 from 0.52.

If we assume that the diversity of power fluctuations due to turbulence is the same as the diversity of the turbulence itself (this is not obviously correct owing to the non-linear nature of the turbine characteristic; a more complete analysis would have to model this effect numerically) then the fractional standard deviation of power fluctuations due to turbulence, for an array of 100 turbines at a spacing of 1 km, will be in the range of 5 - 7% of the mean power output, depending on assumptions about length scales of turbulent wind speed fluctuations.

Conclusions

According to the above analysis, the effect of considering arrays of wind turbines instead of single machines is to reduce the fluctuations due to turbulence by of the order of a factor of 5. It must be stressed that the short term fluctuation in power output from wind turbine arrays will be a non stationary phenomenon, the primary variable being the short term mean wind speed. An additional variable which has not been considered here is wind direction. The effect of wind direction changes on the output of wind turbines is dependent on the design of the array (see Dixon 1982).

Insufficient data is available to allow a treatment of these effects.

The reduction in turbulent fluctuations due to spatial separation of machines in arrays has possibly been over estimated by Farmer et al (op cit) and Bossanyi et al (op cit) due to the neglect of the scaling of wind spectra with height. Nevertheless, turbulent fluctuation of wind turbine array output is likely to be of the order of 2 to 3% of installed power output.The assumption that wind data averaged over a period of 1 hour will represent the effect of clustering of wind turbines into arrays with approximately 100 machines per array, appears rather crude. The resultant errors are likely to be small unless analysis is attempted at the limit of the resolution of the data (periods of the order of 1 hour).

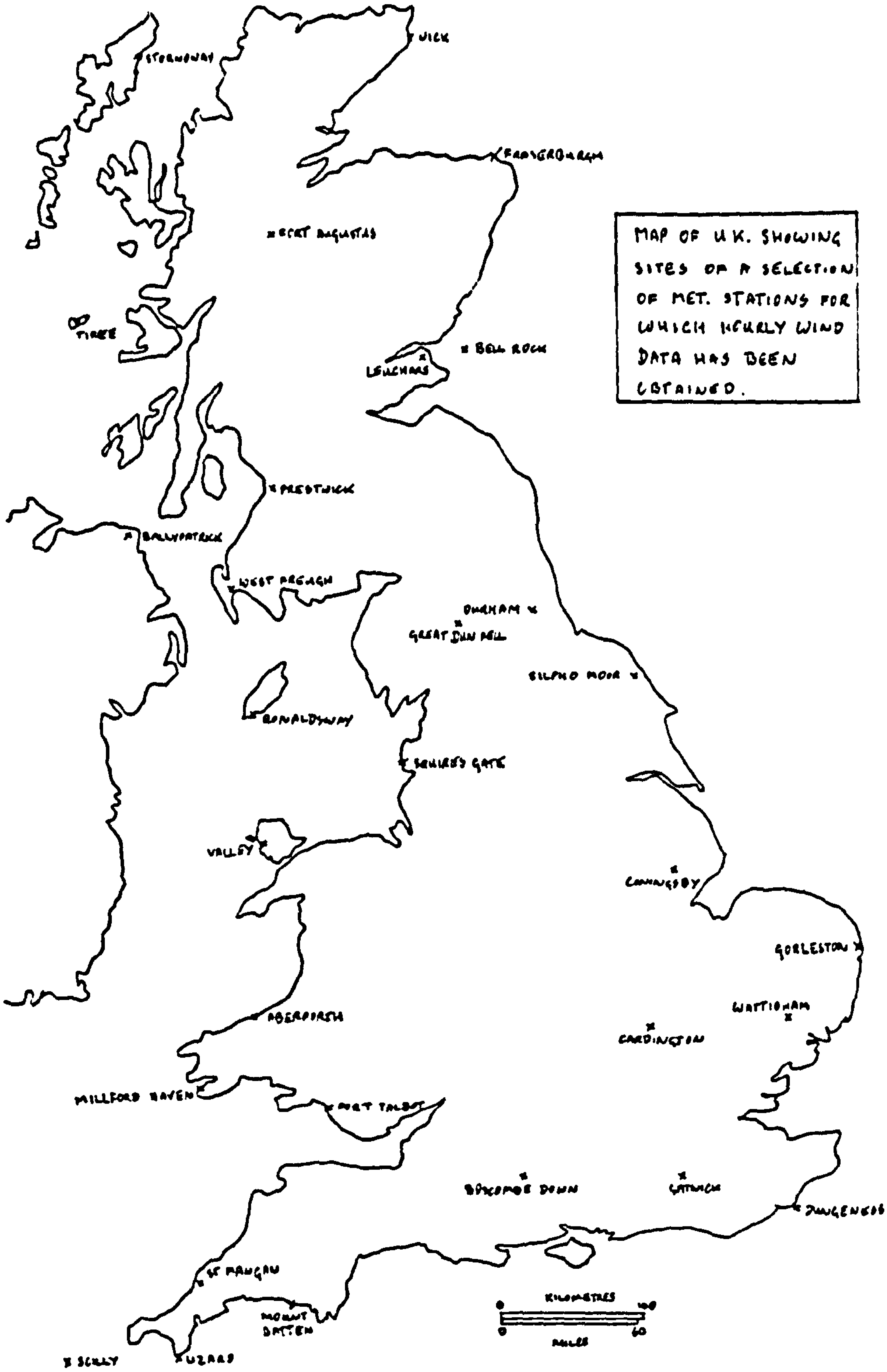


Fig 3.2.1 Map of UK showing sites for which Meteorological Office wind speed data has been obtained.

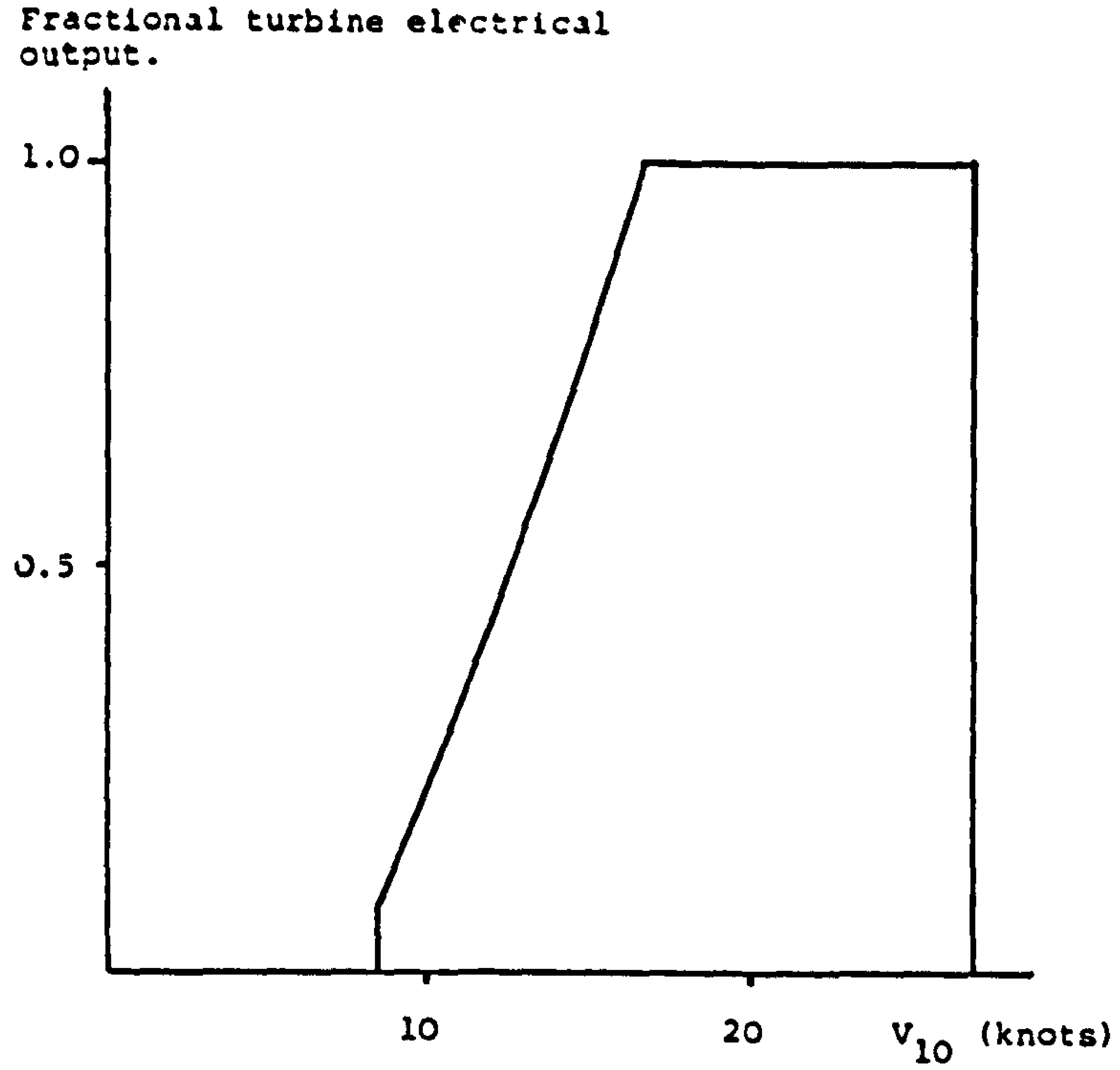
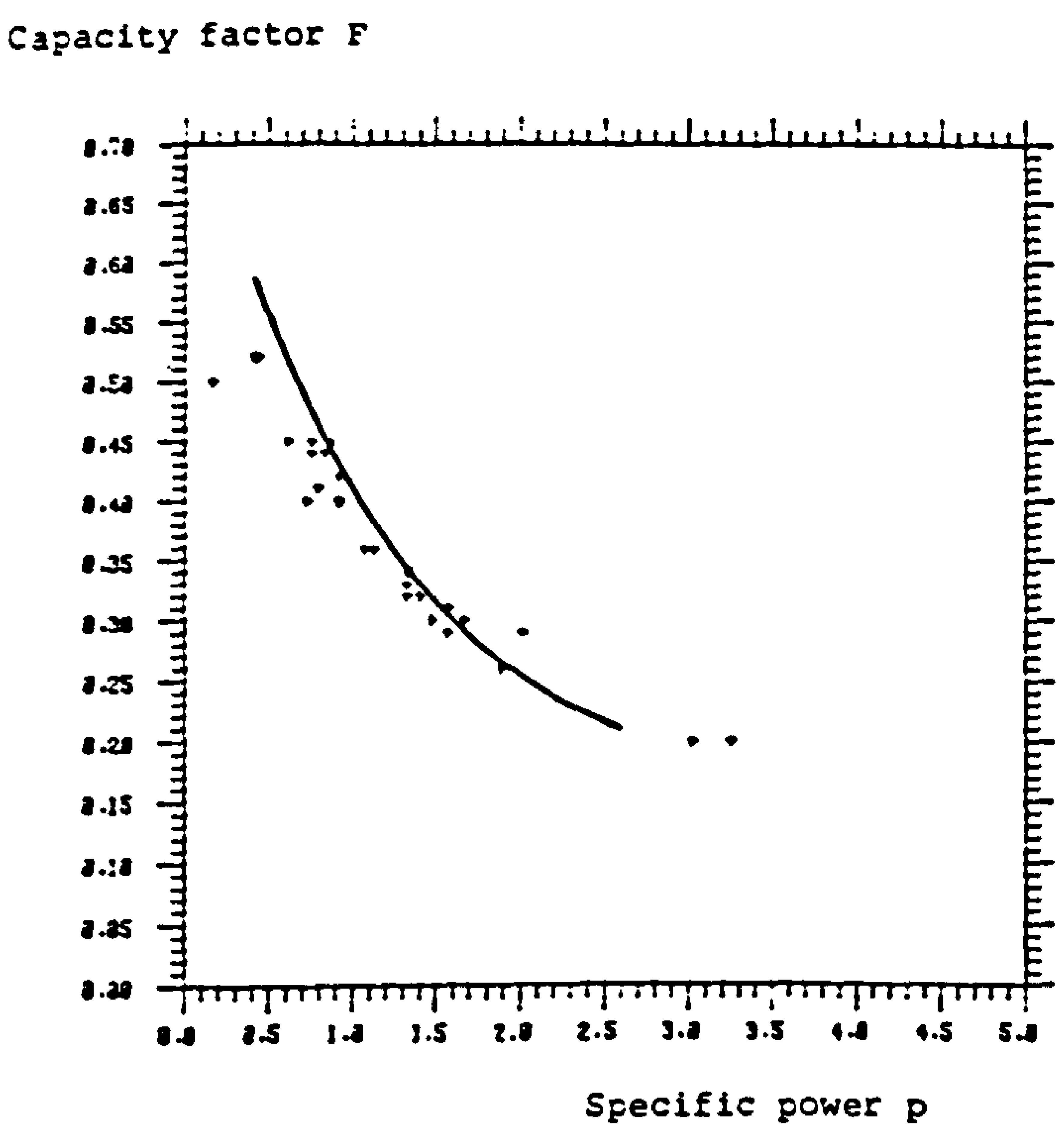


Fig 3.2.2 Wind turbine characteristic used in analysis of hourly wind speed data.



+ based on Met Office data

——— based on Rayleigh wind speed distribution

Fig 3.2.3 Capacity factor vs. specific power for UK wind sites.

TURBULENCE SPECTRA

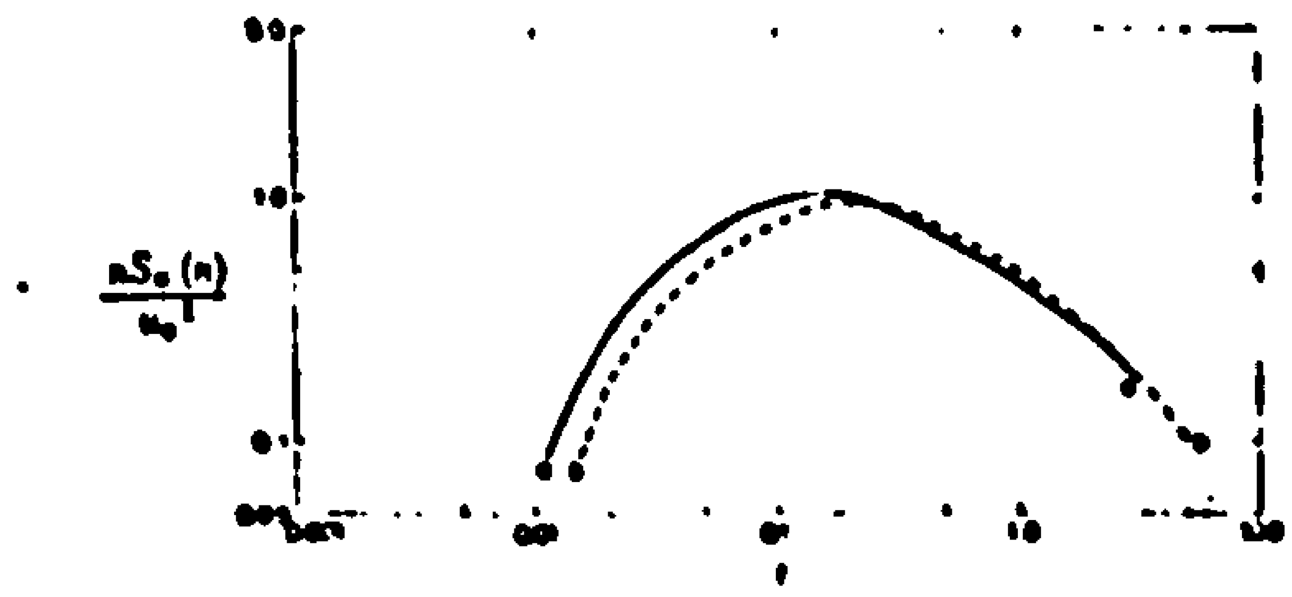


Figure 12. Smoothed and averaged spectra of longitudinal velocity in stable air at tower B, Round Hill. a. 46 m, b. 91 m.

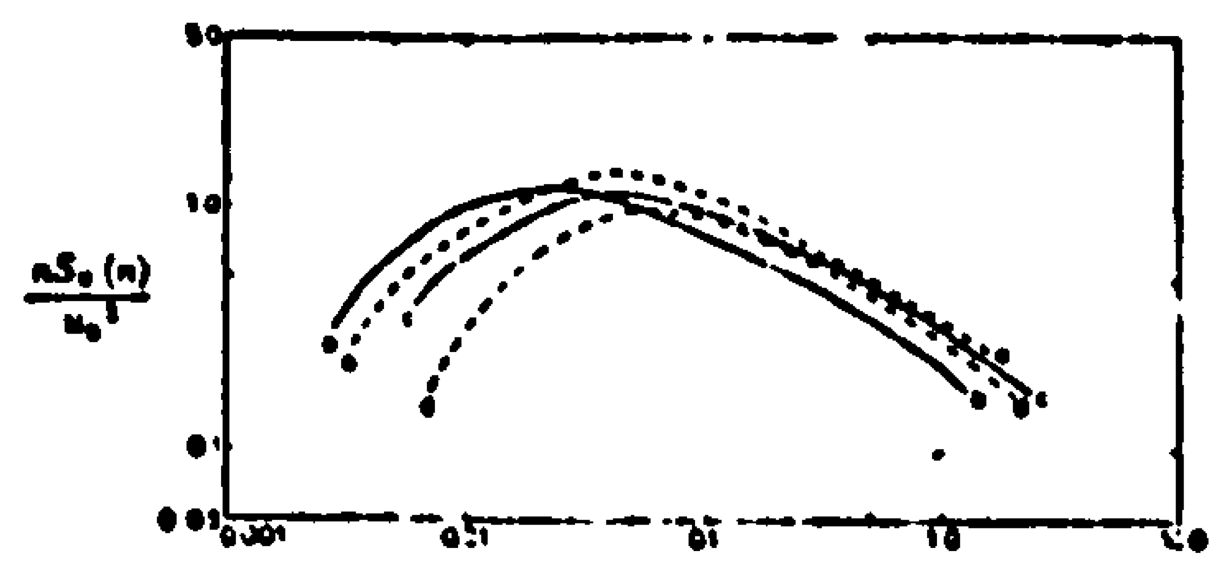


Figure 13. Smoothed and averaged spectra of longitudinal velocity in neutral air at Round Hill. a. 15 m, tower B, b. 16 m, tower A; c. 40 m, tower A; d. 46 m, tower B.

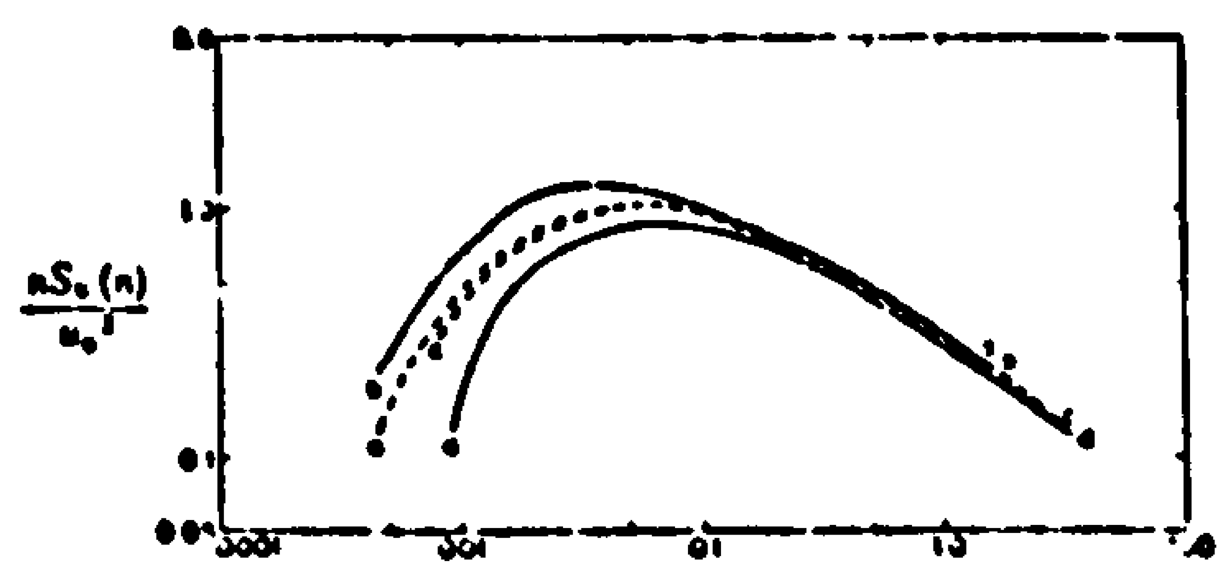


Figure 14. Smoothed and averaged spectra of longitudinal velocity in unstable air at Round Hill. a. 15 m, tower B, b. 16 m, tower A, c. 40 m, tower A; d. 46 m, tower B

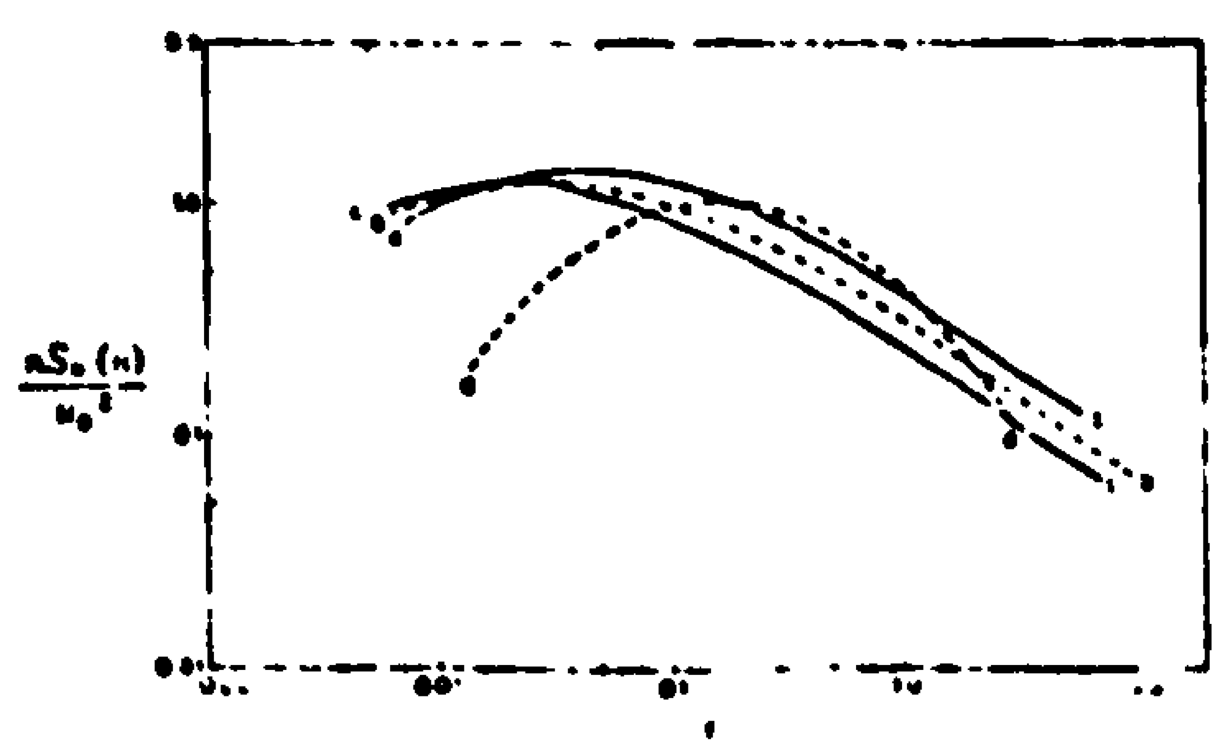


Figure 15. Smoothed and averaged spectra of longitudinal velocity at [illegible] a. stable; b. neutral; c. unstable. d. laboratory spectrum [illegible]

Fig 3.2.4 Turbulence spectra, from Busch and Panofsky, 1968.

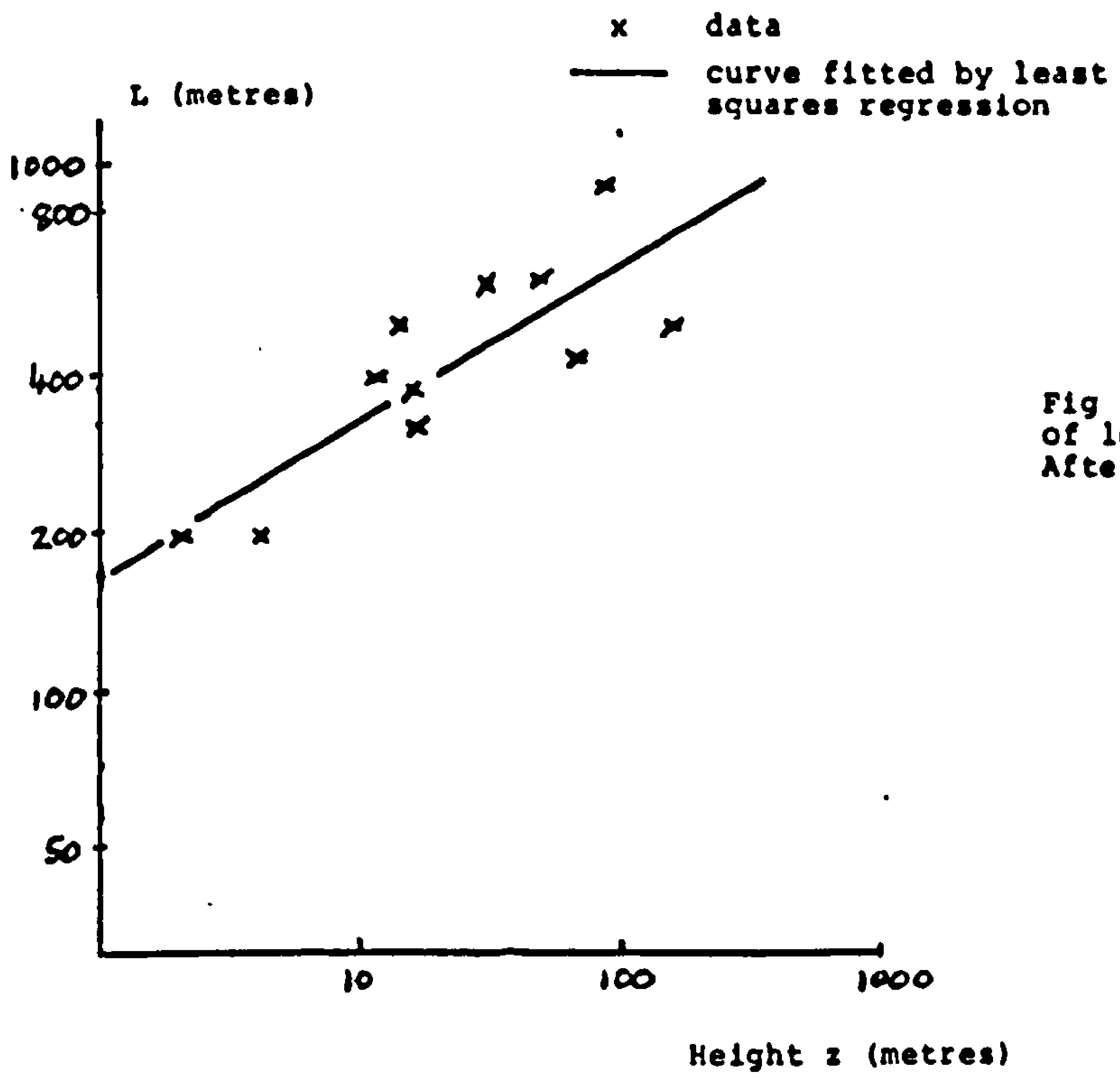


Fig 3.2.5 Length scales of longitudinal turbulence. After Berman, 1965.

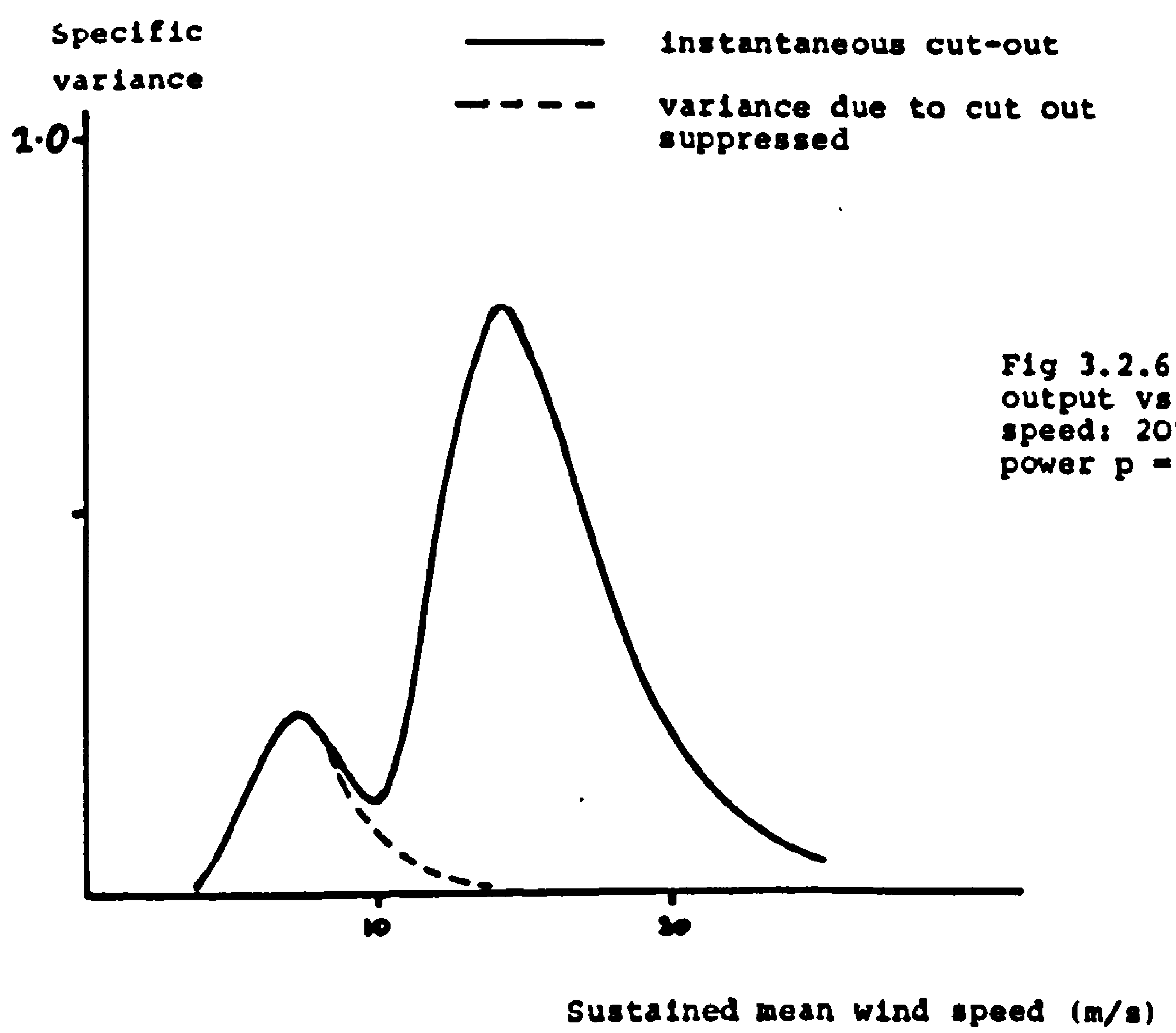


Fig 3.2.6 Variance of turbine output vs. sustained wind speed: 20% turbulence, specific power p = 1.0.

3.3 ANALYSIS

3.3.1 SPECTRAL ANALYSIS

The purpose of this section is to provide the reader who is not familiar with the use of spectral analysis of random variables, with sufficient insight into the theory, to understand the origin purpose and limitations of the techniques used. There is an extensive literature on the subject of spectral analysis for those who require a more rigorous treatment (see eg. Bendat and Piersol 1966, Newland 1975, Lumley and Panofsky 1964).

Wind power is characterised by fluctuations with a very wide range of periods. In meteorological work wind speed fluctuations are often, fairly arbitrarily, divided up into short period fluctuations ($T < 1$ hour) which are classified as turbulence, synoptic fluctuations (1 hour $< T <$ 1 month) otherwise known as weather, annual fluctuations, and fluctuations with periods significantly longer than 1 year, which may be referred to as climatic variation. Where a variable exhibits this type of behaviour, a better understanding of it can often be obtained by Fourier analysis. The most useful form (for the purpose of this section) in which to state the Fourier transform equation is the discrete form:

$$\text{1a.} \quad X_k = 1/N \sum_{r=1}^{N} x_r \exp(-2\pi i k r/N)$$

The inverse discrete Fourier transform is given by:

$$1b. \quad x(t) = \sum_{k=1}^{N} X_k \exp(2\pi ikt/T)\, dn$$

where x_r are the values of the variable $x(t)$ sampled at N equal intervals Δt, and X_k is the Fourier component at the frequency $n = k/N\Delta t$. Note that throughout this thesis I shall use the convention $i=(-1)^{1/2}$.

If the x_r are random, as is the case with wind speed or wind power output, then the X_k derived from different series of x_r will be different. Even if the process $x(t)$ is stationary, any particular X_k will fluctuate from series to series in a random way. In particular, the expectation values of the X_k (the averages evaluated over all possible series of x_r and written $\langle X_k \rangle$) will be zero. As they stand the X_k are not particularly useful statistics. However the squares of the moduli of the X_k are positive definite quantities, and $\langle |X_k|^2 \rangle \geq 0$. For most naturally occurring phenomena $\langle |X_k|^2 \rangle$ are quite smooth and one can usefully define a continuous function S(n)

$$2. \quad S(n)\, \Delta n = 2\langle |X_k|^2 \rangle$$

where $\Delta n = 1/N\Delta t = 1/T$. T is thus the duration of the time series used to calculate the X_k. S(n) is actually equal to the limit of the expression on the right hand side of equation 3.3.2 as T tends to infinity and Δt tends to zero. S(n) thus defined is known as the power spectrum or the spectral density function of x(t).

It is worth noting here that equation 3.3.2 is not the only or even the most common way of defining S(n). I have used it because it parallels closely the way that S(n) is estimated in practice from the discrete Fourier transform defined in equation 3.3.1.

The physical significance of the power spectrum S(n) (I will drop the 'power' in what follows) is not yet clear. Consider:

$$3. \quad \sum_{k=0}^{N-1} X_k X_k^* = 1/N^2 \sum_{r,s=1}^{N} x_r x_s \sum_{k=0}^{N-1} \exp(-2\pi i(r-s)k/N)$$

where * denotes complex conjugate. A geometrical argument shows that the final term in the above is equal to N if r=s, and is zero otherwise. So:

$$4. \quad \sum_{k=0}^{N-1} |X_k|^2 = 1/N \sum_{r=1}^{N} x_r^2$$

The right hand side of the above is simply the second moment of the x_r, equal to the variance of x_r if the mean of x_r is zero. This equation is the discrete form of Parsival's equation.

In order to go further in exploring the significance of spectra, we need to look at a phenomenon known as aliasing, which arises because the sampling interval is not infinitesimal. Consider the following:

$$X_{N/2 - r} = \sum_{s=1}^{N} x_s \exp(-2\pi i(N/2 - r)s/N$$

$$= \exp(-2\pi is) \sum_{s=1}^{N} x_s \exp(2\pi i(N/2 + r)s/N$$

which gives:

5. $$X_{N/2 - r} = X_{N/2 + r}^{*}$$

The implication of the above is that the discrete spectral estimates X_k are only unique up to $k=N/2$, and that the power spectrum is symmetrical about the so-called folding or Nyquist frequency $n = 1/2\Delta t$. This phenomenon appears in many branches of physics, the examples which spring immediately to mind are the effects on waves in crystalline solids of the Brillouin zone boundaries. If we take aliasing into account, equation 3.3.4 can be rewritten:

6. $$2 \sum_{k=0}^{N/2-1} |X_k|^2 = 1/N \sum_{r=1}^{N} x_r^2$$

with no loss of generality. An additional implication of aliasing is that if the continuous process x(t) contains frequencies above the Nyquist frequency, these components will appear in the discrete Fourier transform at frequencies below the the Nyquist frequency. Equation 3.3.5 implies that the high frequency spectrum is simply folded back onto the low frequency spectrum. If we can assume that the spectral estimates above and below the Nyquist frequency are statistically independent, then the expectation value of

$|X_k|^2$ (where $k=N/2-r$) will be the sum of the contributions from the power spectrum $S(n)$ at $n=(n_{ny}-n')$ and $n=(n_{ny}+n')$ where $n'=r/N\triangle t$. This effect is illustrated in fig 3.3.1. The problem is actually slightly worse than outlined here. If the process $x(t)$ contains frequencies in the range $2n_{ny} < n < 3n_{ny}$, this part of the spectrum $S(n)$ will be doubly folded by the discrete Fourier transform. The problem of unravelling multiple folding is likely to be intractable. The solution is to make certain that the spectrum of the quantity which is to be discrete Fourier transformed tends rapidly to zero for $n > 2n_{ny}$. This can be done very simply by averaging the process $x(t)$ over the sampling period $\triangle t$.

If the above condition is satisfied, and we are able to estimate the form of $S(n)$ for $n > n_{ny}$, then correction for the effect of aliasing is possible, and an un-biassed estimate of $S(n)$ in the range $n < n_{ny}$ can be obtained from the $|X_k|^2$, via equation 3.3.2. The correction factor for aliasing will generally be equal to 2 at the Nyquist frequency, and will tend to unity as n tends to zero.

The significance of the power spectrum $S(n)$ should now be clearer. The integral of the spectrum over all frequencies is equal to the variance of the process $x(t)$. The integral of $S(n)$ between frequencies n_1 and n_2 is equal to the variance of the process $x(t)$ which arrives between those frequencies - if $x(t)$ is filtered using a band pass filter with limiting frequencies n_1 and n_2, the variance of the filtered process $x'(t)$ will be equal to the integral of $S(n)$ between the two frequencies. Note that the filter may be considerably more complex than a simple band pass filter, and may be expressed initially in either the time or frequency domains. The connection between these two domains is provided by the convolution theorem. If the operation "Fourier transform" is denoted by "FT" and the operation "convolute" is denoted by "*" (note the difference between this and "complex conjugate" which is superscript *) then:

7. $FT(x*y) = FT(x).FT(y)$

The importance of this is that real problems often revolve around the application of some time domain averaging process to a random variable. We now have a general method for evaluating the variance of the output x'(t) of such filters from the spectrum S(n) of the input variable x(t).

Practical estimation of spectra

I have used equation 3.3.2 above to estimate wind power fluctuations. It is clearly necessary to find some way to estimate the expectation value of $|X_k|^2$. This can be done in a combination of two ways:

1) by averaging over a block of raw spectral estimates obtained from a single Fourier inversion, to obtain a smoothed value

$$S(n)\Delta n \simeq 2/(\Delta k - 1) \sum_{k-\Delta k/2}^{k+\Delta k/2} |X_k|^2$$

This process is known as smoothing in the frequency domain.

2) by averaging the $|X_k|^2$ obtained from non overlapping blocks of data each of duration T. This is known as ensemble averaging.

In mathematical terms these operations are very similar. The estimates of the spectrum S(n) obtained by smoothing will themselves be random variables. It is to be expected and in practice it is found (Newland 1975, p 138) that the variable $|X_k|^2$ has χ^2_2 statistics. An estimate of S(n) obtained by averaging over m estimates of this variable will have χ^2_{2m} statistics. The actual uncertainties in the wind spectra I have calculated will be discussed later.

In the case of wind sampled at an interval of 1 hour, it is possible to estimate a correction for aliasing. This will also be discussed later.

3.3.2 THE EFFECT OF AVERAGING IN THE TIME DOMAIN

The effect of averaging in the time domain is important both intrinsically (the effect of dedicated storage on the output of a fluctuating energy source could be approximated by averaging the output of the source over a suitable time), and because it enables us to examine a number of other effects. Let us assume that x(t) is a random process, which is to be averaged over time intervals J to produce a secondary process x'(t). This operation is mathematically described by the convolution of x(t) with a window function w(t) defined by:

8. $w(t) = 1/J \quad \text{for } -J/2 < t < J/2$

$w(t) = 0 \quad \text{elsewhere}$

and

9. $$x'(t) = \int_0^{\infty} x(t)w(t-t')dt'$$

The Fourier transform of $X'(n)$ of x' is related to the Fourier transforms of x and w by:

10. $$X'(n) = X(n)W(n)$$

The frequency domain filter is given by:

11. $$W(n) = \{\sin(\pi nJ) / \pi nJ\}$$

The power spectrum $S'(n)$ of x' is given by:

12. $$S'(n) = |X'(n)|^2$$

$$= |X(n)|^2 \{\sin(\pi nJ) / (\pi nJ)\}^2$$

$$= S(n) \{\sin(\pi nJ) / (\pi nJ)\}^2$$

If the variance of $x(t)$ evaluated over all time is denoted by σ^2, and the variance of the time averaged process $x'(t)$ is denoted by σ_J^2, then:

13. $$\sigma_J^2 = \int_0^{\infty} S(n) \{\sin(\pi nJ) / (\pi nJ)\}^2 \, dn$$

The function $\{\sin(\pi nJ) / (\pi nJ)\}^2$ is shown in fig 3.3.2, plotted against $\ln(nJ)$. The effect of finite averaging time J on the variance of the process x is (to a first approximation) to cut off the integration of the spectrum $S(n)$ at $n=1/J$.

3.3.3 THE SPINNING RESERVE PROBLEM

We can use the theory presented above to investigate fluctuations of some process x(t) over a finite period J. If we denote averages over infinite periods by $\langle\rangle$ and averages over J by $\langle\rangle_J$, then :

$$x(t) - \langle x\rangle = (x(t) - \langle x\rangle_J) + (\langle x\rangle_J - \langle x\rangle)$$

$$\sigma^2 = \langle (x(t) - \langle x\rangle)^2\rangle$$

$$= \langle\langle (x(t) - \langle x\rangle)^2\rangle_J\rangle$$

$$= \langle\langle (x(t) - \langle x\rangle_J)^2\rangle_J\rangle + \langle\langle (\langle x\rangle_J - \langle x\rangle)^2\rangle_J\rangle$$

$$+ 2\langle\langle (x(t) - \langle x\rangle_J)(\langle x\rangle_J - \langle x\rangle)\rangle_J\rangle$$

(since averaging is a linear operation). Note that the second term in the above is just σ_J^2 as defined in the previous section. The third term is identically zero. The first term represents the the mean standard deviation of the process x(t) evaluted over all periods of length J. Following Pasquill, I will call this quantity $\langle\sigma_J^2\rangle$. The above can now be rewritten:

14. $\sigma^2 = \sigma_J^2 + \langle\sigma_J^2\rangle$

Hence:

15. $\langle \delta_J^2 \rangle = \int S(n) \, [1 - \{\sin(\pi n J)/(\pi n J)\}^2] \, dn$

The significance of the above is that the quantity $\langle \delta_J^2 \rangle$ is closely related to the spinning reserve problem, since it tells us how much on average the process x(t) fluctuates over periods of length J. The limitation of this quantity is that the estimate of the amount of fluctuation is referred to the average of x over the period J. In the spinning reserve problem, the system planner is faced with the problem of trying to estimate the likely fluctuation of some variable from its current value, over the next J minutes. The quantity of interest is:

$\langle (x(t+J) - x(t))^2 \rangle$ not

$\langle\langle (x(t) - \langle x \rangle_J)^2 \rangle_J \rangle$

But a little reflection will show that to a first approximation:

$\langle (x(t+J) - x(t))^2 \rangle \simeq 2\langle \delta_J^2 \rangle$

A second approach to the spinning reserve problem is as follows. From the definition of the inverse discrete Fourier transform (equation 3.3.1b) we can write the change in the value of x over a time interval J as:

16. $x(t+J) - x(t) = \sum X_k \, [\exp(2\pi ik(t+J)/T) - \exp(2\pi ikt/T)]$

which may be simplified to give

17. $x(t+J) - x(t)$

$$= \sum (2i\, X_k \exp(\pi i k J/T) \sin \pi k J/T \exp(2\pi i k t/T)$$

Now from equation 3.3.4 the variance of x, σ^2, is given by

$$\sigma^2 = \sum |X_k|^2 \quad = \int_0^\infty S(n)\,dn$$

Comparison of equations 3.3.1 and 3.3.17 will show that the discrete Fourier transform of the random process x(t+J) - x(t) is given by:

$$2i\, X_k \exp(\pi i k J/T) \sin \pi k J/T$$

Therefore the variance of this process (ie the mean square value of the change in x over a period of J), which I will write $\sigma^2(J)$, is given by:

18. $\sigma^2(J) \quad = \quad \sum 4|X_k|^2 \sin^2 \pi k J/T$

The continuous form of this equation is

19. $\sigma^2(J) \quad = \quad 4\int_0^\infty S(n) \sin^2 \pi n J \, dn$

The filter 4 $\sin^2(\pi n J)$ is shown in fig 3.3.3. The effect of this filter is to cut off the integration of S(n) below $n \simeq 1/10J$. The mean value of the filter above $n \simeq 1/J$ is approximately 2, as expected, and consistent with the remarks about the significance of the quantity $\langle \sigma_J^2 \rangle$.

It is worth while spending some more time clarifying the differences between the two quantities discussed above ($<\sigma_J{}^2>$ and $\sigma^2(J)$). The derivation of these quantities is shown diagramatically in fig 3.3.4. If an estimate of the mean value of some variable of interest, over some critical period measured from the present, is possible, then $<\sigma_J{}^2>$ may well be a useful quantity to estimate. On the other hand if it is more convenient to estimate the current value of the variable of interest, then an estimate of the expected change in this variable over the critical period may be more useful. For the purpose of analysing wind power output fluctuations I have evaluated the latter statistic $\sigma^2(J)$ as a function of J for a number of wind sites and pairs of wind sites. I will use the term "random walk characteristic" to refer to it.

It is necessary to point out that random processes are not always stationary (or even gaussian). In particular although unpredictable variation of consumer demand for electricity in the case of the CEGB can be modelled by a classical random walk, the standard deviation of this process varies greatly through the day (Farmer et al 1980). The random walk characteristic in the form introduced here may need to be modified to take account of this.

3.3.4 THE ADDITION OF PARTIALLY CORRELATED RANDOM VARIABLES.

One of the areas that this thesis is directed toward, is the effect of geographical dispersion of wind turbine arrays on the fluctuations of wind power output. A commonly used method of investigating the degree of dependence of pairs of random variables, as a function of period or frequency, is to estimate the coherence function of the variables. The coherence function γ of two variables x(t) and y(t) is defined by

20. $$\gamma^2(n) = S_{xy}(n) \, S_{xy}^*(n) \, / \, \{ S_x(n) \, S_y(n) \}$$

where $S_{xy}(n)$ is the cross spectrum of the input variables, x and y. The cross spectrum is simply the smoothed product of the discrete spectral estimates X_k and Y_k^*. Thus

21. $$S_{xy}(n) = (2/N\Delta t) \langle X_k Y_k^* \rangle$$

For linear systems, where x(t) is the input to the system and y(t) is the output, $\gamma^2(n)$ "can be interpreted as the fractional portion of the mean square value at the output which is contributed by the input x(t) at the frequency n." (Bendat and Piersol, op cit, p142)[1]. In the case of wind power we are clearly not dealing with a linear system in which wind power output at one site is the "input" to the system, and the output at another site, the "output". The interpretation of the coherence in this case is more difficult. Perhaps the best description is to say that the coherence function $\gamma^2(n)$ is mathematically equivalent to the (square of the modulus of) the Pearson product moment coefficient (correlation coefficient) of the discrete spectral estimates X_k and Y_k, where the averaging process implied by this statement is carried out either in the frequency domain or over ensembles. This incidentally

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[1]. Note that either x or y may be considered to be the input, as the coherence function is symmetrical in these quantities.

implies that the statistics of coherence function estimates are closely analogous to those of correlation coefficients (see Bendat and Piersol, op cit, p194).

The advantages of using the coherence function to study the correlation of stochastic process, are

a) that it has a well defined range - $0 \leq \gamma^2(n) \leq 1.0$ - with the lower end implying statistical independence of the two processes, and the upper limit implying complete dependence, and

b) that its statistics have been investigated and are well documented.

Confidence intervals for coherence function estimates.

As noted above, a coherence function estimate is mathematically equivalent to the correlation coefficient of dicrete spectral estimates of two random processes, x(t) and y(t). As such it may be expected to have the same statistics. The exact statistics of the correlation coefficient are extremely complex, except in the case where the expectation value of the coefficient is zero (Alder and Roessler, 1977). In non-zero cases it has been found that the quantity

22. $z = \tanh^{-1} \gamma$

is approximately normally distributed, with mean and standard deviation given by:

23. $mu_z(n) = (m-2)^{-1} + \tanh^{-1}(\gamma(n))$

24. $\delta_z^2 = (m-2)^{-1}$

where m is the number of degrees of freedom of the estimate γ' of the expectation value of the coherence function γ. The transformation given by equation 3.3.22 is known as Fischer's Z transformation. Note that the distribution implied by equations 3.3.22 to 3.3.24 is skewed, particularly for coherence function estimates with few degrees of freedom. Bendat and Piersol (op cit) state that the Z transformation is useful in the range $0.35 \leq \gamma^2 \leq 0.95$, and for $m \geq 40$. In practice, all of the coherence function estimates presented in this thesis will have $m \geq 90$. They may however not satisfy the former condition. Confidence intervals for the coherence function, with 100 degrees of freedom are shown below:

Confidence intervals for γ^2

The confidence limits of the coherence function are given by:

$$\tanh\{ \tanh^{-1}\gamma' - 1/(n-2) - z/(n-2)^{1/2} \} < \gamma$$

$$< \tanh\{ \tanh^{-1}\gamma' - 1/(n-2) + z/(n-2)^{1/2} \}$$

where z is the abssica value of the standard normal distribution corresponding to a given confidence level.

γ'	0.4	0.5	0.6	0.7	0.8	0.9
1% Upper	0.20	0.30	0.41	0.54	0.68	0.83
Lower	0.58	0.66	0.73	0.81	0.87	0.94
5% Upper	0.24	0.34	0.46	0.58	0.71	0.85
Lower	0.54	0.62	0.70	0.78	0.86	0.93

For completeness, the distribution of coherence function estimates for zero expectation value will also be given. It is found (Adler and Roessler, op cit) that the distribution of

$$25. \quad \gamma/\{ (1-\gamma^2)/(m-2) \}^{1/2}$$

satisfies a Student's t-distribution, with m-2 degrees of freedom.

A second method of presenting the independence of a pair of random variables has been used by Lowe and Alexander (Lowe and Alexander, 1981). This is to calculate the ratio

$$26. \quad R(n) = S_{x+y}(n)/\{2(S_x(n) + S_y(n))\}$$

This quantity is an approximation to the ratio of the actual variance of the sum x(t) + y(t) near the frequency n, to the variance which would result if x and y were completely correlated at this frequency. To the extent that this is only an approximation, it would be better to define

$$27. \quad R(n) = S_{x+y}(n)/\{S_x(n) + S_y(n)\}$$

which is the ratio of the actual variance of the process x + y to the variance which would result if x and y were completely uncorrelated at this frequency. In this second definition, $R(n) = 1.0$ if x and y are uncorrelated at frequency n, and $R(n) \simeq 2.0$ if x and y are completely correlated at this frequency. The variance R(n) has perhaps the advantage of simplicity of interpretation over the coherence function. This was the reason for its introduction in the paper referred to above (Lowe and Alexander, op cit). Its disadvantages are that it has an ill-defined range, and there is no background of theory behind it (for instance on confidence limits). No more will therefore be said of it.

3.3.5 THE DIVERSITY FACTOR

We can examine the effect of the degree of correlation of groups of wind power sites on the random walk characteristics of their combined outputs. A convenient way of doing this is to define a diversity factor D(n,t) which is the ratio of the actual standard deviation of fluctuations in the combined power output of the n sites, at lead time t, to the standard deviation of fluctuations which would result if the wind sites were all 100% correlated with each other. As described above D may be written:

$$28. \quad D(n,t) = \delta_n(t) \;\; / \;\; \left(\sum_{i=1}^{n} \delta_i(t)\right)$$

where $\delta_n^2(t)$ is the random walk characteristic of the combined output from the n sites and $\delta_i^2(t)$ is the random walk characteristic of the output from site i. Interpretation is easiest if D is evaluated for pairs of sites. Extension to a large number of sites can then be made by use of the cross correlation coefficients r_{ij} of fluctuations at pairs of sites. This may be done as follows (the dependence on t is implicit):

$$29. \quad \sigma_n^2 = \sum_{i=1}^{n} \sigma_i^2 + \sum_{i \neq j} r_{ij} \sigma_i \sigma_j$$

Now if $\sigma_i = \sigma_0$ for all i

$$\sigma_n^2 = \sigma_0^2 \{ n + r n(n-1) \}$$

where r is equal to the mean of the r_{ij} . Hence:

$$30. \quad D = \sigma_n / (n \sigma_0) = (1/n \{1 + (n-1)\bar{r}\})^{1/2}$$

If the standard deviations of the x_i vary then equation 3.3.30 will under estimate D. It can be shown that if the σ_i are normally distributed with mean σ_0 and standard deviation s, and if the r_{ij} are not correlated with the magnitudes of the σ_i then D may be approximated by the equation:

$$31. \quad D^2 \simeq 1/n \{e + (n-e) r \}$$

where

$$e = (\sigma_0^2 + s^2) / \sigma_0^2$$

Hence D will be increased by increases in e. In practice for wind power at sites in the UK we find that $s \simeq 0.2\sigma_0$. Hence D(n,t) can be estimated from data for site pairs using equation 3.3.31 with $e \simeq 1.04$.

In order to be able to estimate diversity factors for different groups of UK wind sites, I have calculated the cross-correlation coefficients of fluctuations of power output at chosen pairs of wind sites, as a function of lead time. The calculation method is the direct application of the equation

32. $$r_{ij}(J) = \langle (x_i(t) - x_i(t+J))(x_j(t) - x_j(t+J))\rangle \,/\, \sigma_i(J)\ \sigma_j(J)$$

to the series representing the wind power outputs at each site. It is possible to calculate the cross correlation coefficients from the discrete spectra of wind power output at each wind site. However, this involves a separate filtering and inverse Fourier transform for each lead time, and although intellectually satisfying, represents an inefficient method.

3.3.6 THE RELATIONSHIP OF SPECTRAL ANALYSIS TO THE PROBLEM OF ENERGY STORAGE.

Spectral analysis provides a method of separating the fluctuations of a random process into bands of frequency or period. It is intuitively obvious that the information in the original random process, thus presented, tells us what we need to know in order to determine the effects of energy stores of different sizes on the output of a random energy source such as wind. In particular it enables us to evaluate the size of store that is needed to provide a continuous power output with a given level of reliability. Simply, if the dispersion of the output from some random source of energy is σ and the fourier spectrum is peaked around a frequency $n=1/T$, then the storage requirement is related to δT . Intuitively, if the standard deviation of wind power

$\delta \simeq \langle w \rangle$ (where $\langle w \rangle$ is the mean power output)

and the frequency spectrum of wind speed peaks at

$T \simeq 4$ days

that an energy store capable of storing about 4 days average output of wind power should enable most of the fluctuations of wind power output to be smoothed. Statements of this type can be made more rigorous in the following way. Consider a system which consists of a flucuating energy source, an infinite energy store and a constant energy demand equal to the mean output of the energy source:

$x(t) \longrightarrow |\ \underline{\text{STORE}}\ | \longrightarrow \langle x \rangle$

I have already introduced the quantity δ_S, which is the dispersion of the process $\langle x(t) \rangle_S$. If the energy level of the store is E (measured from some arbitrary zero point) then the change in the level of the store over a time period T is given by:

$$33. \quad \triangle E = \int_0^T x(t) - \langle x \rangle \, dt$$

$$= T \, (\langle x(t) \rangle_T - \langle x \rangle)$$

The dispersion of $\triangle E$ is therefore simply:

34. $\sigma_E = T\,\sigma_T$

Before going further it is worth making the point that in the case of wind power the probability distribution of ΔE is not gaussian, for the simple reason that wind power output is bounded by zero and the installed capacity. But for $\Delta E < \langle x \rangle$ and for T of the order of 1000 hours or more the gaussian approximation is useful.

To illustrate the conclusions that we can draw from the analysis presented above consider the case where σ_E is a monotonically increasing function of T. In this case it is clear that no level of storage will be sufficient to provide a firm energy demand from the fluctuating energy source. If on the other hand σ_E tends to some upper limit for $T \gg T'$ then long term energy storage and completely wind powered energy systems may be a possibility.

An example plot of $T\,\sigma_T$ versus T for a high average wind speed site is shown in fig 3.3.5. The ordinate axis has been normalised by the mean wind power output at the site. The plot peaks at approximately 500 , with $n \simeq 10^{-4}$ hr^{-1}. The lower bound of ΔE over this period is therefore approximately 20 greater than the standard deviation of ΔE. The duration of the data is insufficient to allow estimates to be made of the probability function of ΔE for T greater than about 1000 hours.

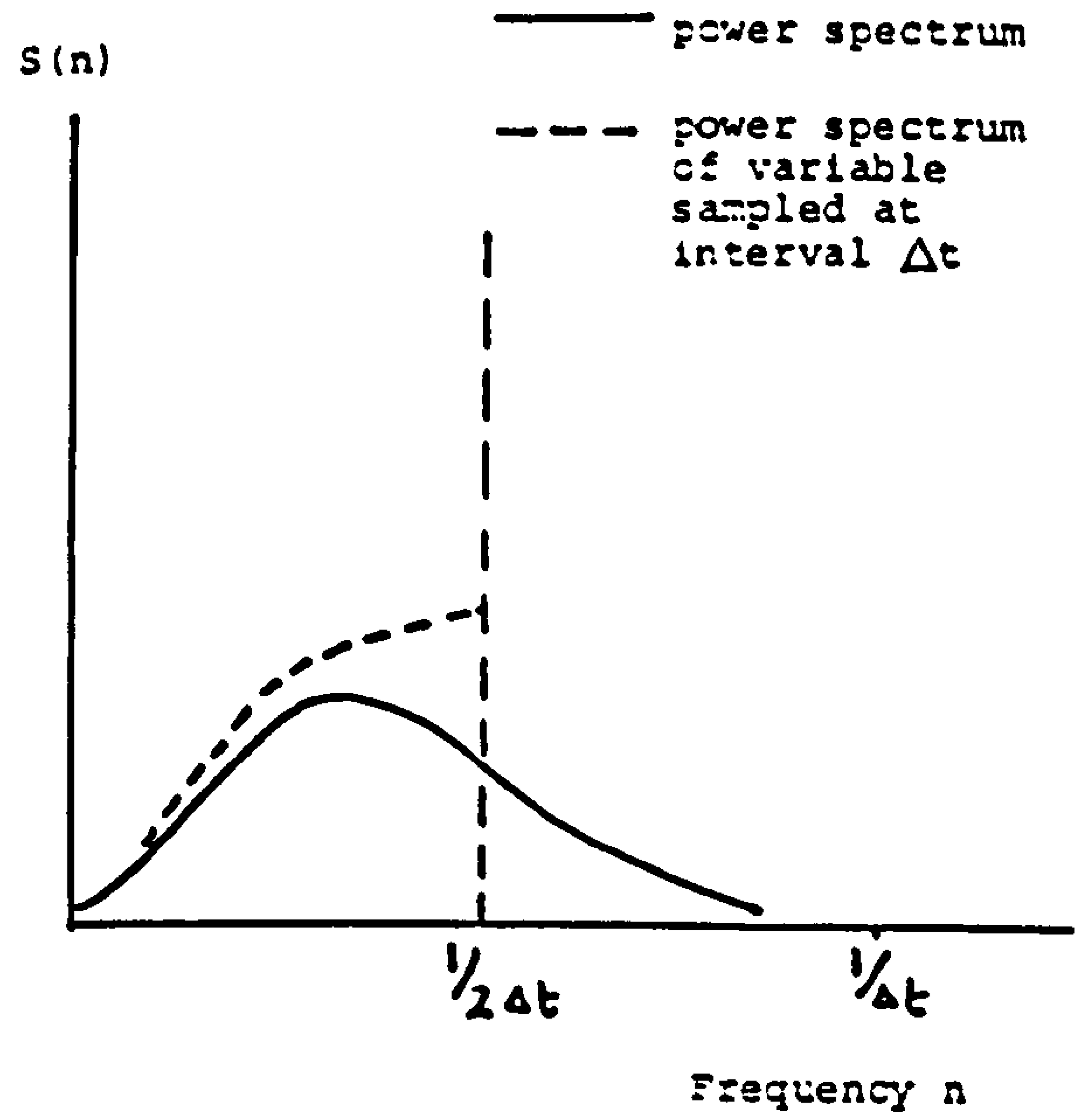


Fig 3.3.1 The effect of aliasing on spectra.

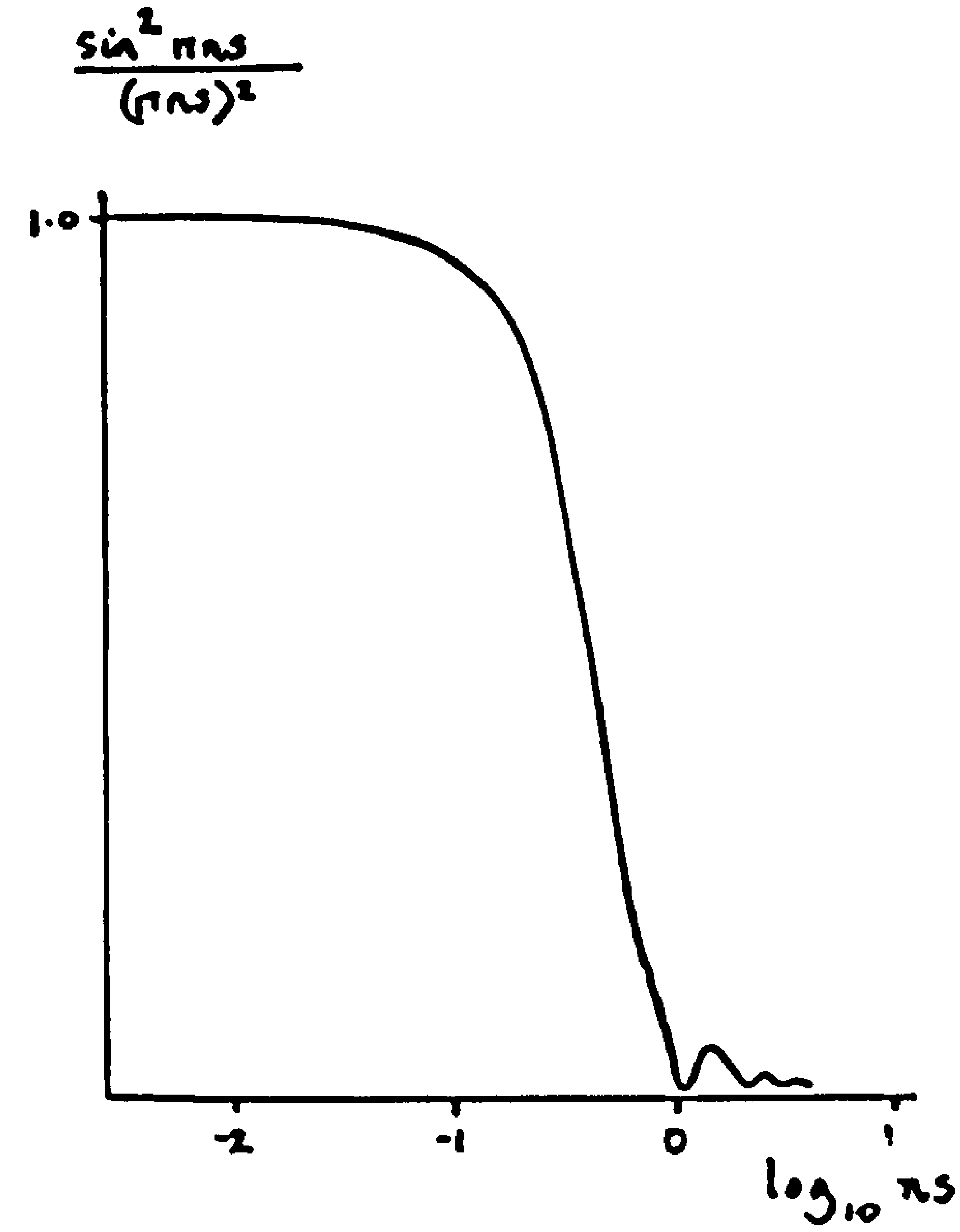


Fig 3.3.2 The effect of finite averaging time s, on spectra.

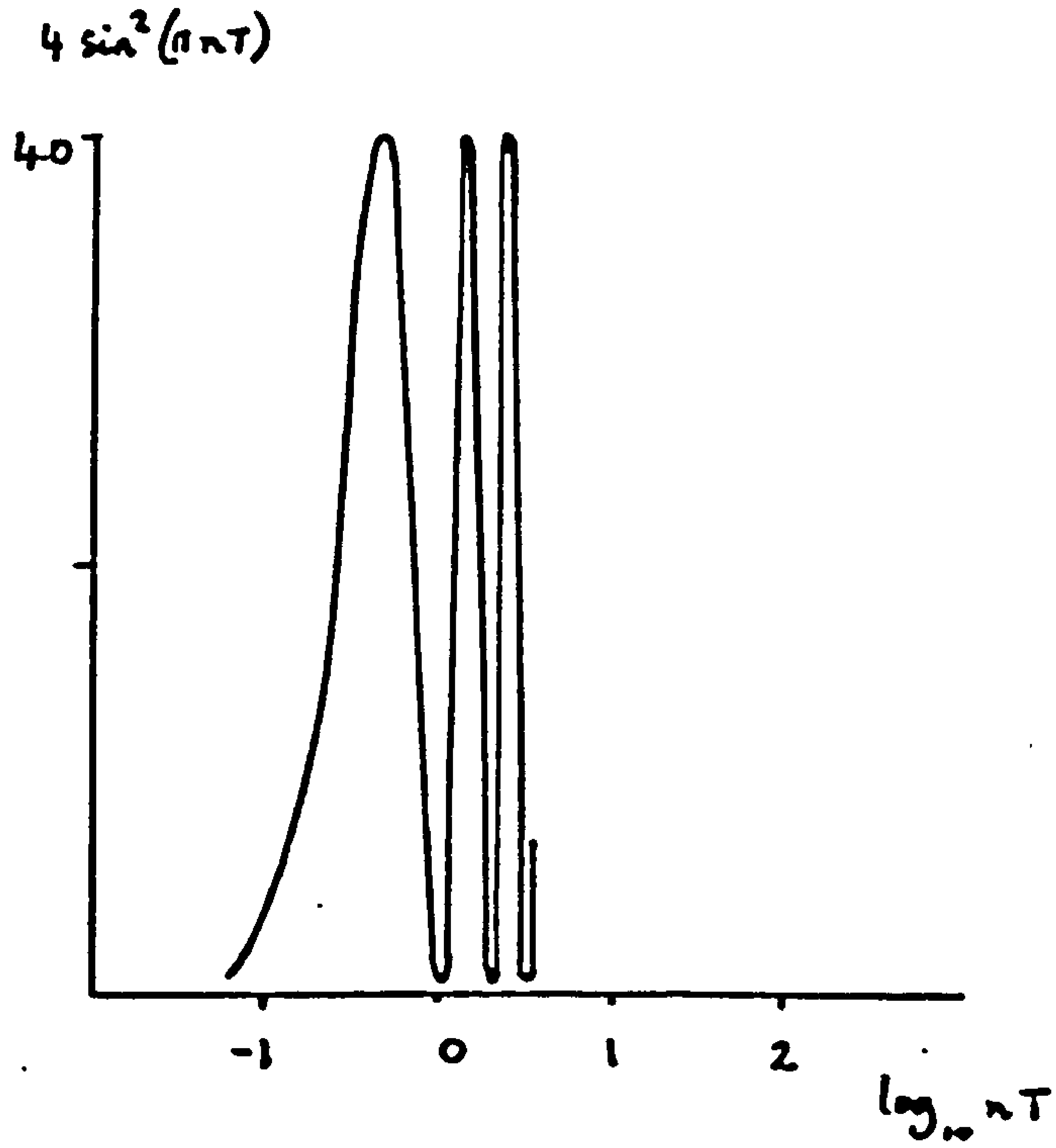
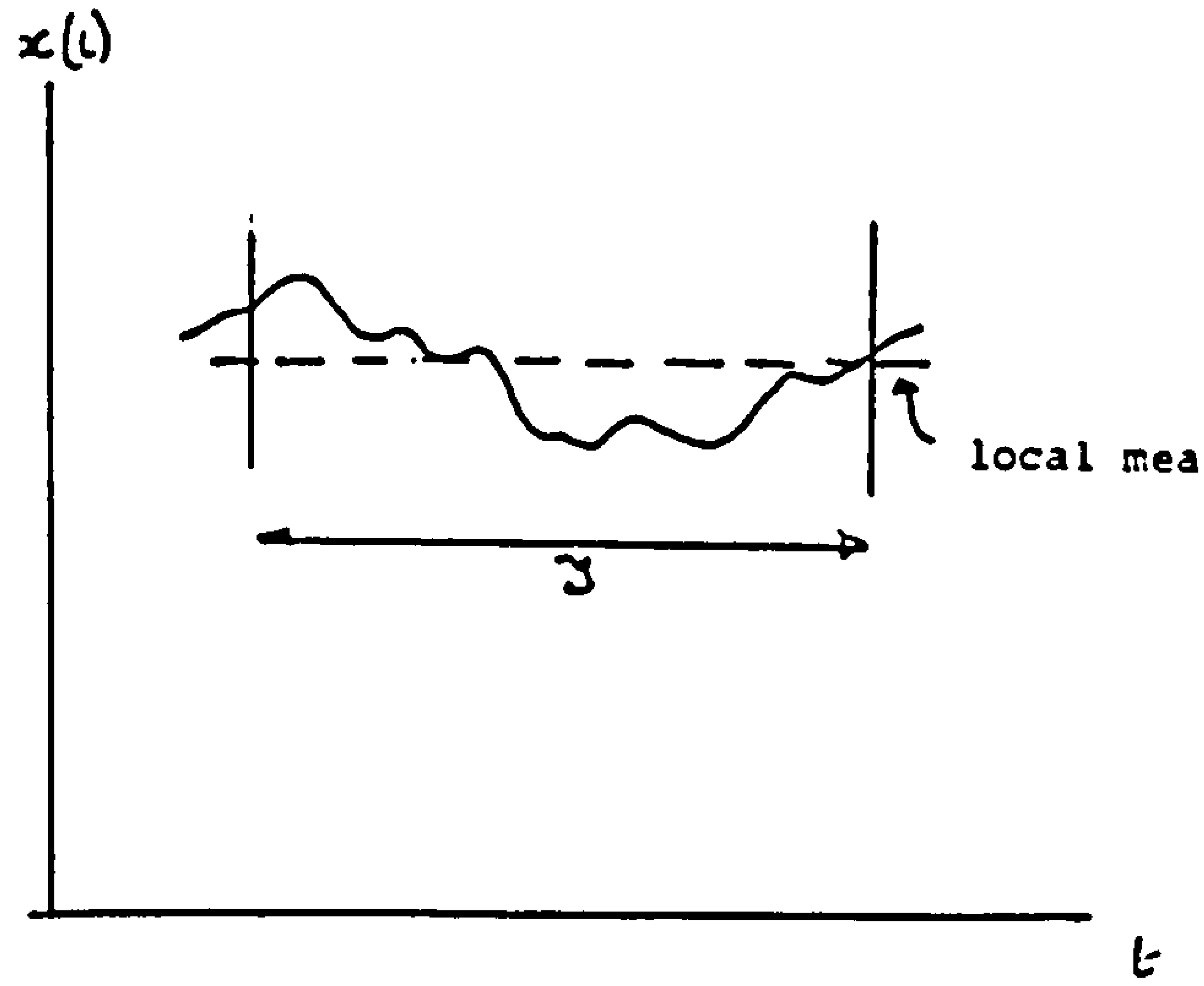


Fig 3.3.3 The filter for variance of changes in value of random variable over time period T.

Derivation of $\langle\sigma_\tau^2\rangle$ the mean variance about the local mean, for periods of length τ.



Derivation of $\sigma^2(\tau)$ the variance of changes of x over periods of τ.

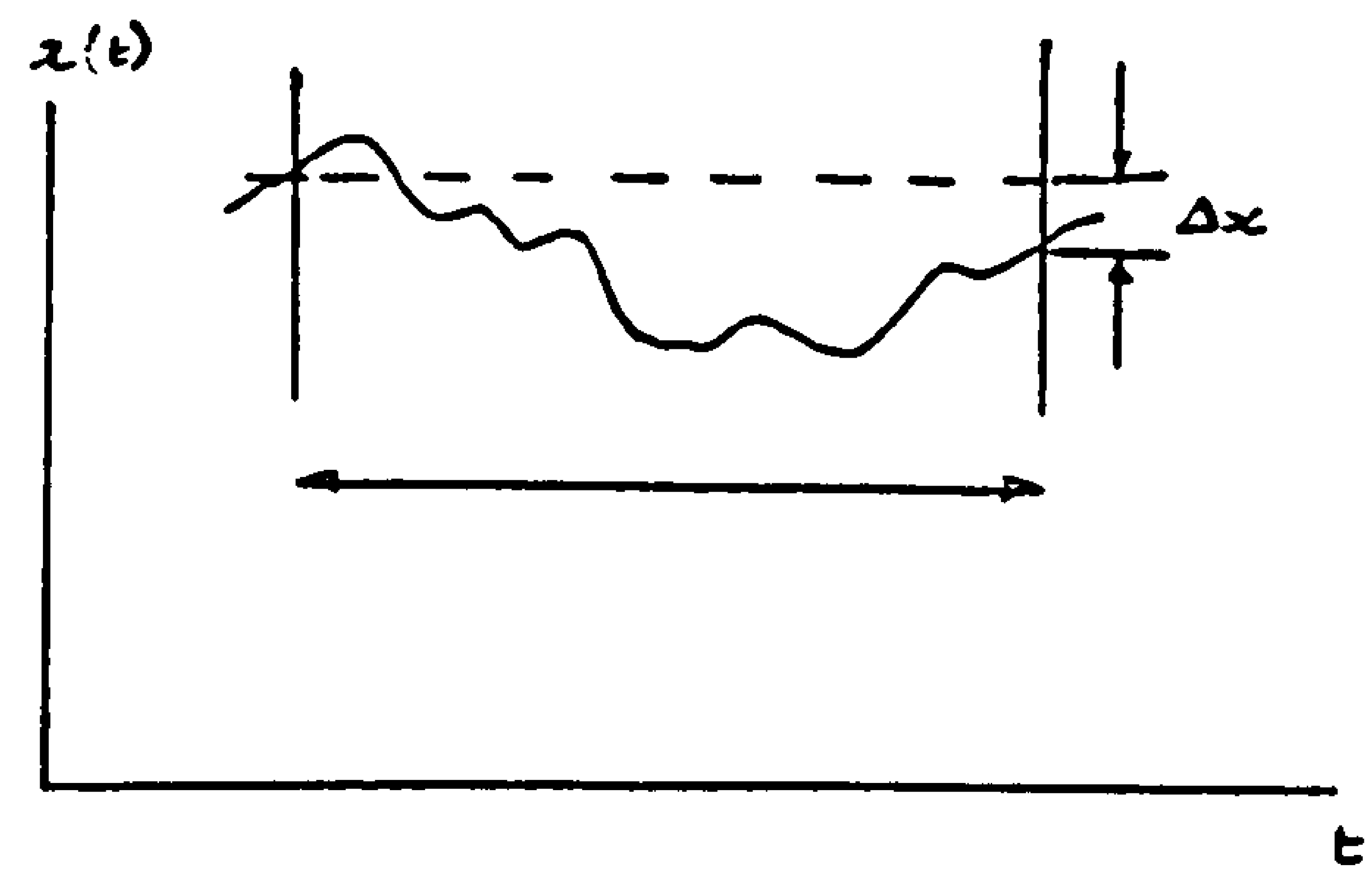


Fig 3.3.4 Comparison of two methods for estimating the change in the value of a random variable over time periods T.

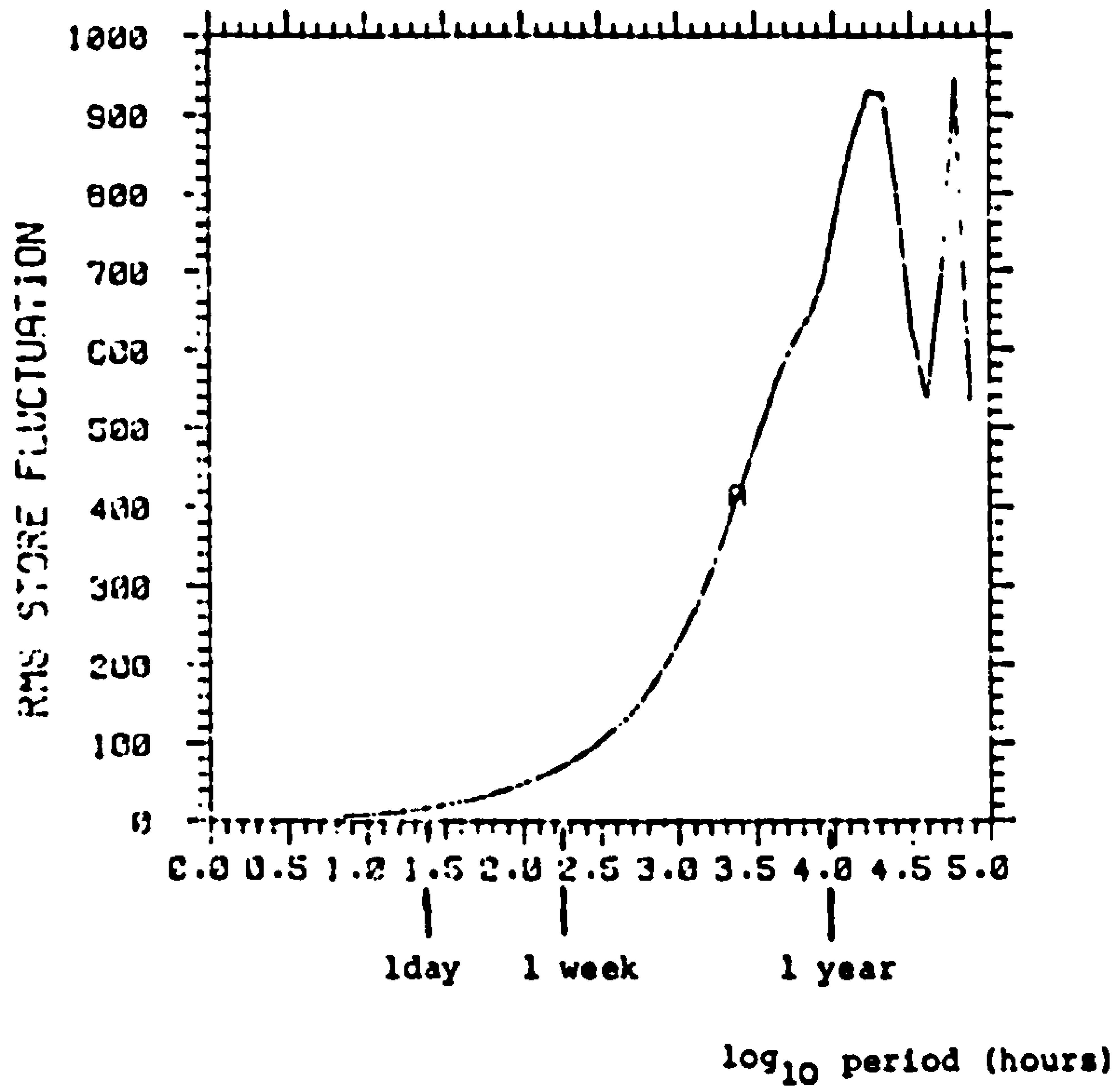


Fig 3.3.5 Fluctuation in the level of an infinite energy store as a function of lead time.

Note the use of the logarithmic frequency axis. This is made necessary by the wide range of frequencies of interest. The units of the vertical axis are rms store fluctuation divided by the mean wind power. The result is a time constant characterising the size of store required to convert wind power into a steady output over given periods of time.

3.4 ANALYSIS PROGRAMS

The foregoing section described the mathematical basis for the statistical analysis of wind power. I now wish to descibe in some detail the programs written to perform most of the computation involved in the analysis, showing how the above theory was applied.

My original objectives in writing these programs were:
1) to produce spectra of wind power output for a large number of geographical sites in the UK, covering a range of frequencies from 1 hr^{-1} down to 1 $year^{-1}$.
2) to filter the spectra thus obtained to produce estimates of random walk characteristics, and storage fluctuations for each site.
3) to produce estimates of the effects of geographical dispersion of wind power sites in terms of coherence functions and cross-correlation coefficients (based on the analysis set out in sections 3.3.4 and 3.3.5.
4) to present all of the above information in tabular and graphical form.

The response to the above specification was two sets of programs written and run originally on the Cambridge IBM 370 computer (this has recently been replaced by an IBM 3081). The programs use the Cambridge high level graphics package HIGRAPH for graphical output, the numerical analysis library SYS2.NFORTLIB (for the discrete Fourier transform subroutine) and where necessary a specially written library of subroutines.

The specification separates clearly into a requirement for analysis of single site characteristics, and analysis of site pairs. Limitations of computer storage also forced a second separation. In order to make any estimate at all of the magnitude of seasonal fluctuations it is desirable to analyse as much data as is available for each site - in my case 9 years of data. In order to make estimates of the effects of short term fluctuations in wind power output it is necessary to analyse data at the shortest sampling interval possible - in the case of most Meteorological Office data a period of 1 hour. Storage limitations effectively impose a bandwidth limitation on spectral analysis, and meant that it was impractical to produce estimates of spectra over the whole of the frequency range in one Fourier inversion. The analysis for single sites was therefore split into:

1) programs which analyse time series of 9 years' duration, consisting of data averaged in the time domain over adjacent blocks of 6 hours.

2) programs which perform Fourier inversion of time series of one year's duration and sampling interval of 1 hour, and which average spectra thus obtained over the 9 available years.

Single site analysis programs

There are 2 single site analysis programs:

1) F1S1Y9A This calculates a spectrum for a single site, by averaging the spectra obtained by the successive Fourier inversion of 9 adjacent blocks of one year of data. The output of the program consists of

a) graphical output of a smoothed spectrum over the frequency range 3 yr^{-1} to 0.5 hr^{-1}, for a single site.

b) graphical and tabular output of the random walk characteristic of wind power output. This is relevant to the spinning reserve problem (see section 3.3.3).

2) F1S9Y This averages wind power estimates for a single site over adjacent blocks of 6 hours. Nine years of data at a sampling interval of 6 hours are then Fourier inverted. The output consists of

a) a spectrum substantially overlapping the one produced by F1S1Y9A, but extending over a frequency range of 1/3 yr^{-1} to 1/12 hr^{-1}. This enables the annual cycle to be resolved, albeit coarsely.

b) an estimate of the magnitude of storage fluctuations as a function of lead time (in fact an estimate of $T\bar{\sigma}_T$ as a function of T, see section 3.3.6). This is relevant to the storage problem.

The graphical output for a single site is shown on the following pages. The program listings for F1S1Y9A and F1S9Y are given in appendix 2.

Degrees of freedom and confidence intervals.

The statistics of estimates of spectral density, made from discrete Fourier series, have been discused briefly in section 3.3.1 . Spectral estimates have χ_{2m} statistics, where m is simply the number of raw spectral estimates which are combined to produce the final smoothed estimate of spectral density. All spectra presented in this thesis have been smoothed using the subroutine SMOOT3, which following Petersen (Petersen 1975) averages spectral estimates in the frequency domain, over a band of varying width. Specifically,

1. $m = \{25 + (k/24)^2\}^{1/2}$

(where k is the discrete frequency index, see equation 3.3.1). For small values of k, m = 5, and for large values of k, m $\simeq$ k/24. In the case of the spectra which are also ensemble averaged, the number of degrees of freedom of the spectral estimates, as a function of the frequency n ($hours^{-1}$), is given by:

2. $2m \simeq 18\,\{25 + (365\,n)^2\}^{1/2}$

This function is tabulated below:

1/n (hours)	2	10	30	100	300	1000	3000
Degrees of freedom.	3290	663	132	111	93	90	90

The confidence limits of spectral density functions which have been treated in this way, are therefore determined by χ^2 with approximately 100 degrees of freedom, over the first two decades of the frequency range. The 95% confidence limits of spectral estimates in this range are therefore given (approximately) by

$$0.86\ S'(n) < S(n) < 1.14\ S'(n)$$

Site pair analysis

The site pair analysis has two objectives. The first is to produce estimates of the coherence function for each site pair. The second is to produce estimates of the cross correlation coefficient of fluctuations in power output at the two sites, as a function of lead time. There are two main programs to do this.

1) COHERE this program proceeds in the same way as the single site analysis program F1S1Y9A as far as the discrete Fourier transform for each of the pair of sites over the frequency range 3 yr^{-1} to 0.5 hr^{-1}. It is not practical or particularly useful to correct the single site transforms for aliasing, before calculating the coherence functions for the following reasons:

a) while the form of the spectral density function for a single site can be estimated at frequencies above the Nyquist frequency, the assumptions required for this proceedure do not in general apply to cross spectral densities.

b) in practice $\gamma^2(n) \simeq 0$ for $n \gtrsim 0.25$ hr^{-1}, and so the folding back of the small high frequency part of the spectrum of each site will not have a significant effect on the actual value of the coherence function. The interesting part of the frequency domain is found to be in the region $n < 0.25$ hr^{-1}, which is essentially free from the effects of aliasing (as discussed in section 3.3.1).

The discrete spectra for the two sites are multiplied to give the discrete cross spectrum. The discrete cross spectrum and the two single site spectra are then ensemble averaged over the available 9 years of wind data, to give preliminary estimates of the spectral density functions. These preliminary estimates are then further smoothed in the frequency domain using the subroutine SMOOT3. Finally the smoothed spectra and cross spectrum are combined to give estimates of the coherence function. Graphical output from this program is shown on the following page.

Degrees of freedom and confidence intervals.

The confidence intervals of the coherence function depend in a complex way on the magnitude of the estimates (in the range $0 < \gamma^2 < 1.0$) and on the number of degrees of freedom, as has been discussed in section 3.3.4 . In the program that I have used to estimate coherence functions, the number of raw discrete spectral estimates combined to give each smoothed estimate, is a function of frequency (see above). I have therefore estimated the 95% confidence intervals for the coherence function of the single site pair presented below, as an illustration to the reader of what to expect. I have preferred to keep the rest of the graphical output as uncluttered as possible. Those who require, have sufficient information to estimate confidence intervals in other cases.

2) CROSSCOR as was noted in section 3.3.5, estimation of cross correlation coefficients of power output fluctuations at a pair of sites can be calculated from discrete Fourier series for the two sites, by a separate filtering and inverse transformation for each lead time. It is far simpler and quicker to estimate these coefficients by direct application of equation 3.3.32 above, especially where output for a limited number of lead times is required. The drawback of this is that for some site pairs the resultant coefficients become rather unstable for lead times around 24 hours. This may be due to the effects of sharp diurnal peaks at some of the wind sites. CROSSCOR evaluates the coefficients for lead times in the range 1 hour to 24 hours, at intervals of 1 hour. Output for each site pair is graphical and tabular. Coefficients for each pair of sites are appended to a disc file for later analysis. Output for a single site pair is shown below.

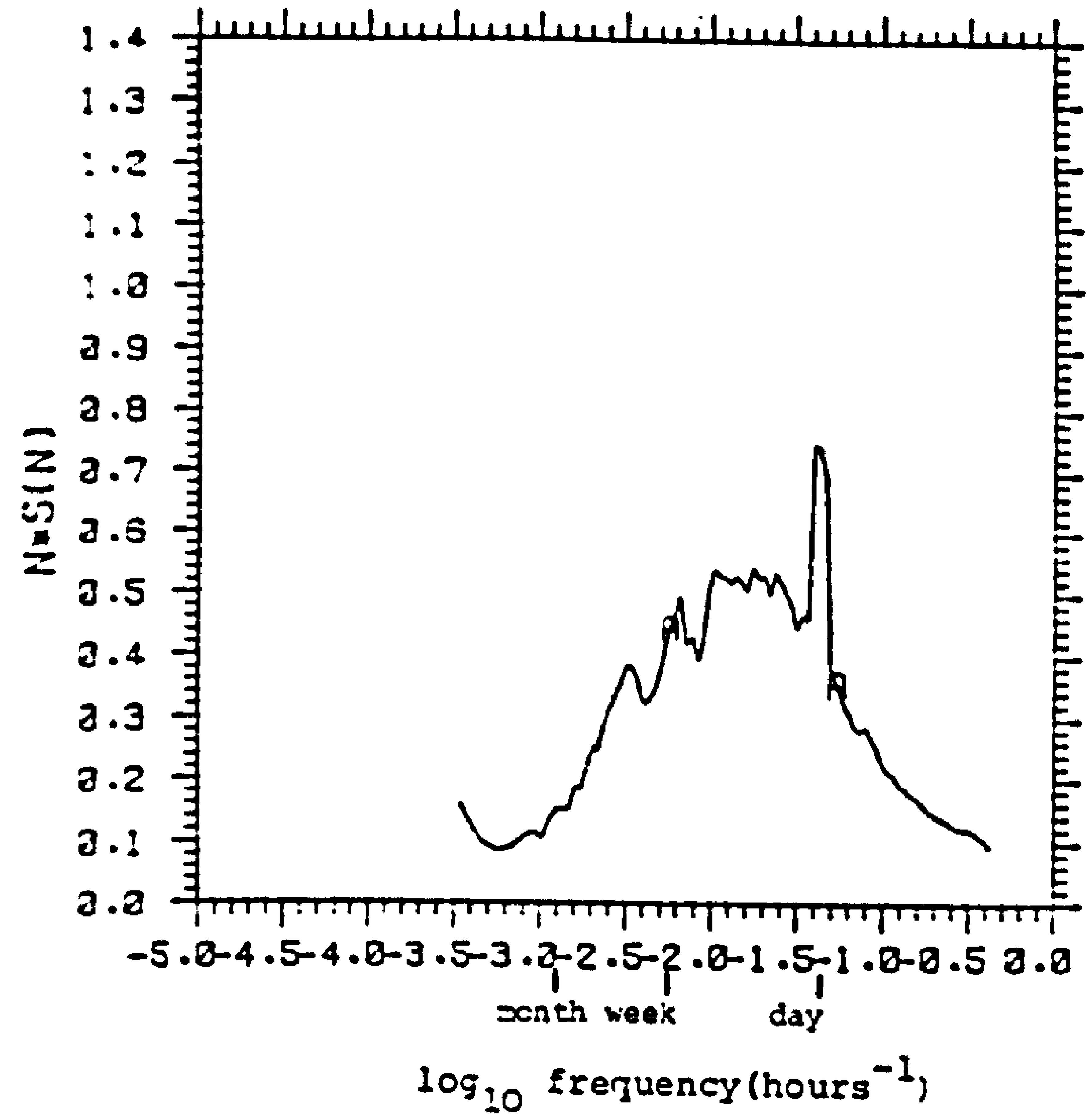


Fig 3.4.1 Wind power spectrum over frequency range 3 yr^{-1} to 0.5 hr^{-1}.

Note the large spike at the diurnal frequency.

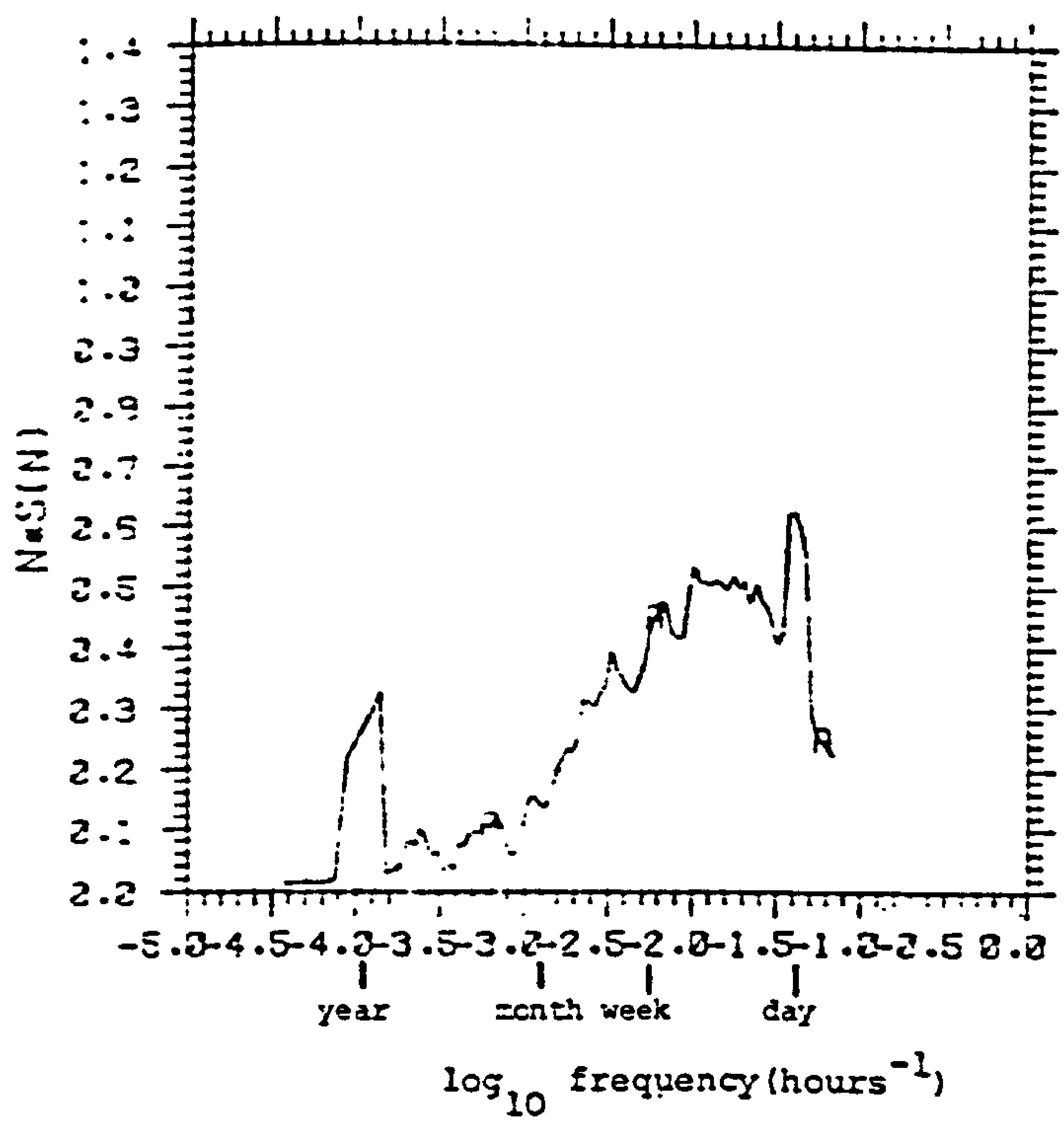


Fig 3.4.2 Wind power spectrum over frequency range 1/3 yr^{-1} to 1/12 hr^{-1}.

Note the diurnal and annual spikes. The resolution and accuracy of the annual spike is poor due to the short duration of the wind dataset used.

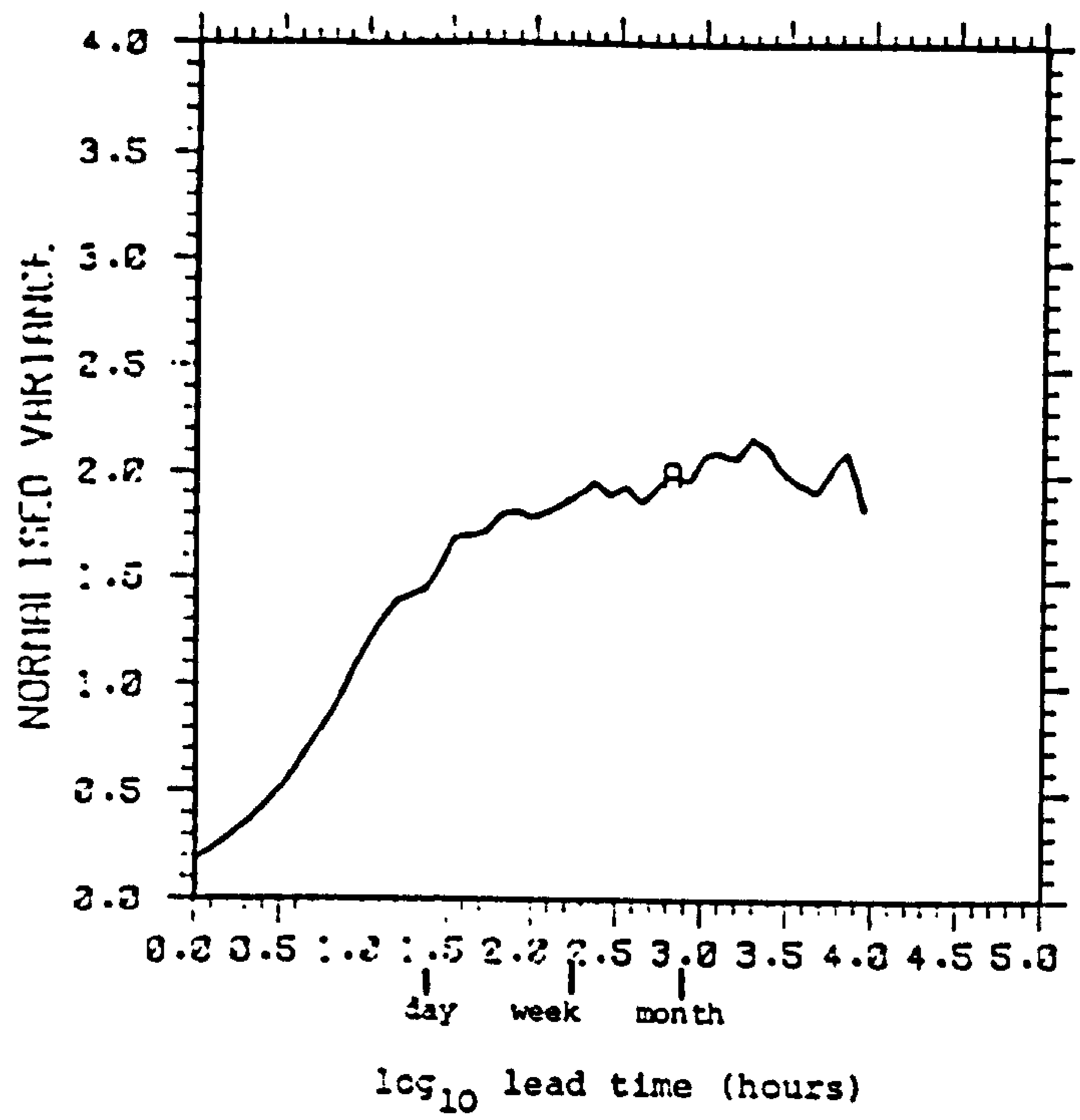


Fig 3.4.3 Variance of changes in wind power as a function of lead time.

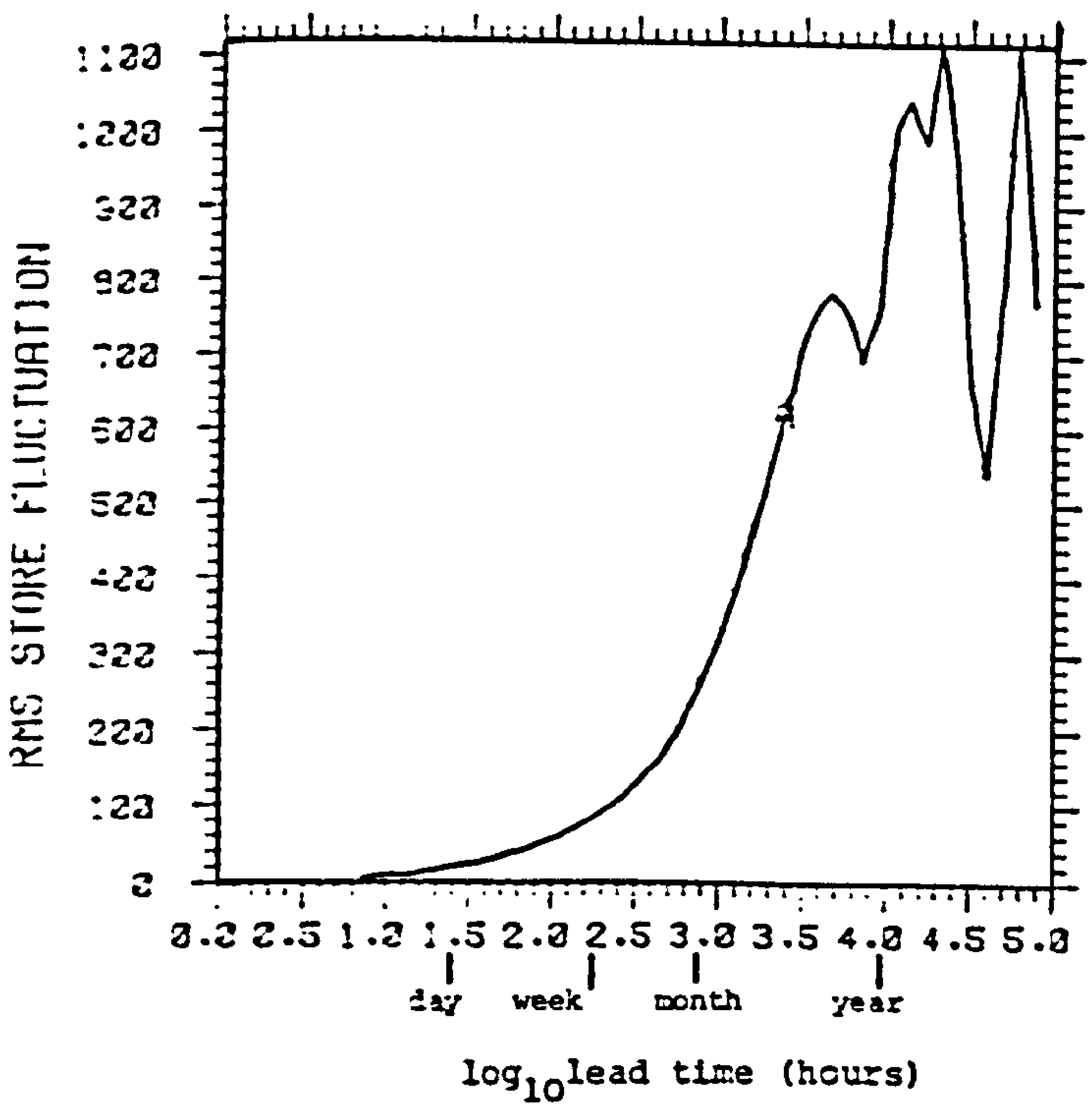


Fig 3.4.4 Fluctuation in the level of an infinite energy store (rms) as a function of lead time.

Note that values for lead times greater than 1 year are not statistically significant.

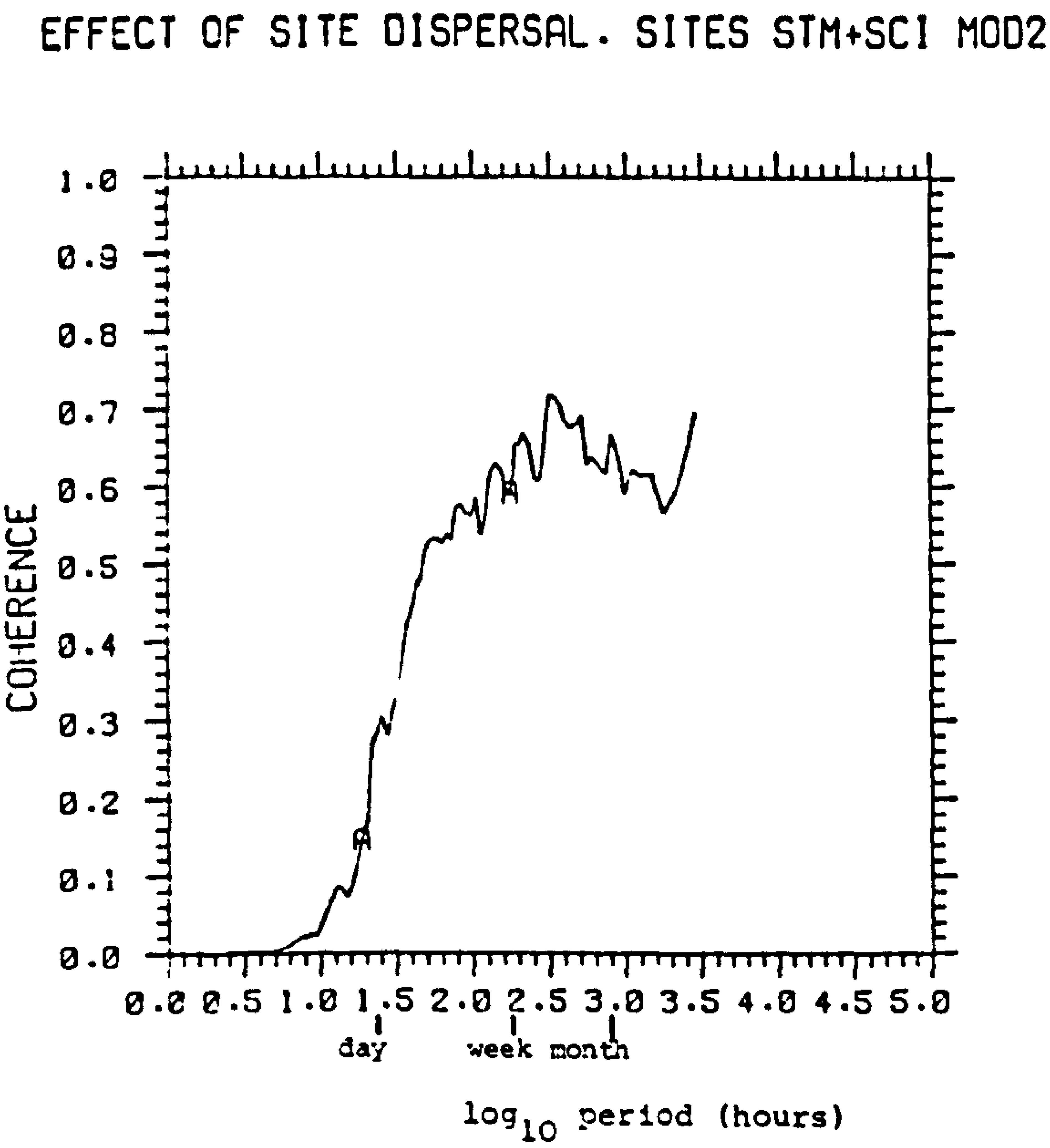


Fig 3.4.5 Coherence function for wind power from turbines at St. Mawgan and Scilly. The plot is against the logarithm of the period, rather than frequency, and runs from 2 hours to 1/3 years.

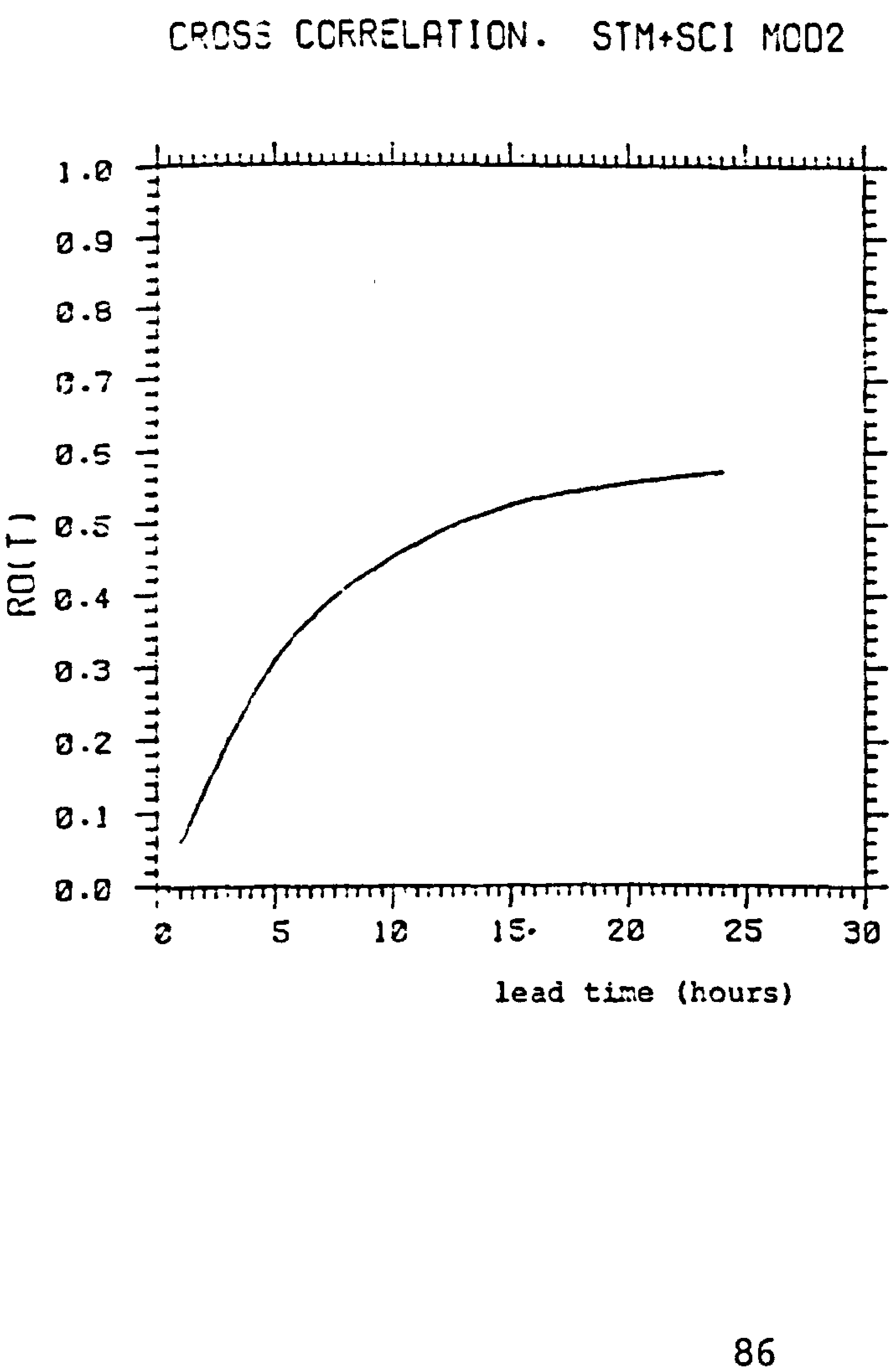


Fig 3.4.6 Cross correlation coefficient of changes in turbine power, for sites St. Mawgan and Scilly, for lead times 1 to 24 hours.

3.5 RESULTS OF ANALYSIS

Results of the analysis of wind data, based on the programs discussed in section 3.4 are presented here. Single site statistics are presented first, followed by a more lengthy section on site pair statistics.

3.5.1 SINGLE SITE ANALYSIS RESULTS

The first 30 figures in appendix 1 show normalised spectra for the 30 wind sites listed in section 3.2. The most important point to make regarding these spectra is that the general shapes of all of the spectra are very similar. The differences in most cases are in the total area under the curve, the behaviour of the curve at low frequencies, and the height relative to the rest of the curve of the diurnal spike. The variation in the variance of power at different wind sites will be dealt with in more detail in chapter 4, but run 25 and run 29 illustrate the range. The behaviour of the low frequency end of the spectrum depends mainly on the summer to winter variation in wind speed, the variation of which is again discussed in chapter 4. Variations in the magnitude of diurnal fluctuations are discussed in more detail at the end of this chapter and in chapter 4.

Statistics of changes in power.

The next thirty figures show the random walk characteristics for these thirty sites. These figures emphasise the similarity between the behaviour of wind power at the different sites. A surprisingly good general fit to these curves is provided by the following equation:

1. $\sigma(t)^2 = 2\,\sigma_0^2\,[1 - \exp(-t/12)]$

where $\sigma(t)$ is random walk characteristic of the site, and σ_0 is the normalised standard deviation of power at the site, evaluated over the full 9 years of available data. This equation was suggested obliquely by Farmer et al (Farmer et al 1980). The constant, 2 in the above has been fitted by least squares regression of data for 25 sites, for lead times t from 1 to 16 hours inclusive. The estimate of this coefficient returned by the regression program was 1.97 ± 0.012, with a correlation coefficient of 0.96.

A useful simplification of the above equation can be made by noting that for a specific power of 1.0, $\sigma_0 = P$. If the equation is non-dimensionalised by the mean power, and the above approximation is made, then

2. $b(t)^2 = 2.0\,[1 - \exp(-t/12)]$

where b(t) is the specific standard deviation of changes in wind power at a lead time of t hours. Equation 3.5.1 breaks down quite badly for t greater than about 18 hours, but in the above range 1 to 16 hours provides a useful rule of thumb for the behaviour of typical wind sites. In any case this is the region of greatest importance to operators of grid systems, since it determines the requirements for operating reserve. A graph of equation 3.5.2 is shown in fig 3.5.1 below.

It was noted in section 3.3 that the mean variance of power over periods of duration t, $\langle\sigma^2{}_t\rangle$, is one half of the variance of changes in power over the same period, to a good approximation. An independent check has been done on this, and the result confirms that

$$\langle \sigma^2_t \rangle = \sigma_0^2 \; [1 - \exp(-t/12)]$$

to within 10%.

3.5.2 SITE PAIR ANALYSIS RESULTS

Site pair analysis was carried out on two sets of pairs of sites. 13 included the common site St Mawgan, and 13 included the common site Gorleston. The locations of the sites used is shown in section 3.2. The site pairs are listed below:

Common site STM		separation (km)
1.	SCI	107
2.	VAL	314
3.	DNG	417
4.	WAT	451
5.	GOR	517
6.	LZA	55
7.	MNT	66
8.	PRT	151
9.	MHN	134
10.	ABP	190
11.	BDN	238
12.	CON	441
13.	WIK	893

Common site GOR		separation (km)
14.	WAT	69
15.	CAR	152
16.	GWK	203
17.	DNG	190
18.	CON	138
19.	SMR	245
20.	DRH	338
21.	LCH	510
22.	FGH	610
23.	PWK	517
24.	WIK	713
25.	STY	800
26.	WFR	503

Coherence functions.

I have found coherence function data rather difficult to summarise, without simply stating the obvious. Qualitatively there are a number of points to be made.

1) coherence function estimates are low for periods $T \lesssim 10$ hours for all sites examined. Wind power fluctuations at periods in this range are therefore uncorrelated, even at site separations as small as 55 km. This is illustrated by fig 3.5.2, which is the coherence function for St Mawgan and Lizard.

2) diurnal fluctuations stand out in nearly all pairs of sites, as more highly coherent than frequencies immediately above or below. The diurnal spike in the coherence function becomes increasingly important as the site separation increases, and the coherence at other frequencies falls off. This is well illustrated by the coherence plots for Gorleston and Wick (separation 713 km) and Gorleston and Wattisham (separation 69 km) see figs 3.5.3 and 3.5.4. The effects of this on cross correlation coefficients will be discussed later.

The general shape of the coherence function can be expressed in a number of ways. The clearest is to look at the variation in the maximum value of the coherence plot, excluding the diurnal spike, and the high frequency edge of the annual spike (which is just visible in the plots already referred to above).

Fig 3.5.5 shows a plot of the maximum value of the coherence function versus site separation for the 26 site pairs examined. The points are quite scattered, but fall quite clearly into 2 groups depending on the "common site". The group based on Gorleston suggest quite strongly an exponential relationship between γ^2_{max} and site separation of the form

1. $\gamma^2_{max}(x) = \exp(-x/L)$

Curves of this form have been fitted by least squares regression to data for both sets of site pairs. The results are tabulated below:

Common site	L(km)	r^2
STM	465$\pm$9%	0.75
GOR	254$\pm$4%	0.93

The characteristic lengths for the two sets of site pairs are significantly different. The fit for the second set of pairs is particularly good. The curves resulting from the above analysis are superimposed on the data in fig 3.5.5.

Perhaps the most important point to make about the coherence function plots, is that they do not suggest a simple relationship between time and space variables in the ranges examined (1 hr < T < 1 yr and 50 km < x 1000 km) for wind power data. By analogy with the Taylor hypothesis for instance, one might perhaps expect that coherence functions could be expressed in non-dimensional coordinates of vT/x , where v is a constant characteristic velocity for weather systems in the Northern hemisphere temperate zone. Evidence for this would be that coherence plots for different site separations would differ by simple left/right translation, when plotted against the log of the frequency. This is simply not the case (see eg. figs 3.5.3 and 3.5.4 above). This behaviour is partly, but by no means wholly caused by diurnal fluctuations, which are coherent over great distances.

Cross correlation coefficients.

Cross correlation coefficients are of more direct significance than coherence functions, and because of their relationship to coherence functions and spectra, are considerably smoother than either, when expressed as a function of lead time. There are actually two distinct correlation coefficients of interest here. One is the cross correlation coefficient of changes in wind power over a lead time of t hours, which is pertinent to short term control problems of the utility. The second is the cross correlation coefficient of instantaneous power outputs,[1] which is important in assessing capacity credit.

1. Cross correlation coefficients of short term fluctuations.

Fig 3.5.6 shows a plot of correlation coefficients for 25 of the 26 site pairs listed above, in the range 1 hr $< t <$ 6 hrs. Qualitatively this figure demonstates two points - first the decline in correlation with increased site separation - and second the increase of correlation with increases in t. These points are intuitively obvious.

Unlike the coherence function data, the correlation data does not separate neatly into site pairs based on St Mawgan

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[1] This is mathematically identical to the cross correlation coefficient of power fluctuations at infinite lead time.

and site pairs based on Gorleston, though there is definitely a tendency for the latter site pairs to have higher coefficients. This can be seen more clearly in figures 3.5.7, 3.5.8, 3.5.9, and 3.5.10 which show correlation coefficients as a function of site separation only.

There are clearly variations in cross correlation coefficients which are not explained by lead time and site separation. Three interesting points in the above figures are those for STM/LZA, STM/MNT and GOR/WAT, at separations of 55 66 and 69 km. At a lead time of 1 hour there is almost exactly a factor of 2 between the highest and lowest of these pairs. The data for STM/LZA is consistently lower than for these other sites, as can be seen in fig 3.5.6 above. Simple explanations, such as compass bearings, are not satisfactory. My conclusion is that generalised statements about cross correlation coefficients based on lead time and site separation, may typically be in error for any particular site pair by a factor of 1.5 .

While not claiming that such a process does more than make it easier to perceive qualitatively the behaviour of the data, I have attempted to fit a general surface to the data in fig 3.5.6. A variety of surface types were tested:

a) $r(x,t) = \exp[-x/L(T)]$

b) $r(x,t) = \exp[-x/L(t)]^a$

c) $r(x,t) = [(x/L(t)) + 1]^{-a}$

The first two of these are not particularly helpful. The third is capable of fitting the data reasonably well with $a = 0.6$, and gives a graph of $L(t)$ vs t which passes reasonably close to the origin (ideally in all three of the above one would like the length scale to tend to zero as t tends to zero). The fits for each value of t were done by eye. The equation which results from this surface fitting is:

2. $$r(x,t) = [(x/2.6t-1.6) + 1]^{-0.6}$$

A sketch of this surface is shown in fig 3.5.11. Perusal of the cross correlation plots in appendix 1, reveals non-monotonic behaviour in the range 12 to 24 hours for many of the site pairs. This effect increases with site separation, and is particularly evident in the plots for STM/WIK and GOR/STY. The effect is what one would predict from knowledge of the behaviour of coherence functions as site separation is increased. The extreme case is when the power outputs at two sites are only coherent at the diurnal frequency. One can then write the power outputs

3. $$x(t) = a \sin 2\pi t/24 + x'(t)$$

$$y(t) = b \sin 2\pi t/24 + y'(t)$$

where x' and y' are uncorrelated. In this case the covariance of fluctuations at the two sites becomes:

4. $$\delta_{xy} = ab(1 - \cos 2\pi t/24)$$

and for $t = 24$ hours, δ_{xy} and hence r are equal to zero. This is best illustrated by the site pair GOR/STY.

The conclusion from this is that for wind sites separated by more than about 400 km, correlation coefficients estimated from wind speed data measured at around 10 metres are likely to be highly misleading. This is because diurnal fluctuations in wind speed are a strong function of measurement height (among other factors). This is discussed in more detail in chapter 4. The problem of diurnal fluctuations in wind power output crops up again and again. The real answer to this problem is to collect data at heights equivalent to those where wind turbines will ultimately operate. Multi-megawatt machines will almost certainly operate at hub heights of the order of 100 m, where 10 m data on diurnal fluctuations is probably irrelevant.

2. Cross correlation coefficients of instantaneous power.

Fig 3.5.12 shows a plot of cross correlation coefficient of instantaneous power for the pairs of sites discussed above. This figure bears a close similarity to the plot of γ^2_{max} in fig 3.5.5. The plot is noisy, but quite linear over the range of site separations considered. For obvious reasons one would expect the plot to sweep sharply upward for site separations less than 50 km, approaching 1.0 as the separation tends to 0. This linearity is useful, in that it makes the relationship between the mean site separation and the mean cross correlation coefficient quite simple.

The cross correlation coefficient of instantaneous powers is less affected by the diurnal spike than the correlation coefficient of power changes over short periods, since it is related to integrals of spectral density over the whole range of frequency.

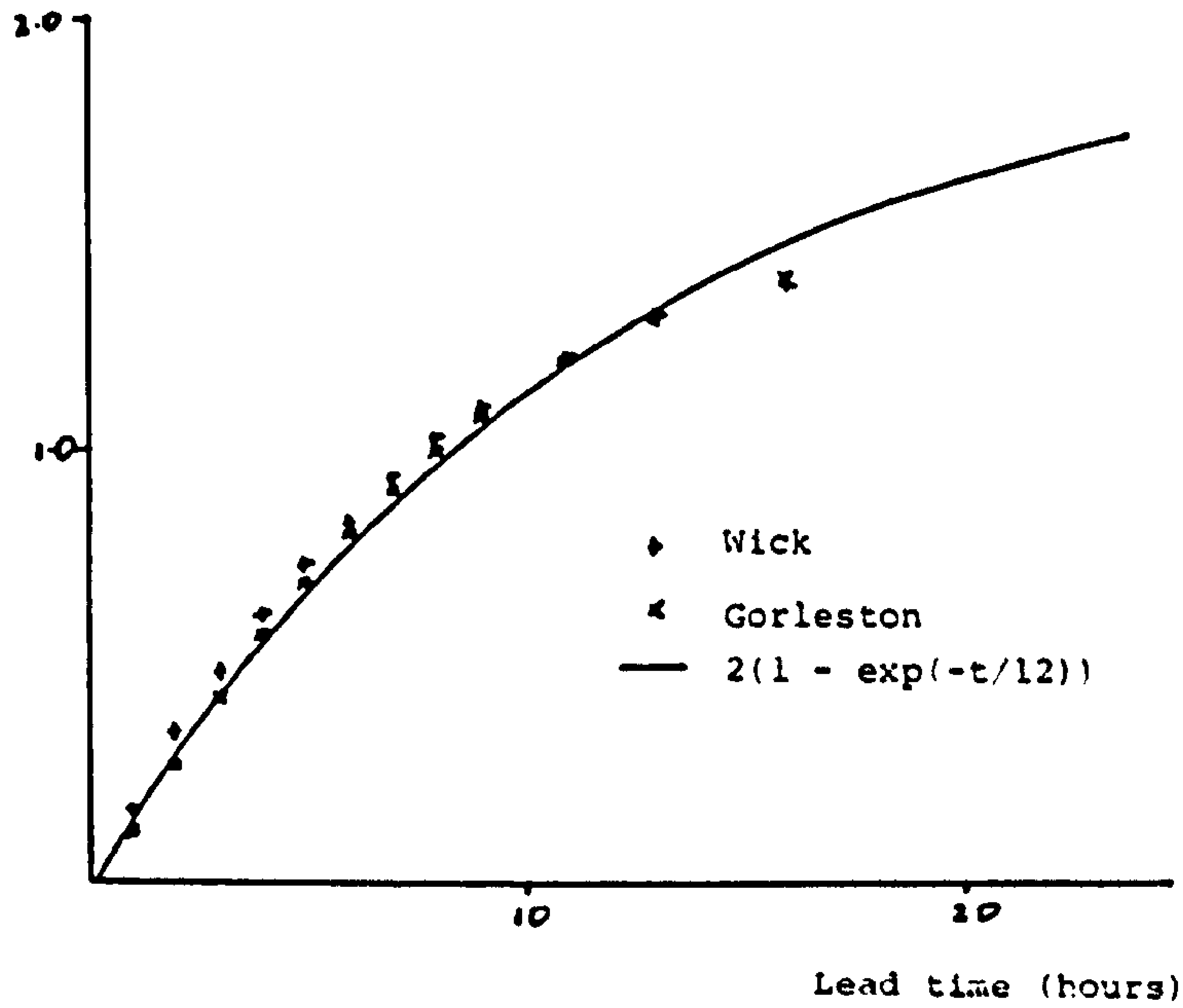


Fig 3.5.1 Variance of changes in wind power: fitted curve and representative points.

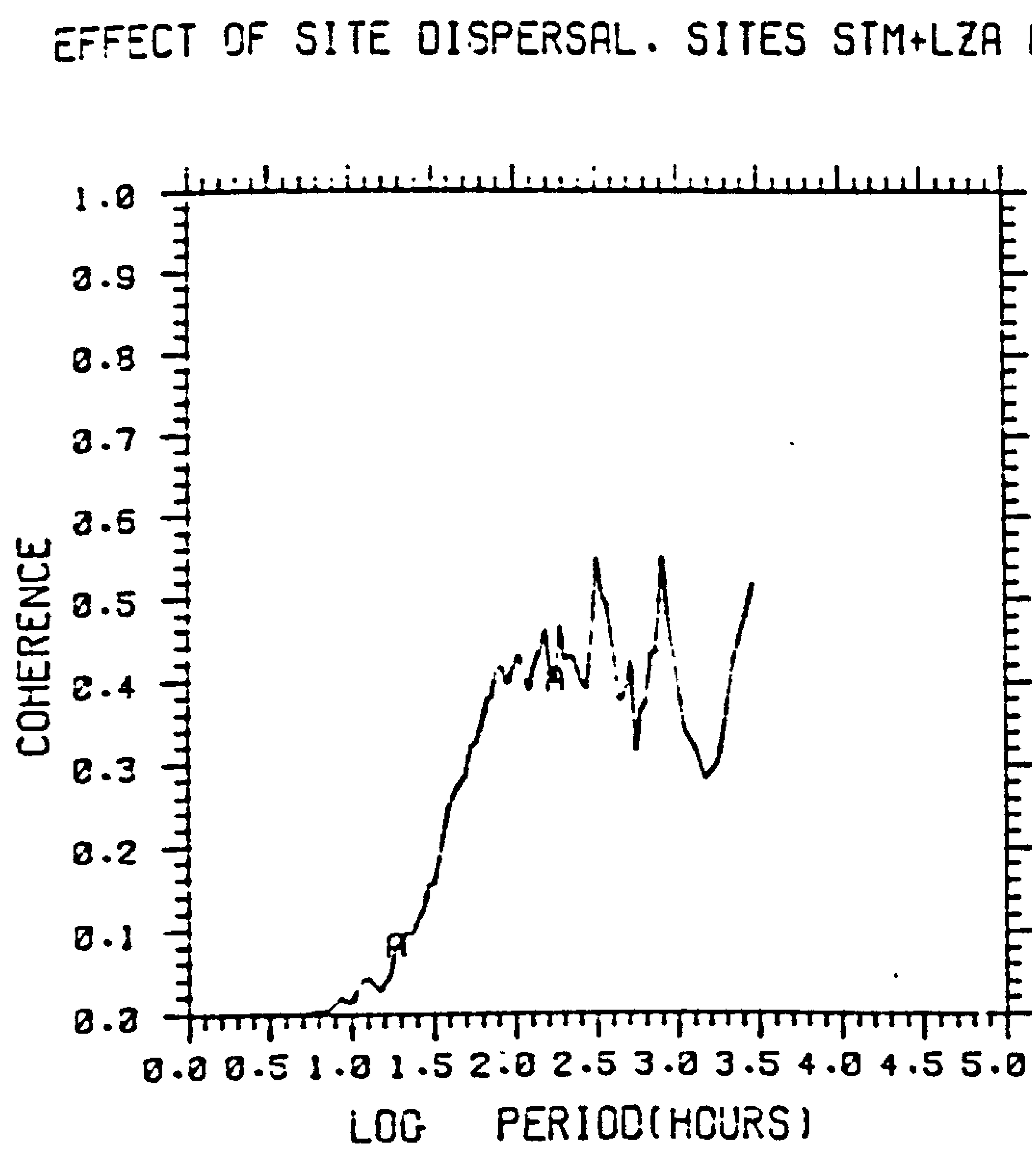


Fig 3.5.2 Coherence function for wind power from turbines at St. Mawgan and Lizard.

EFFECT OF SITE DISPERSAL. SITES GOR+WIK MOD2

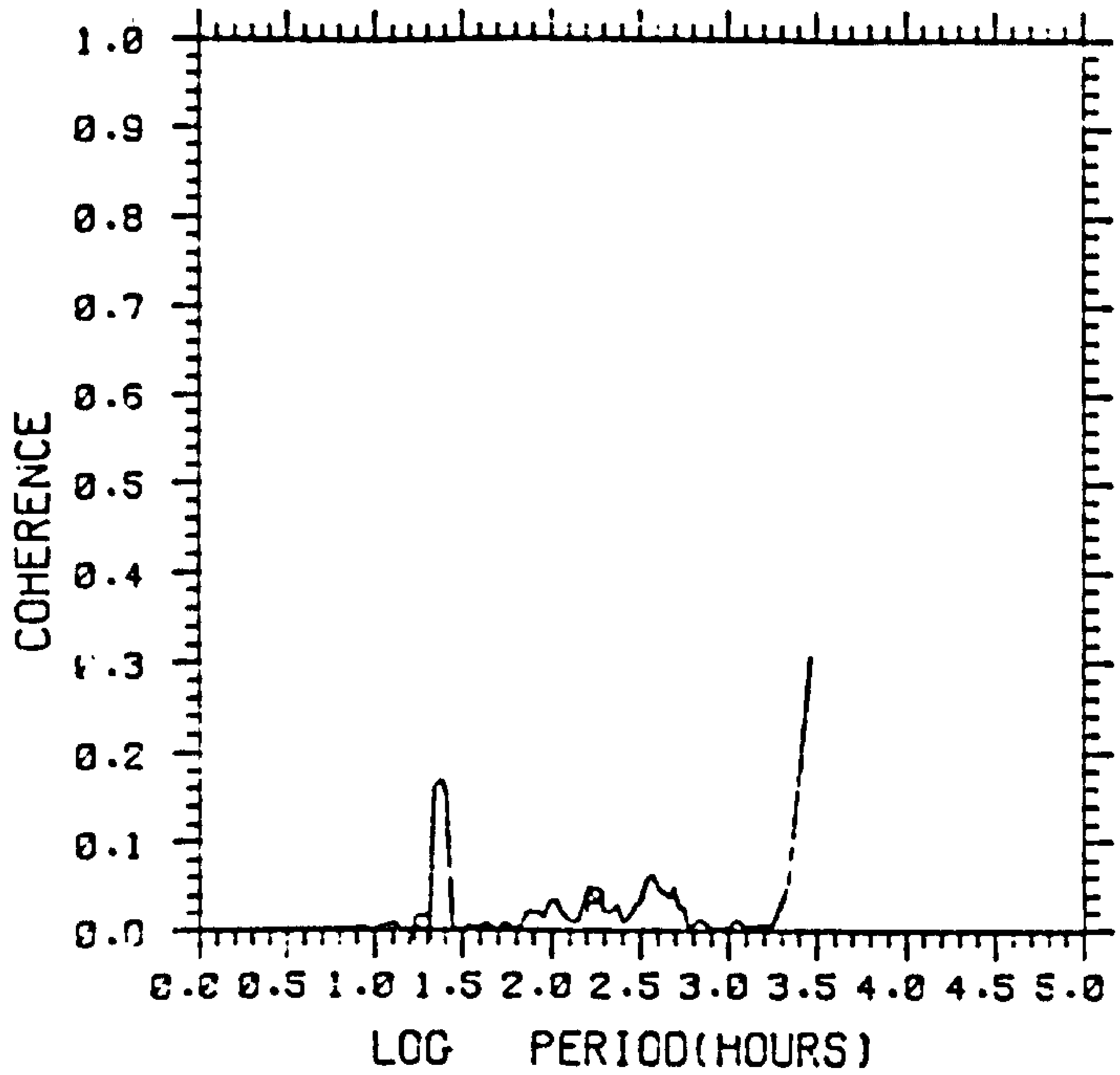


Fig 3.5.3 Coherence function for wind power from turbines at Gorleston and Wick.

EFFECT OF SITE DISPERSAL. SITES GOR+WAT MOD2

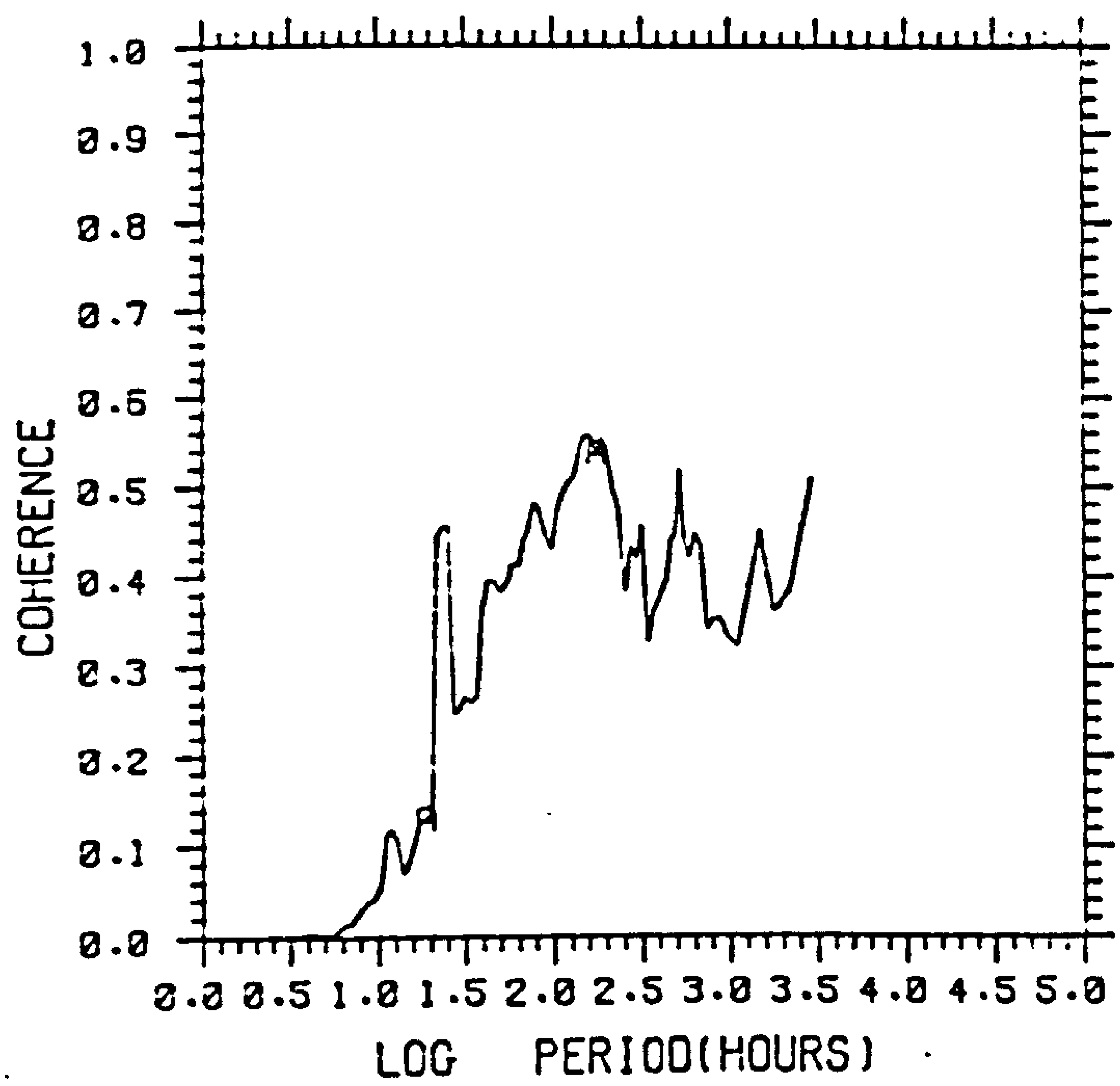


Fig 3.5.4 Coherence function for wind power from turbines at Gorleston and Wattisham.

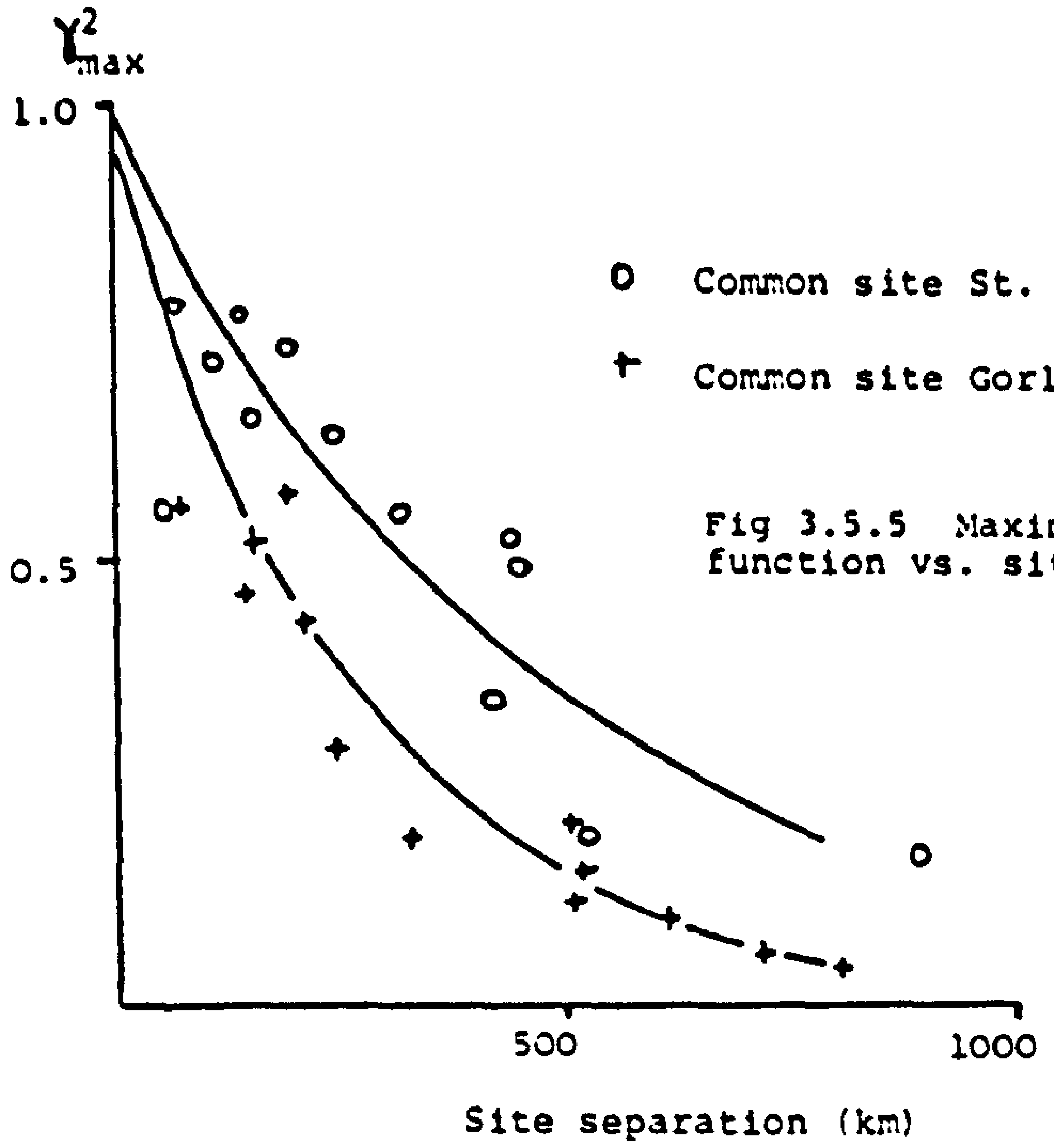


Fig 3.5.5 Maximum value of coherence function vs. site separation.

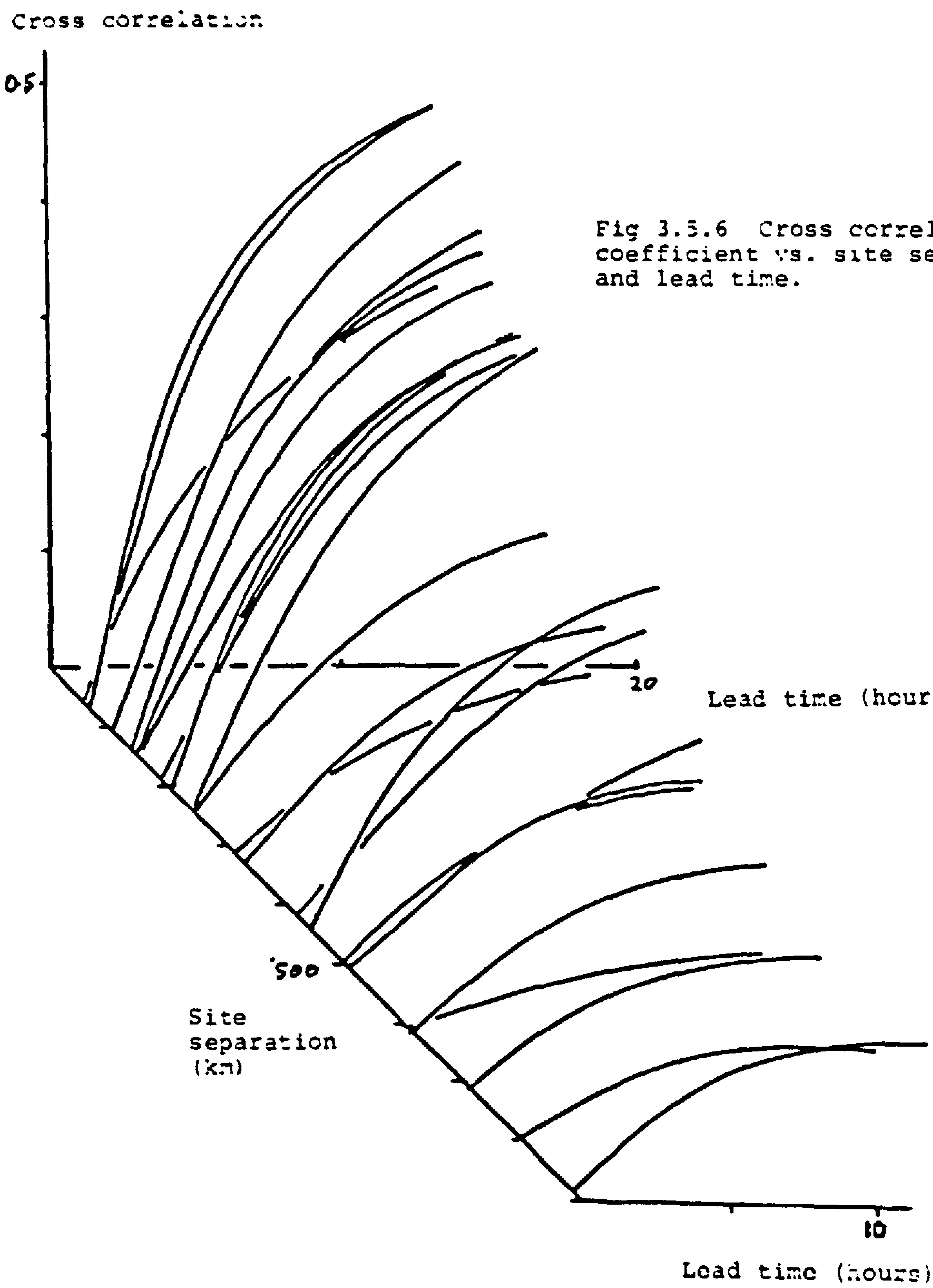


Fig 3.5.6 Cross correlation coefficient vs. site separation and lead time.

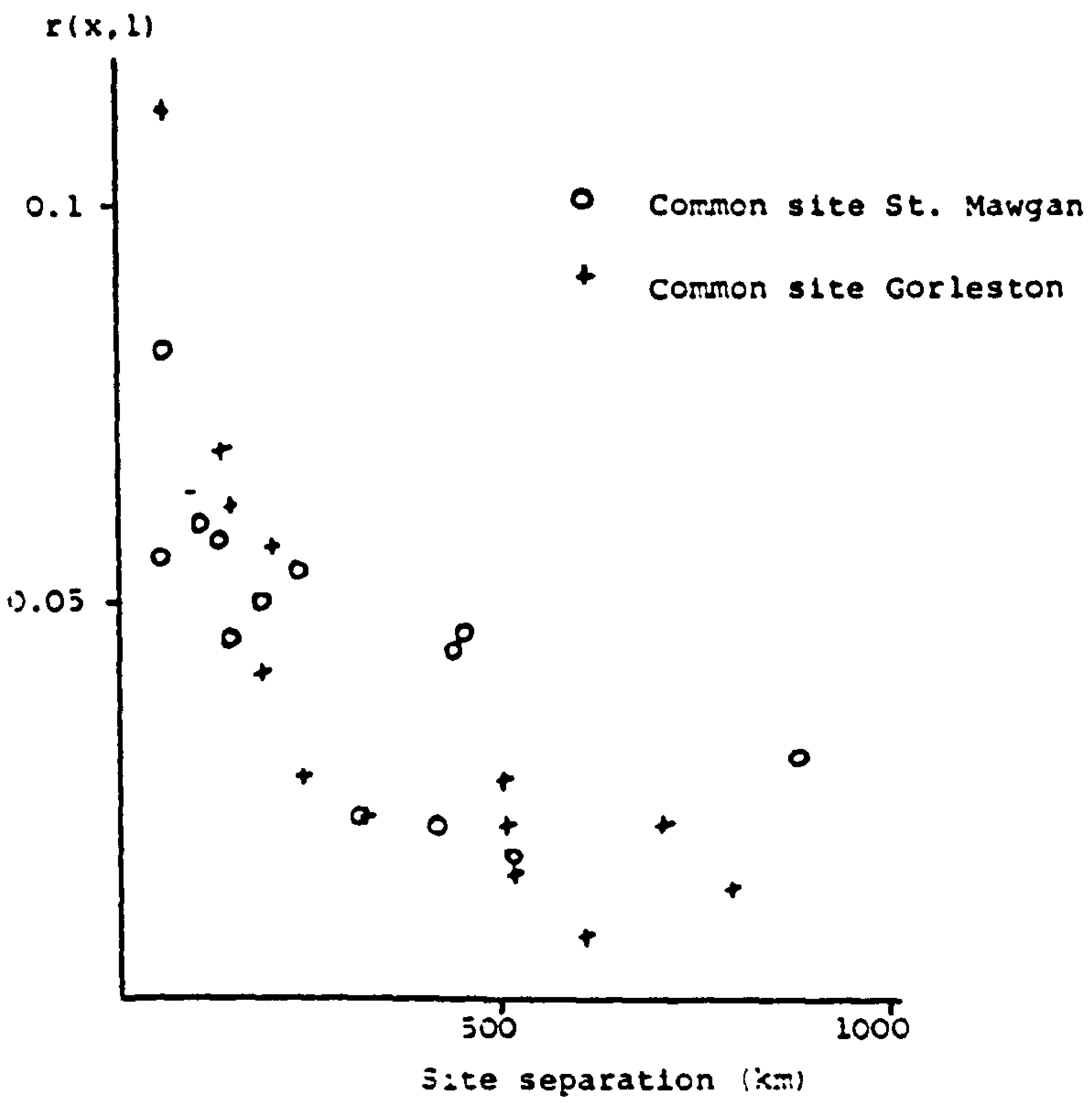


Fig 3.5.7 Cross correlation coefficient at a lead time of 1 hour vs. site separation.

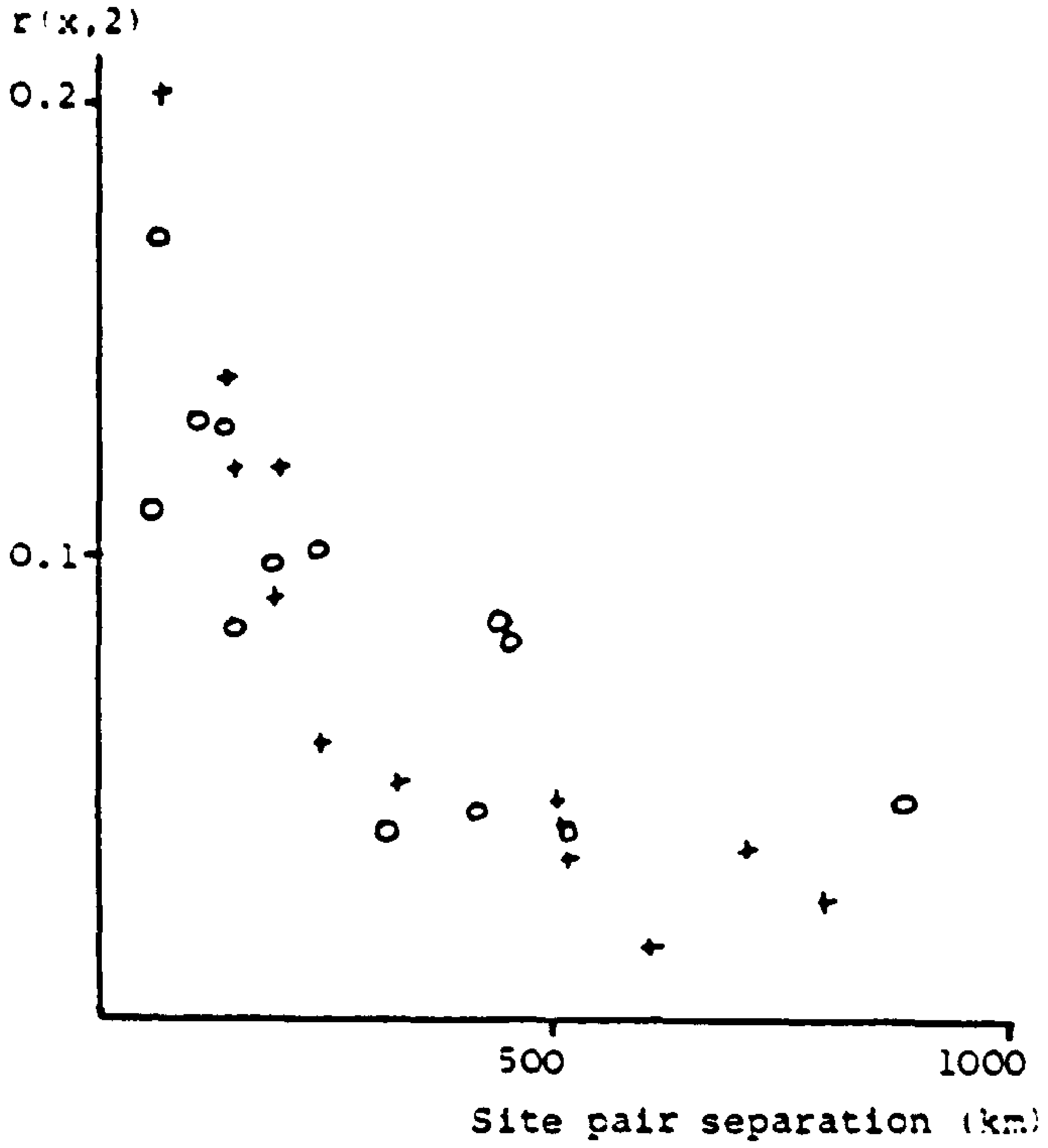


Fig 3.5.8 Cross correlation coefficient at lead time of 2 hours vs. site separation.

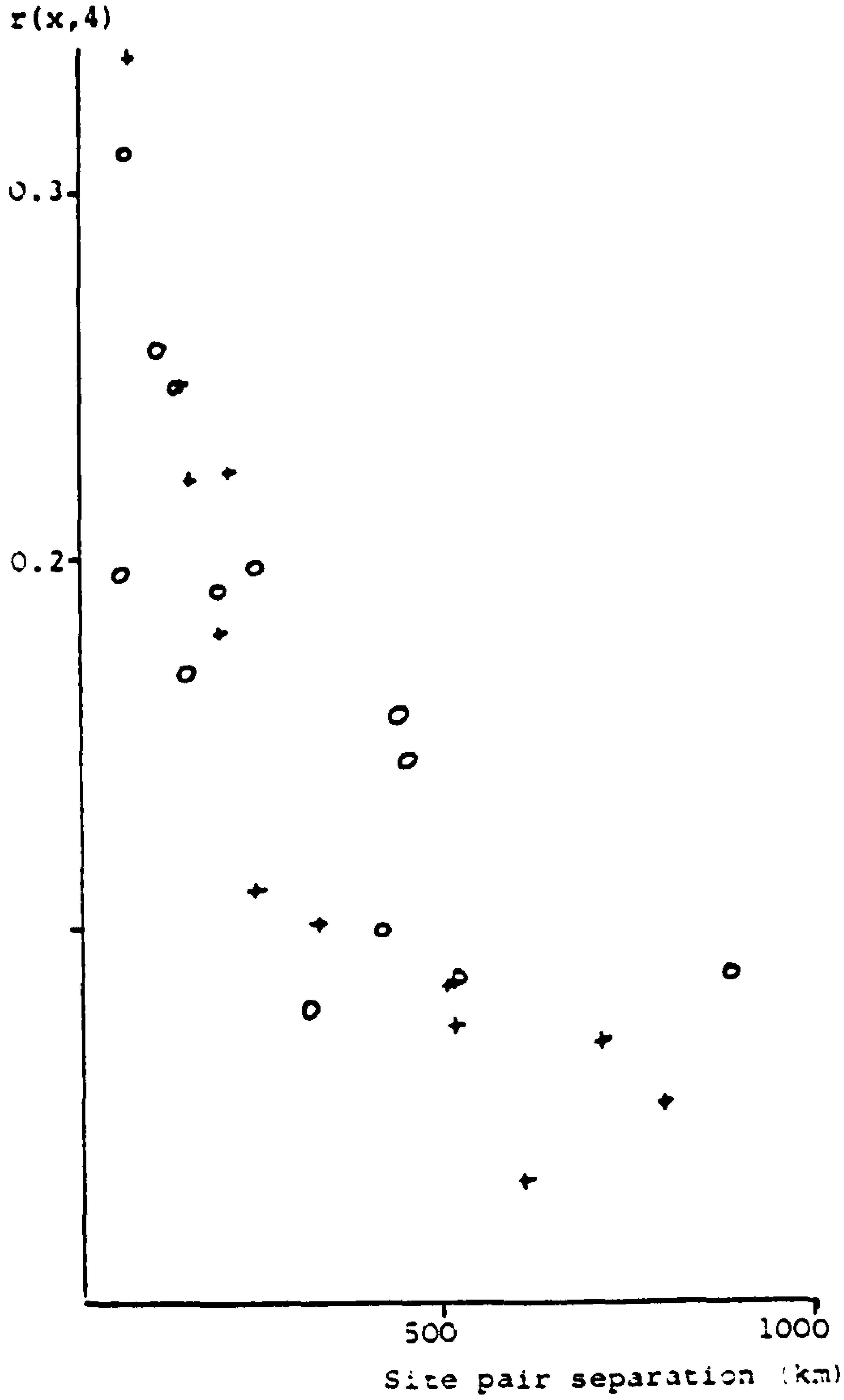


Fig 3.5.9 Cross correlation coefficient at lead time of 4 hours vs. site separation.

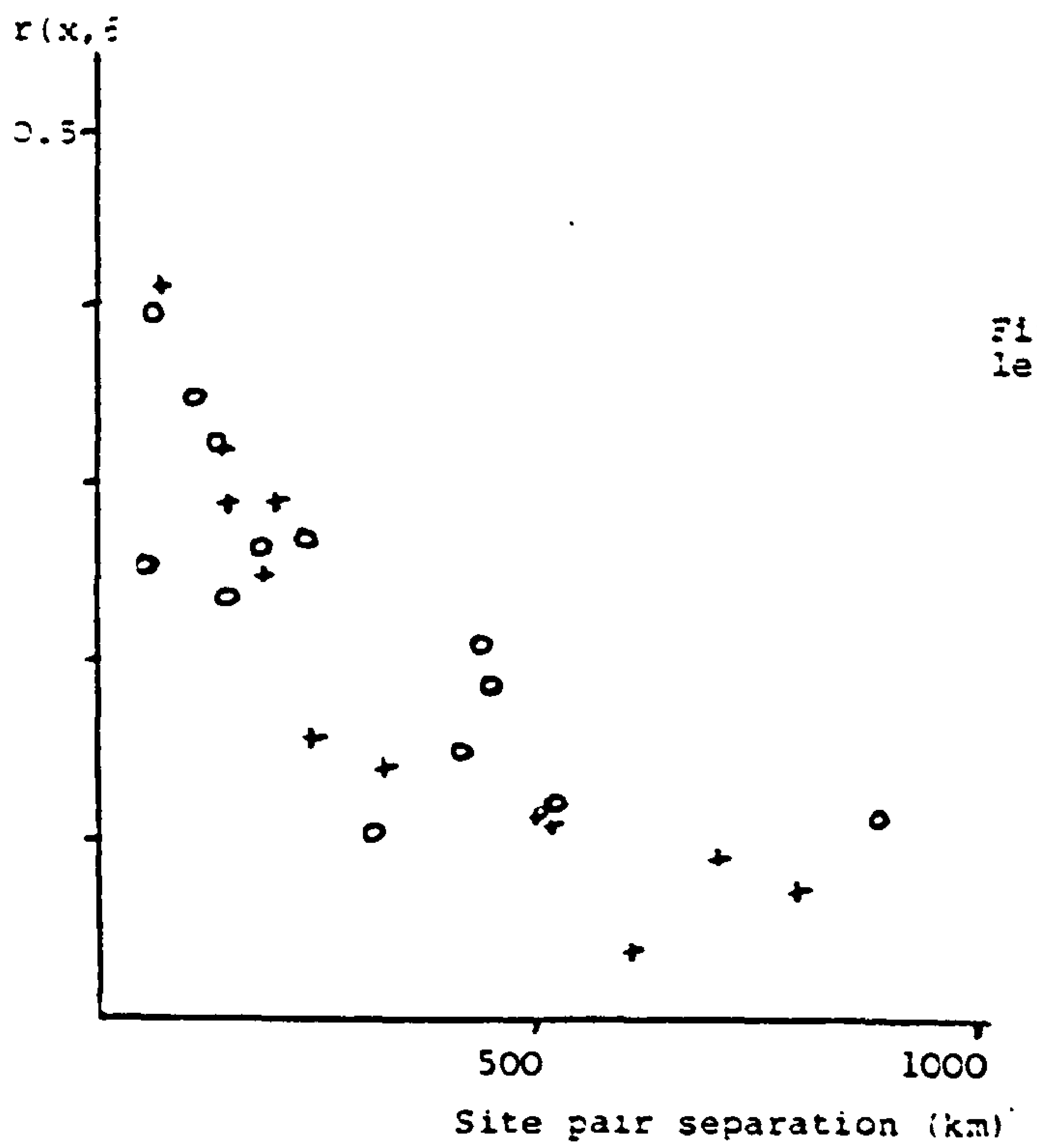


Fig 3.5.10 Cross correlation coefficient at lead time of 6 hours vs. site separation.

o Common site St. Mawgan

+ Common site Gorleston

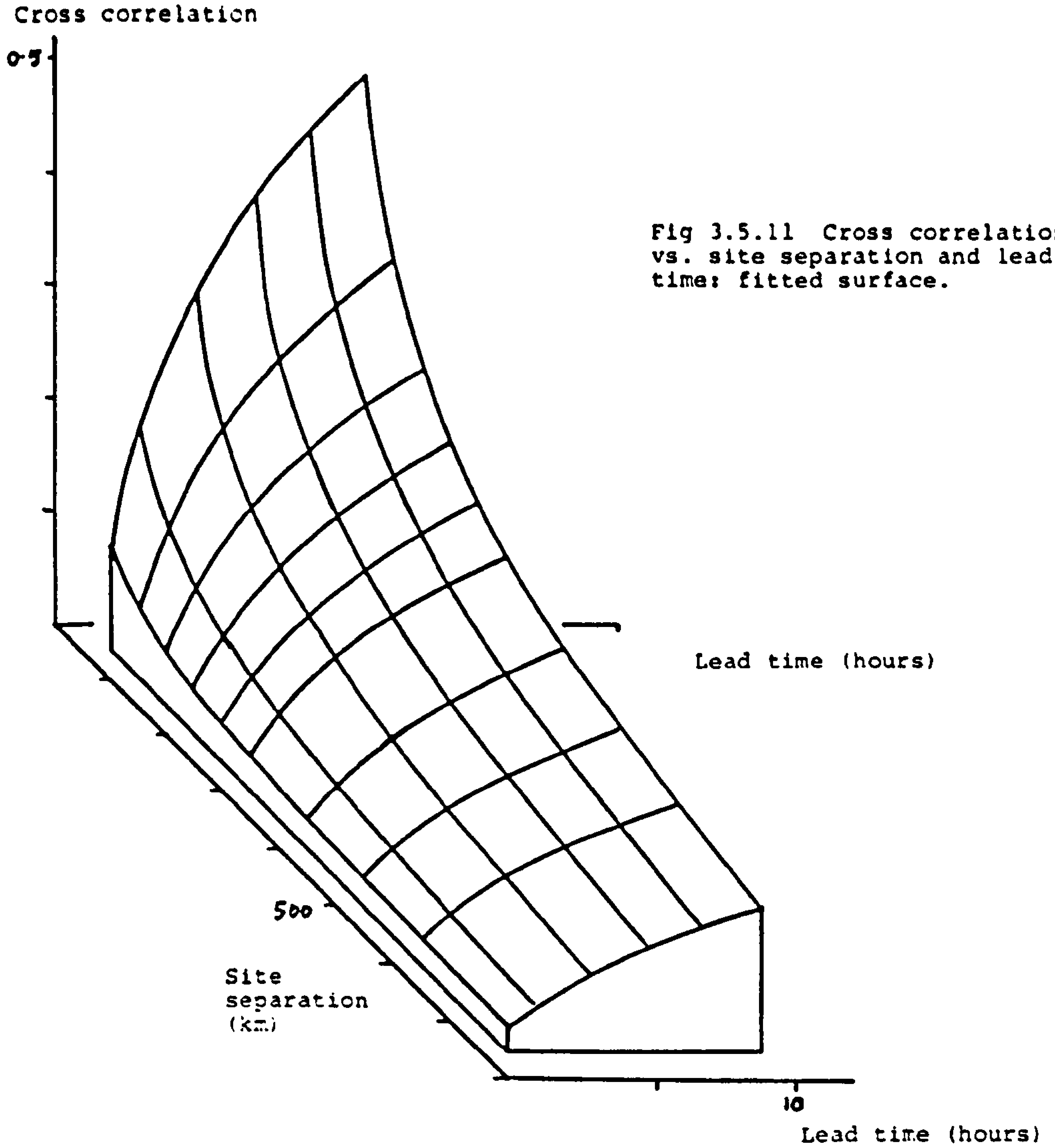


Fig 3.5.11 Cross correlation vs. site separation and lead time: fitted surface.

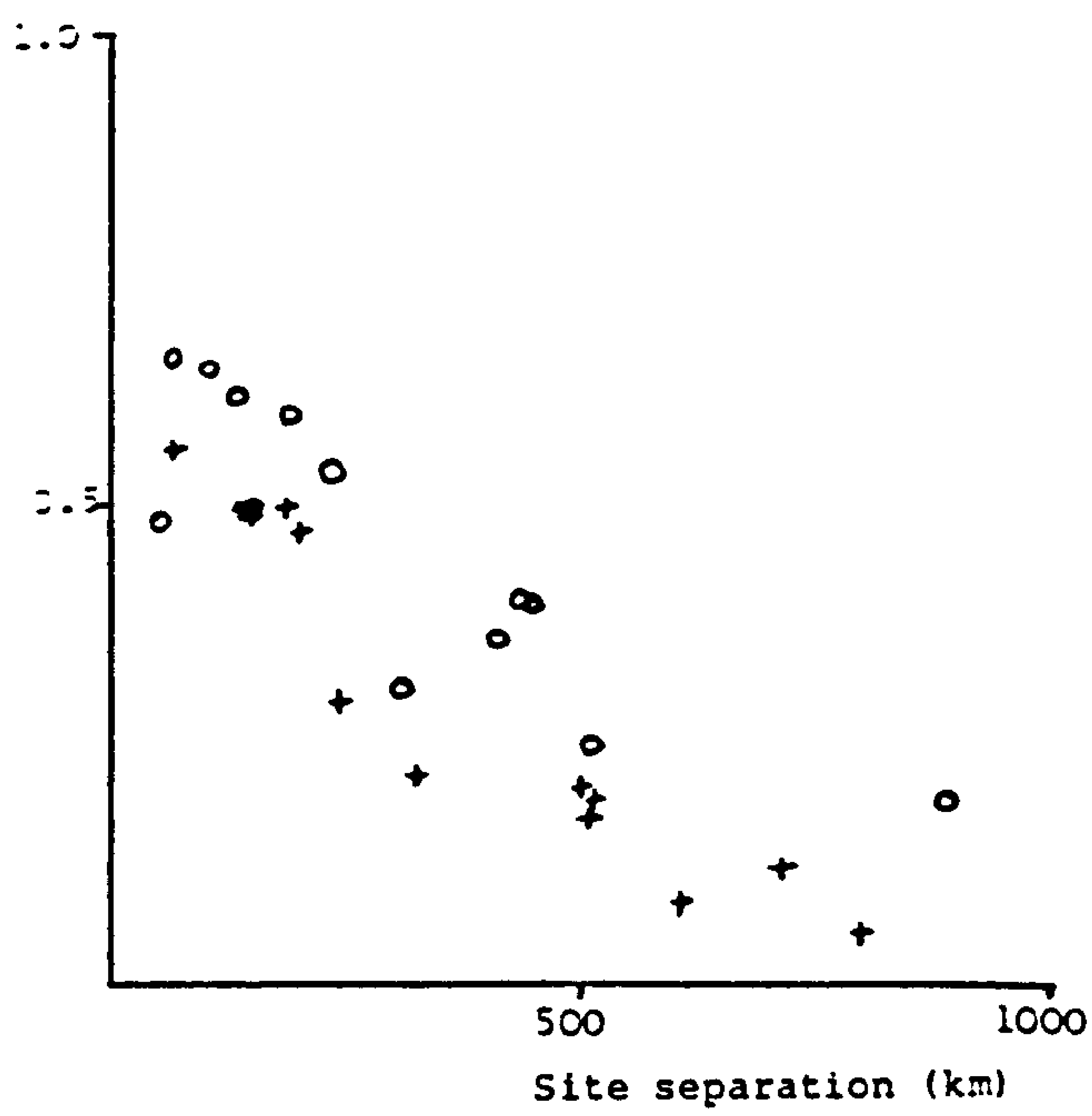


Fig 3.5.12 Cross correlation coefficient of power vs. site separation.

o Common site St. Mawgan

+ Common site Gorleston

4. CAPACITY CREDIT

4.1 INTRODUCTION

Capacity credit may be defined as the amount of extra peak demand a utility system can supply due to the addition of new plant of a particular type. Capacity credit can be expressed either as a dimensionless "peak load factor" - xGW per GW of installed capacity - or as yGW. One can also refer to marginal capacity credit - the rate of change of the peak capacity that the system is able to meet, with installed capacity of a particular type. Having determined capacity credit for a given type of plant one may wish to determine the economic value of this credit. The problems associated with doing this are not dealt with in this section.

The simplest approach to determining capacity credit is to estimate the power output from the new source of electricity that is available for 100% of the time in the months when peak demand is likely to occur. This was the approach used by Vimukta (Vimukta 1977) , admittedly with reservations, to estimate the technical effects of installing wave power generators in the UK, and has been used frequently in the past by other authors (Anderson et al. 1978, Allen and Bird 1977) sometimes implicitly. The result has been the generation of a great deal of confusion. (cf. correspondence between Anderson et al. and Leicester et al. in Nature 279 28 1979).

The work of Anderson et al. is instructive in illustrating the lengths to which it is necessary to go in order to demonstrate non-zero capacity credit within this simple framework. These authors looked at the long term reliability of a domestic heating system based on a dispersed array of wind turbines operating through heat stores, and backed up by the CEGB grid. This system was found to have a very high reliability, with the effective availability never falling below 50% over a period of 17 years. The main fault with this work was the highly unrealistic model of the house used in the study. This model diregarded fortuitous heat gains completely and assumed a thermal time constant of 10 hours. the level of insulation which it is currently economic to install in an electrically heated house in the UK, would yield a thermal time constant of the order of a week, and little or no space heating load (ie. the heat demand would be dominated by the hot water load). Had these factors been taken into account, the numerical results would undoubtedly have been different, but in which direction it is hard to say. The point here, is that where arguments are advanced to claim capacity credit for "non-firm" sources of electricity on the basis of consideration of a particular class of demand, the question is immediately begged of whetner still greater capacity savings could be obtained simply by re-optimising the demand and disregarding the new source of electricity completely.

Until 1980, much work in the area of assessing the value of wind generated electricity was devoted to determining the extent to which the statistics of wind power output resemble those of certain classes of conventional power station. An excellent study of this type was reported by Sørensen (Sørensen 1978). This dealt with the following:

1) the shape of the wind power output duration curve as a function of the size of the energy store dedicated to each machine .

2) the ability of a wind plus storage system to follow fluctuating loads, as a function of the size of the energy store dedicated to each machine.

3) the comparison of wind systems with nuclear power plants.

4) the effects of very long term energy storage on wind system reliability.

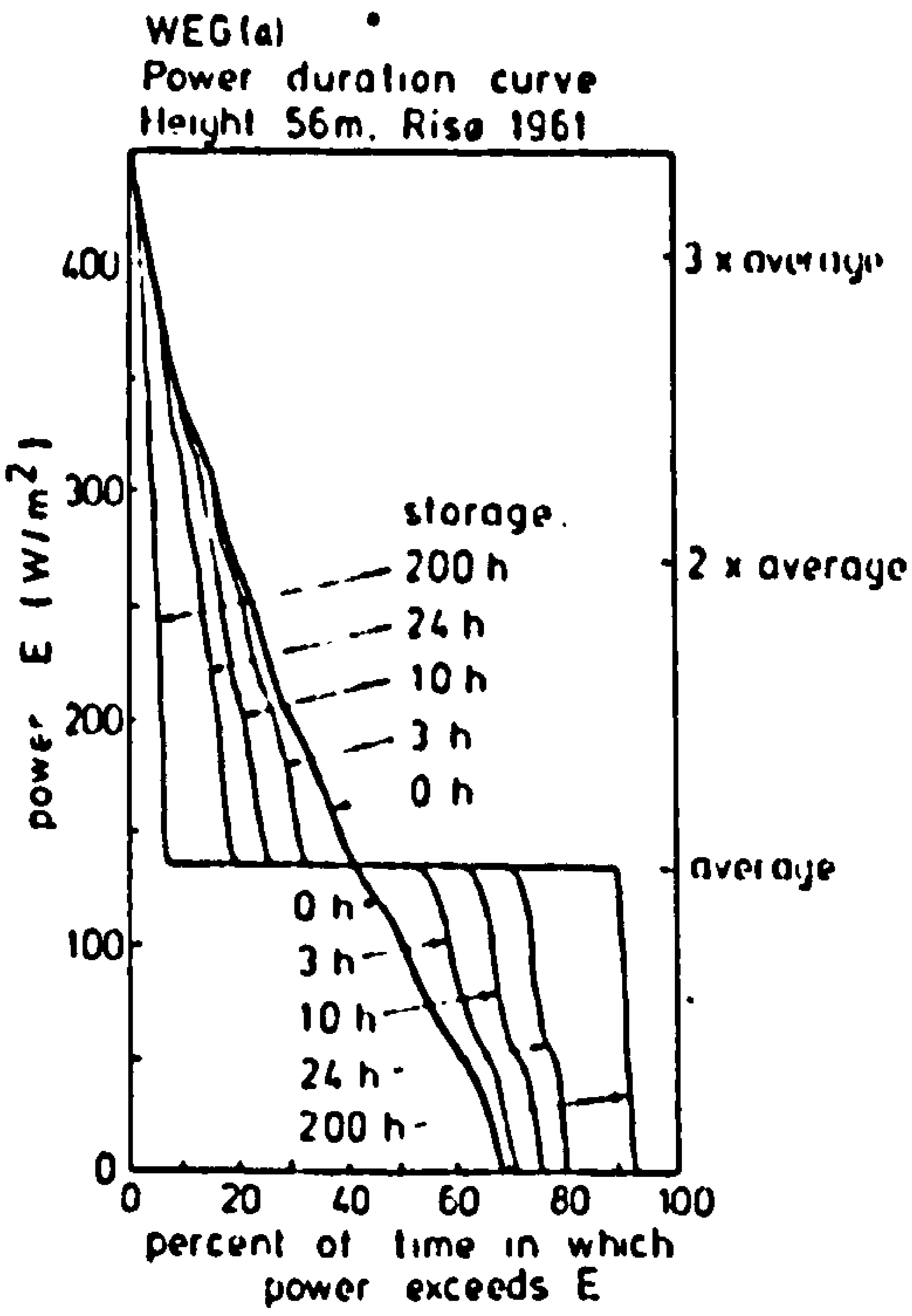
Sørensen's conclusions are:

1) that small amounts of storage result in windpower systems being as reliable as individual nuclear power systems (figs 4.1.1 and 4.1.2).

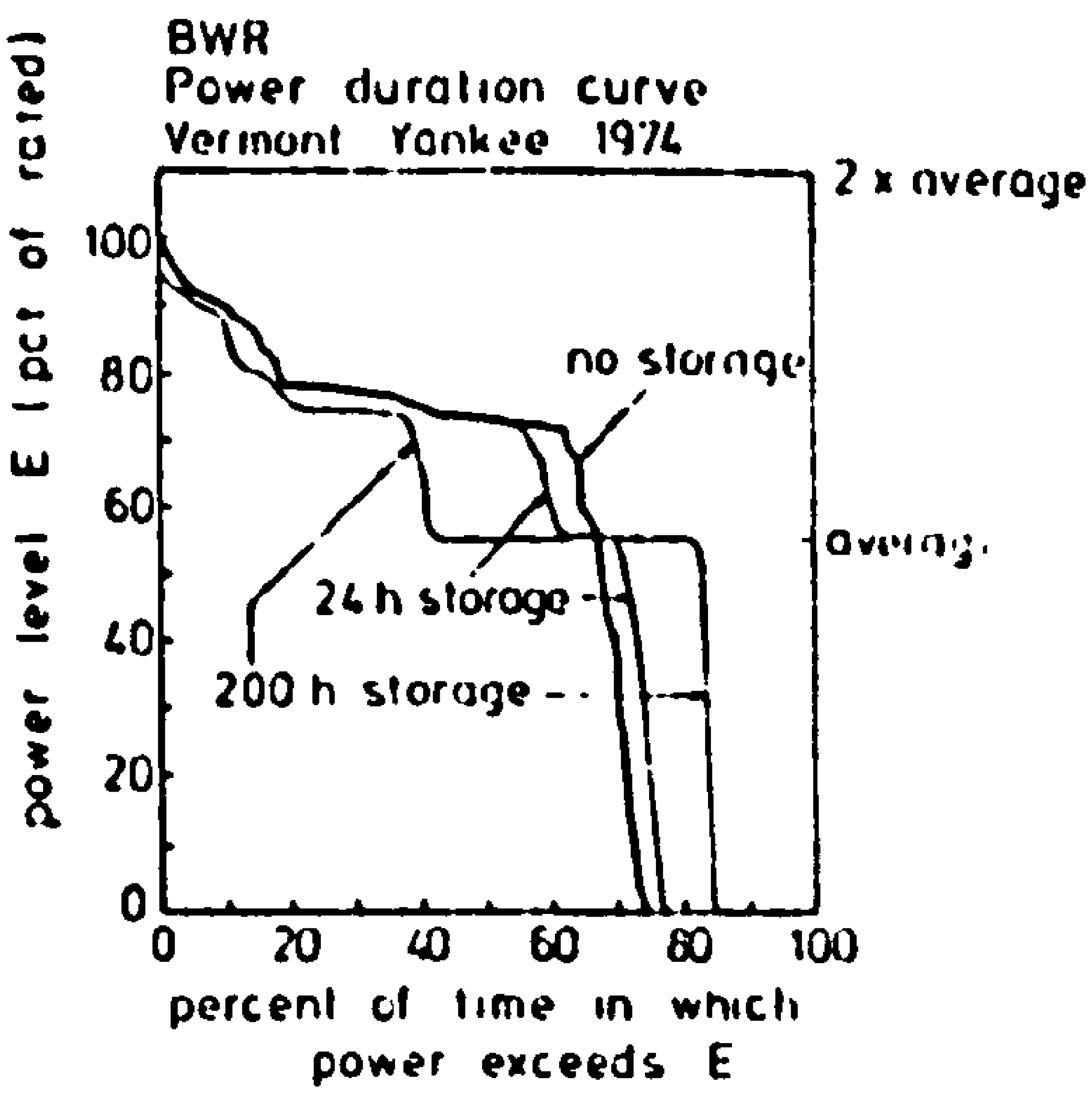
2) that the reliability of a system based on nuclear plant increases more rapidly with the number of stations than does the reliability of a wind power system as more sites are used.

A number of criticisms of Sørensen's work can be made. The most fundamental is that the elctricity systems he studies are not globally optimised. (This is the same criticism I levelled at Anderson et al.) No attempt was made to find the most economic way in which wind power or storage could be integrated into a grid. (Sørensen actually used the Danish grid for his studies). In optimising a system one first has to define it. The position of the system boundary is arbitrary - it is largely for one's own convenience that it is made to coincide with institutional boundaries. It should be noted that optimisation of the whole UK energy system on the basis of the criteria used in the nationalised supply sector, would probably change the whole system out of recognition. A change of the order of a factor of 3 in energy intensity of UK GNP has been posited by Leach (Leach et al 1979), while Olivier et al (Olivier et al 1983) suggest a change of the order of a factor of 5. One can tentatively conclude from this work and from parallel work in other industrialised countries (CONAES, Nørgaard 1979) that present energy systems as a whole are rather a long way from optimality, in the sense of the criteria used in analysis of energy supply in the UK. This brings into question the value of marginal analysis used in this thesis and elsewhere as a tool for rationally deciding questions of technical policy.

Sørensen's work was therefore clearly not valueless. A later study (Sørensen 1980) which opens up the very interesting prospect of supplying all Danish and Norwegian electricity demand from a combined wind and hydro system, clearly derives directly from this earlier work. What the 1978 study did was to make the need for a more consistent assessment of the capacity credit of wind power unavoidable, by demonstrating how arbitrary the distinction between firm and non-firm sources of electricity could be.



Power durations curves for a WEG (a) system with or without (thick curve) storage, placed at Risø (56 m, 1961 data).



1974 power duration curve for the Vermont Yankee nuclear power plant (boiling water reactor, with and without a hypothetical storage (based on [8])

Figs 4.1.1&2 The effects of short term storage on the power output of wind and nuclear plant. From Sørensen, 1978.

4.2 PROBABILISTIC APPROACH TO CAPACITY CREDIT I

The first paper published in the UK which presented a coherent analysis of capacity credit for "non firm" energy sources, was that by Rockingham (Rockingham 1980). The results and theoretical method of this paper are summarised in the following. The notation for this section will be as follows:

$p_d(x)$=probability density function of demand
$p_w(x)$=probability density function of wind power
$p_T(x)$=probability density function of net demand (demand - wind)
P=annual mean wind power
L=annual mean demand power
P'=mean wind power at times of peak grid demand
L'=mean peak grid demand
C'=mean conventional plant at times of peak demand
I=wind power penetration P/L

δ_w=annual standard deviation of wind power
δ_L=annual standard deviation of demand
δ'_w=standard deviation of wind at times of peak demand
δ'_d=standard deviation of demand in peak periods

a'=standard deviation of total system uncertainty in peak periods, normalised by mean peak demand L'
b=annual fractional standard deviation of wind power
d=annual fractional standard deviation of demand
b'=fractional standard deviation of wind power in peak periods
d'=fractional standard deviation of demand in peak periods
c'=fractional standard deviation of conventional plant in peak periods

v_d' = actual peak demand as a fraction of L'
v_c' = actual wind power in peak period as a fraction of P'
v_c' = actual conventional plant in peak periods as a fraction of C'
X = net available capacity in peak periods $(v_c \cdot C' - v_d \cdot L')$.
Z = equivalent firm capacity credit for wind power

The method is based on the fact that utilities aim to meet consumer demand for electricity with a finite reliability. The factor which determines whether or not all of consumer demand will be met is the net available capacity X. Rockingham assumed that the probability density functions for $v_c{'}P'$ and $v_d{'}L'$ during periods of peak consumer demand are gaussian. This is illustrated in figs 4.2.1 and 4.2.2. The probability that load will have to be shed at peak is given by R :-

1. $R = PROB(X < 0)$

It can be shown that for two gaussian variables with means $\langle x_1 \rangle$ and $\langle x_2 \rangle$ and variances $\delta_1{}^2$ and $\delta_2{}^2$, the distribution of the sum of the two variables is gaussian with mean $\langle x_1 \rangle + \langle x_2 \rangle$ and variance $\delta_1{}^2 + \delta_2{}^2$. The probability density function of net available plant in peak periods is therefore easily determined from the probability density functions of $v_c{'}C'$ and $v_d{'}L'$. This function is shown in fig 4.2.3. The reliability criterion can now be expressed in the form:-

2. $\langle X \rangle / \delta = \theta$

where

3. $X = (v_c{'}C' - v_d{'}L' + v_w{'}P' - Z)$

$$\delta = (c'^2C'^2 + d'^2L'^2 + b'^2P'^2)^{1/2}$$

where Z is an equivalent firm demand (or negative firm capacity) which can be added to (or subtracted from) the system after the addition of wind energy and balances equation 4.2.2. In the present formulation this capacity has no uncertainty associated with it. Rearranging equation 4.2.2 and substituting 4.2.3 we get:

$$C' - L' + P' - Z = \theta(c'^2C'^2 + d'^2L'^2 + b'^2P'^2)^{1/2}$$

and for the case where P'=0,

$$C' - L' = \theta(c'^2C'^2 + d'^2L'^2)^{1/2}$$

Hence:

4. $$Z = P' + \theta(c'^2C'^2 + d'^2L'^2)^{1/2} - \theta(c'^2C'^2 + d'^2L'^2 + b'^2P'^2)^{1/2}$$

and writing $a'^2L'^2 = (c'^2C'^2 + d'^2L'^2)$

5. $$Z = P' - \theta[(a'^2L'^2 + b'^2P'^2)^{1/2} + a'L']$$

This can be differentiated with respect to the annual mean wind power P to give the marginal capacity credit dZ/dP:

6. $$dZ/dP = (P'/P) - \theta b'^2 (P'/P) P' (a'^2L'^2 + b'^2P'^2)^{-1/2}$$

This is the result which Rockingham derived, although expressed in terms of mean wind power instead of installed wind power. It is instructive to look at the behaviour of these expressions for capacity credit in the limit of small and large P. For small penetrations P/L:

7. $$Z \simeq P' - \theta b'^2P'^2/2aL'$$

$$dZ/dP \simeq (P'/P) - \theta b'^2 (P'/P) P'/aL$$

while for large penetrations:

8. $Z \simeq P'[1 - \theta b']$

$$dZ/dp \simeq (P'/P)[1 - \theta b']$$

For typical wind turbines, at a single UK site $b \simeq 1.0$ and $P'/P \simeq 1.0$. For the CEGB, $a \simeq 0.1$, $L' \simeq 40$ GW, $L \simeq 25$ GW and $\theta \simeq 0.88$ (Monopolies and Mergers Commission, 1981). Hence:

$$dZ/dP \simeq 1 - 5.5\ I$$

The above expression needs to be modified in the light of a more detailed study of the parameters which determine b (in particular it is important to take account of site diversity and annual variation in wind power in estimating b). The conclusion from the above is that marginal capacity credit is likely to approximate to a linear function of wind penetration I, for small penetrations. The large penetration limit should not be taken too seriously, since the gaussian approximations used in the above analysis break down when wind power supplies a substantial fraction of electricity demand.

It is necessary to point out that there is a certain amount of arbitrariness in the definition and estimation of some of the quantities which appear in the analysis presented above, and that in practice this analysis may break down due to basic deficiencies in the model which has been used. The main problems are:

1) The model assumes that system uncertainty can be defined and estimated. The main contributor to system

growth of peak demand over the planning period of seven years. This is illustrated by the following table taken from the Monopolies and Mergers Commission Report on the CEGB (Monopolies and Mergers, 1981).

TABLE 4.6 Changes in the planning margin and the four factors

Period	Planning margin %	(i) Expected winter peak availability %	(ii) Expected variation in winter peak availability %	(iii) Expected variation in demand because of weather %	(iv) Expected variation in demand because of forecasting error %
1959-62	14	90	3	2·4	2·7
1962-64	14	90	3	3·3	2·7
1964-70	17	90	3	3·8	6
1970-71 1976-77	20	88	3	3·8	6
1976-77 (to date)	28	85	3·8	3·8	9

Source: The CEGB and Electricity Council

Derivation of the planning margin. Source: Monopolies and Mergers Commission report on the CEGB.

There is evidence that the CEGB's demand predictions have been consistently too high for some years (see fig 4.2.4 taken from Crabbe and Lowe 1981). Better forecasting methods could probably reduce the uncertainty in the seven year forecast. It has been suggested that the CEGB could reduce the planning margin by using two year forecasts of peak demand (Monopolies and Mergers, 1981, p61) on the basis that uncertainties in the growth of demand could be met by delaying the decommissioning of old plant rather than by building new plant. There would seem to be some merit in this argument. The purpose here is not however to get embroiled in arguments about CEGB corporate planning, but to illustrate the extent to which there may be legitimate argument over one of the parameters which enters into the above analysis. The uncertainty of demand growth may itself be uncertain by a factor of 3.

2) The model assumes independence of wind power output and electricity demand. This is a poor assumption for low wind penetrations in the CEGB grid, and presumably any grid supplying a large amount of space heating. This will be discussed in more detail later.

3) The model assumes that the probability distribution function for wind power output is gaussian. In fact it is highly non-gaussian, for single sites and combinations of sites, as figs 4.2.5, 4.2.6 and 4.2.7 show. The effect of the shape of the wind probability distribution function will be discussed in the following sections.

A final problem with the above model of wind power capacity credit is that it calculates the "equivalent firm capacity" Z. This capacity is assumed to have no uncertainty. It may be more interesting to estimate the conventional capacity, with its associated uncertainty, which is equivalent to wind. If marginal conventional capacity has the same characteristics as average conventional plant [1], then we can expand equation 4.2.2 as follows:

$$
\begin{aligned}
C' - P' - L' &= \theta\sigma \\
&= \theta(c'^2 C'^2 + d'^2 L'^2 + b'^2 P'^2)^{1/2}
\end{aligned}
$$

---

[1] Note that marginal conventional plant referred to here is not rapid response plant, but plant which would be added to the system in response to long term growth of demand. Note also that the characteristics in respect of which this plant is assume to be the same as average existing plant are only the mean output and the variance of output at peak. In practice these conditions may not be met.

This equation can be solved for C' (or L') to give us the total conventional plant requirement (or total demand) with the average uncertainties associated, which would be equivalent to a given level of wind power P':

$$C' = [(L'-P') + \theta\{(L'-P')^2c'^2 + (d'^2L'^2+b'^2P'^2)(1-\theta^2c'^2)\}^{1/2}] \;/\; (1-\theta^2c'^2)$$

This equation is considerably more complex than Rockingham's. To first order in P', C' can be approximated by:

$$C' \simeq [(L'-P') + \theta L'\{(c'^2 + d'^2(1-\theta^2c'^2)\}^{1/2} - \theta P'c'^2\{(c'^2 + d'^2(1-\theta^2c'^2)\}^{-1/2} + O(P'^2) \;...] \;/\; (1-\theta^2c'^2)$$

This equation is similar in some ways to equation 4.2.7 (eg. C' = constant + coefficient x P'), but note that b' does not appear in the coefficient of P' in this formulation. It is not proposed to carry this line of reasoning any further, since its complexity undermines the main function of an analytic approach which is to clarify.

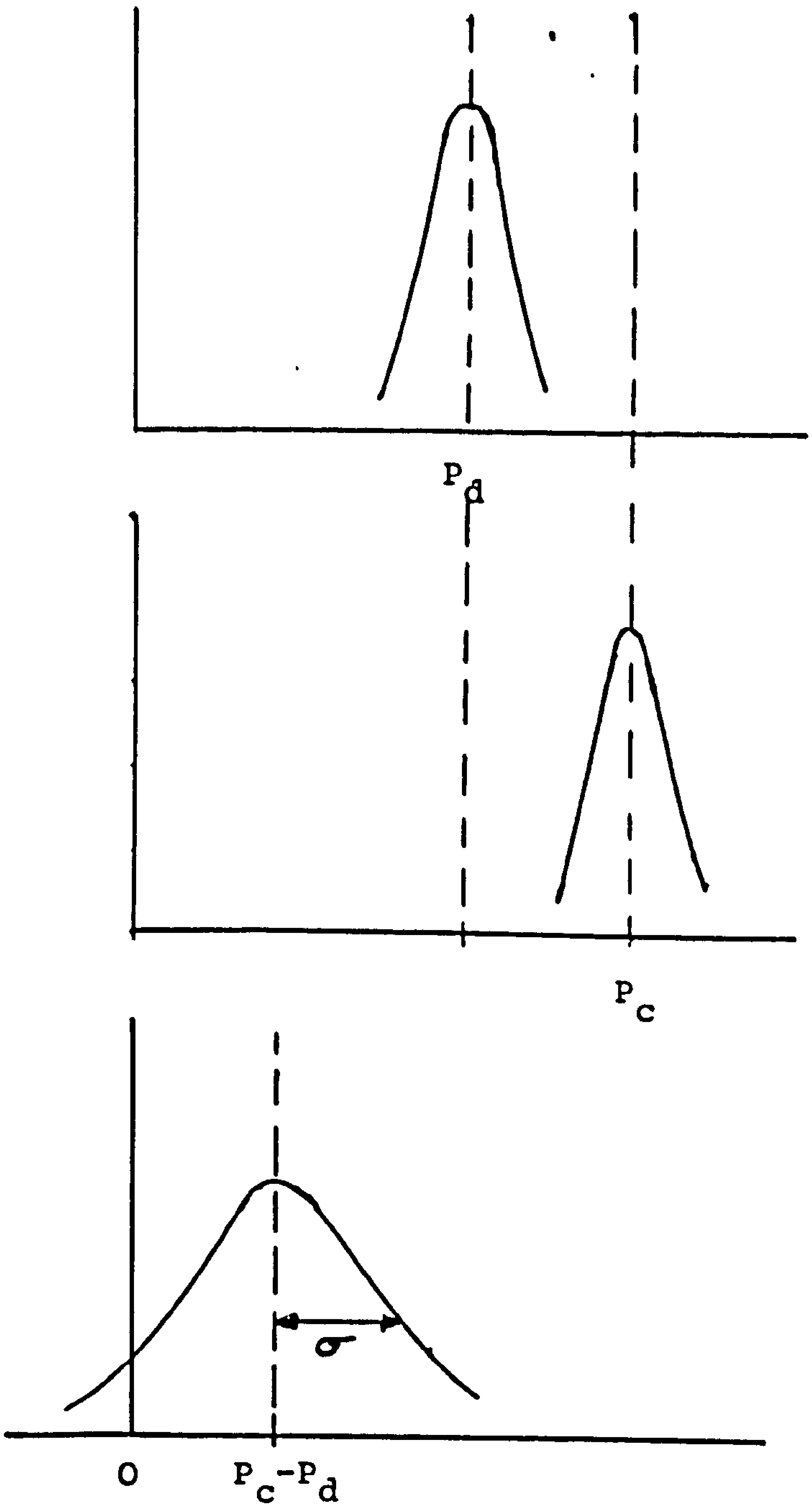


Fig 4.2.1 Probability density function for electricity demand (sketch).

Fig 4.2.2. Probability density function for conventional plant output (sketch).

Fig 4.2.3 Probability density function for net available capacity, showing finite probability of failure to meet full demand (sketch).

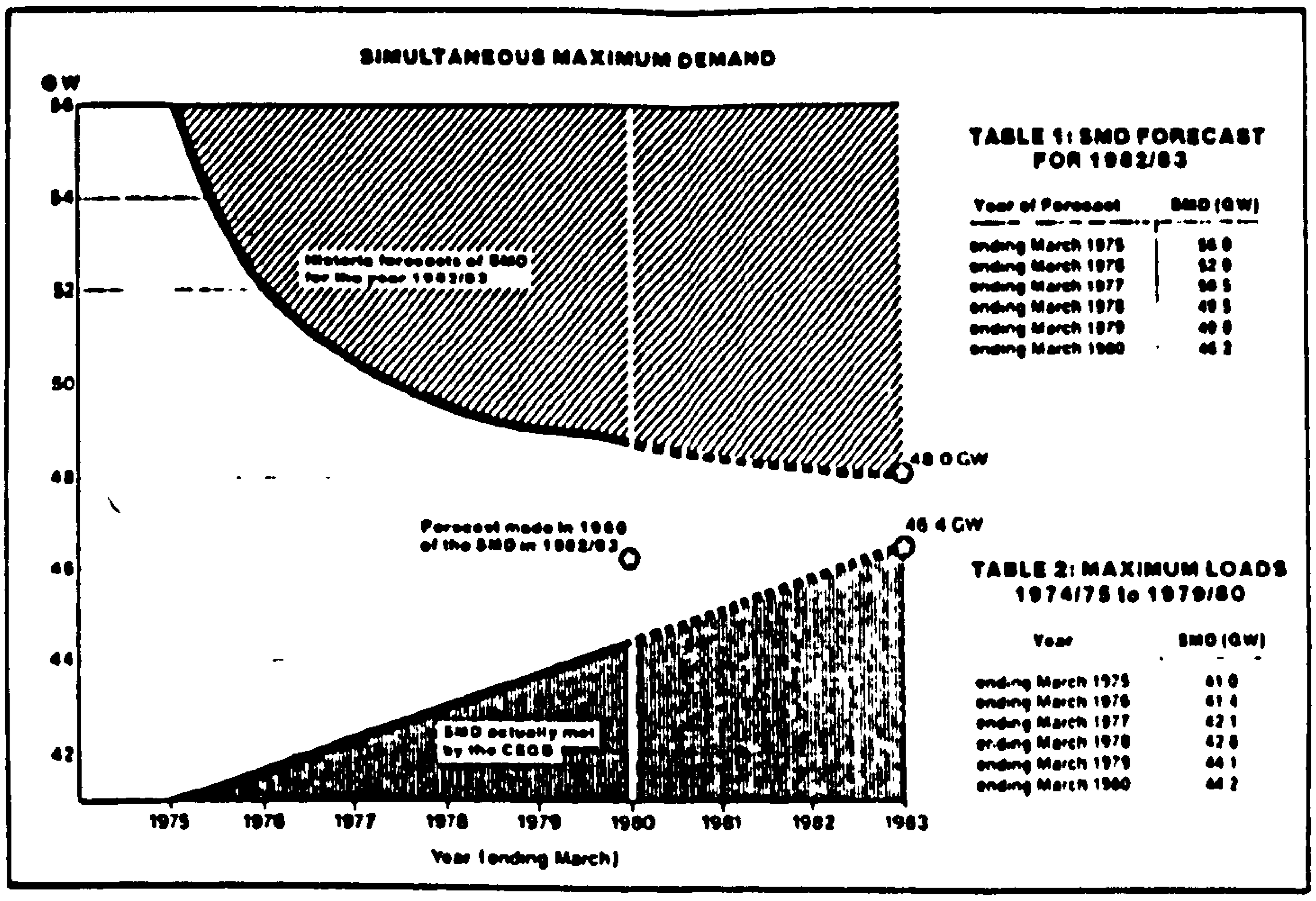


Fig 4.2.4 Evidence for CEGB demand forecasting bias.
From Crabbe and Lowe, 1981.

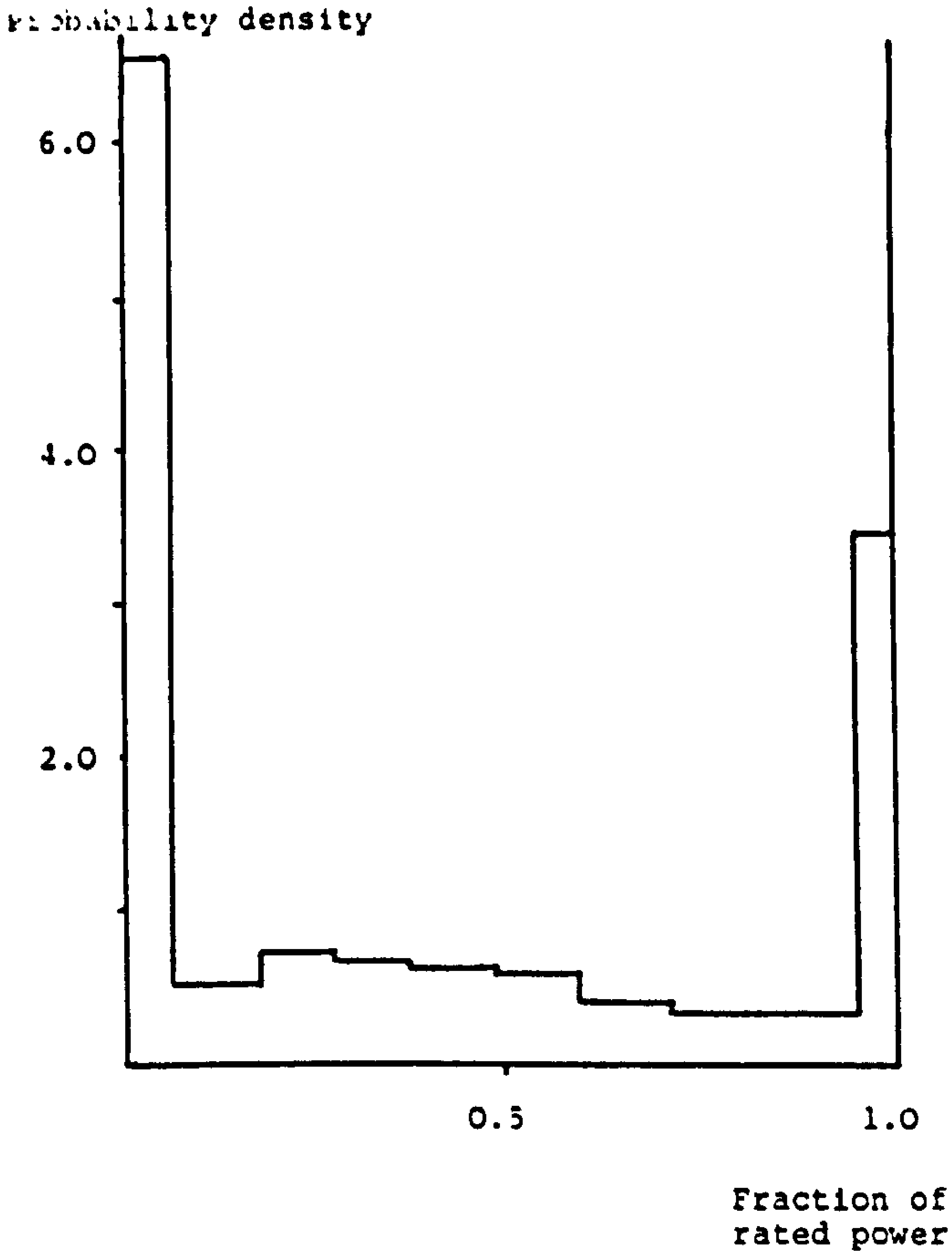


Fig 4.2.5 Probability density function of wind power for a single UK site.

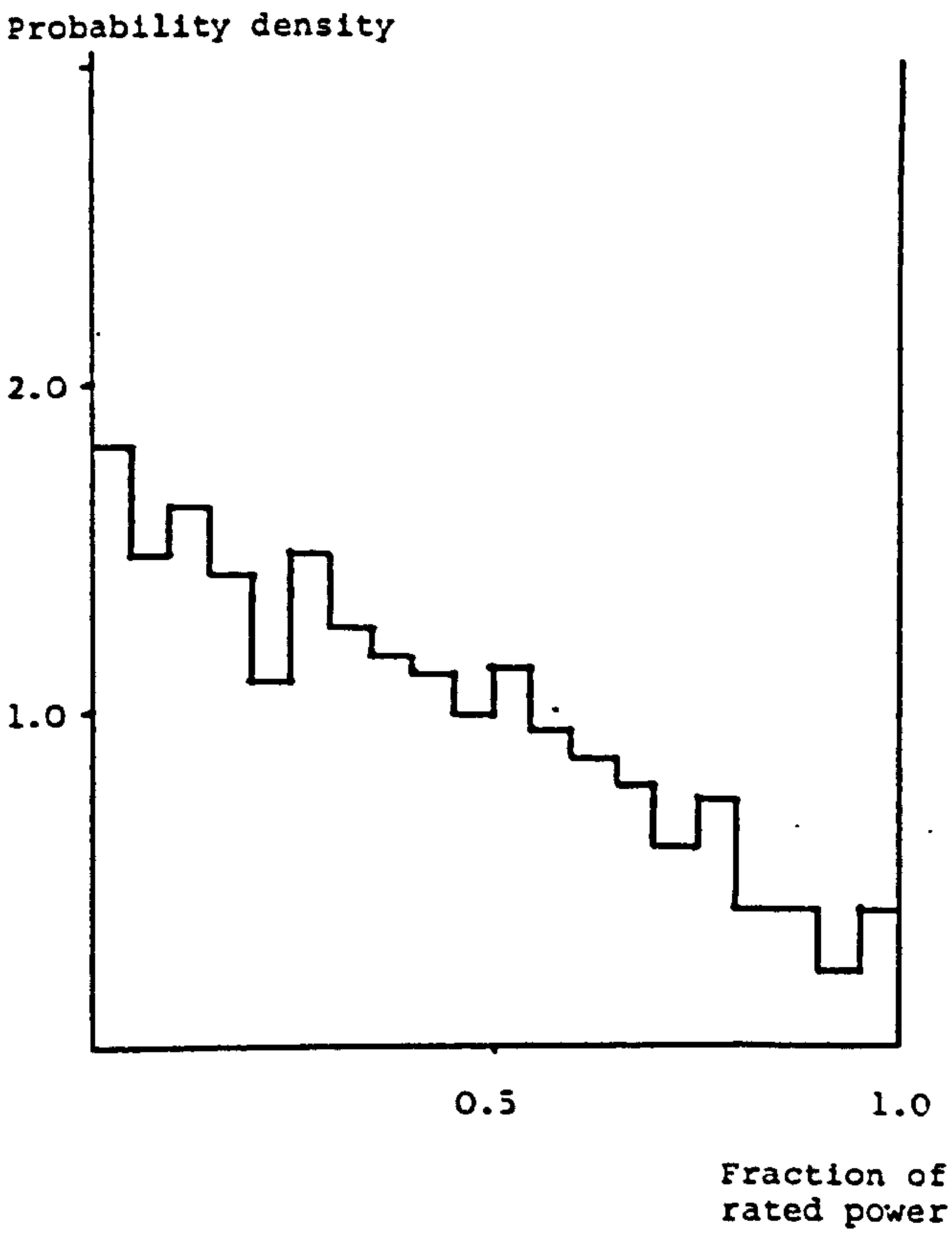


Fig 4.2.6 Probability density function for wind power for a group of 4 UK sites.

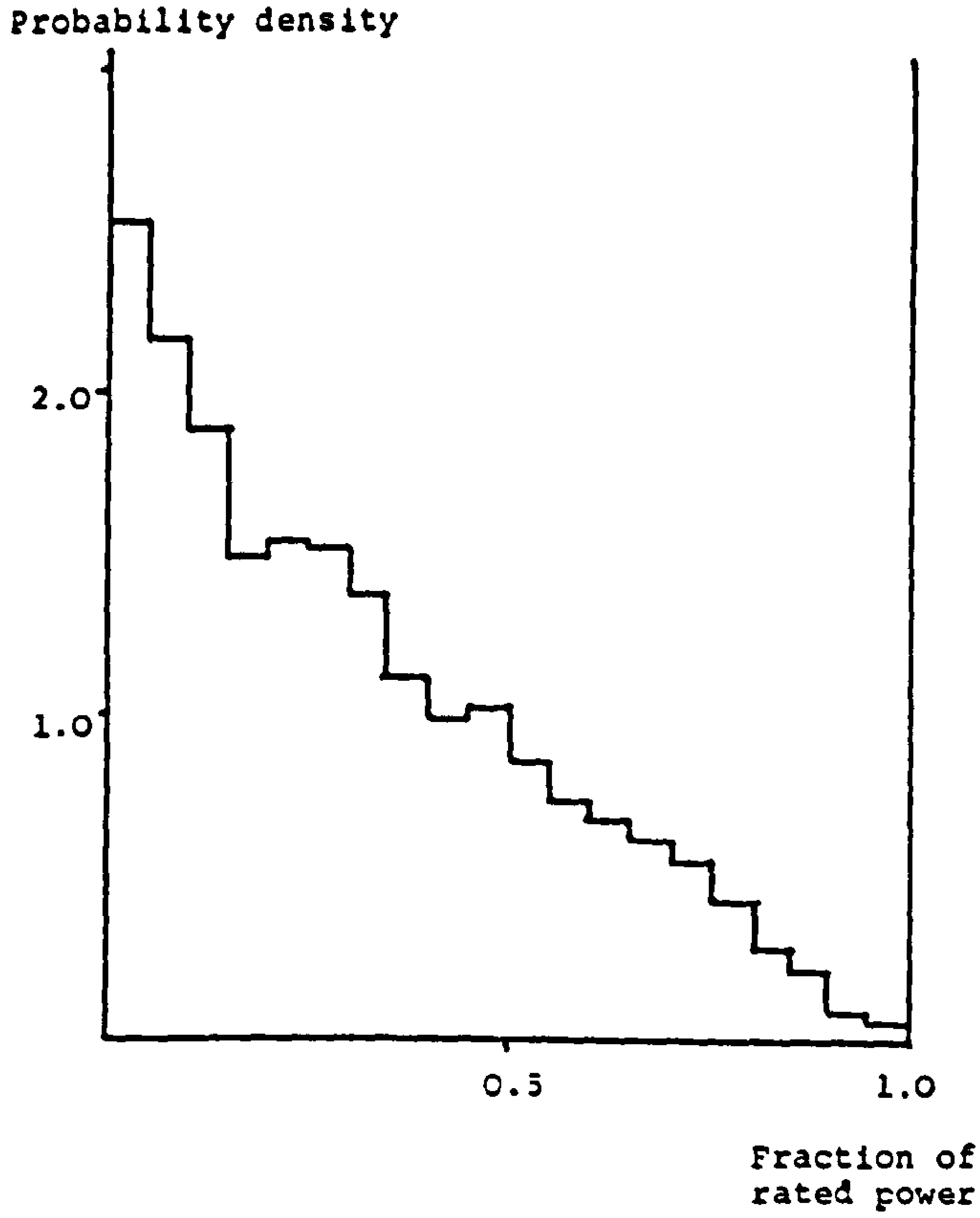


Fig 4.2.7 Probability density function of wind power for a group of 9 UK sites.

4.3 PROBABILISTIC APPROACH TO CAPACITY CREDIT II

Having discussed Rockingham's analysis of capacity credit, I want to present a slightly different approach which has the advantage of making plain the effect on the load duration curve, of adding new sources of electricity to an existing grid.

Electricity demand can be treated as a random phenomenon with non-stationary statistics. Viewed in this way, the load duration curve for a given utility can be reinterpreted as an estimate of the probability exceedence curve for demand. The intensity of a given level of load is simply the probability that the load at any instant will be greater than that level. The probability exceedence curve is a function of time. The load duration curve as measured in any one year is the only direct estimate available for the probability exceedence curve in that year. There may, however be quite simple empirical relationships between the load duration curves in successive years, which enable several years' data to be used to obtain a good estimate of the probability exceedence curve in any one year.

As we have seen, utilities plan to meet consumer demand for electricity with a given level of reliability. This level of reliability may or may not be expressed as a probability of failure to meet demand. Capacity credit for new plant may therefore be defined as the change in peak load which the utility can meet , at a constant level of reliability, when the new plant is added to the system. In order to estimate capacity credit one has to study the statistics of electricity demand minus available conventional plant capacity. To simplify the mathematics in the next four pages, I have assumed that conventional plant availability has a standard deviation of zero. The problem then, is to estimate the change in the load probability exceedence curve when new plant of a particular type is added to the system.

The simplest cases to study are: 1) where the power output of the new source of electricity is completely un-correlated with the existing system demand, and 2) where the correlation is complete. It is the first of these which is of interest in the study of wind power as the power output of wind turbines is, to a first approximation, uncorrelated with electricity demand (in the UK at least). Given two independent random variables, it is easy to show that the probability density function of the difference of the two variables is given by the convolution of their separate probability density functions. A proof of this is given at the end of this section. If we apply this to the present problem, we get:

$$1. \quad p_T(x) = \int p_w(x+x')\,p_d(x')\,dx'$$

where $p_T(x)$ is the probability density function of electricity demand minus the output of the new source of electricity, $p_w(x)$ is the probability density function of the output of the new source, and $p_d(x)$ is the probability density function of the existing electricity demand. The probability exceedence curve of demand on the existing system is the integral of the function $p_d(x)$. This curve is sketched in fig 4.3.1.

The probability density function of the new source of electricity is sketched in fig 4.3.2. The approximate result of convolving p_w with p_d in the case where $\sigma_w << \sigma_d$ (and therefore, since $\sigma_w \simeq P$, $P << \sigma_d$) is simply to shift the graph of p_d to the left:

2. $p_T(x) \simeq p_d(x+P)$

I shall prove this result shortly, but for the moment I wish the reader to accept it. Given the above equation, the probability exceedence curve for demand minus the output of the new source of electricity is also given by:

3. $F_T(x) \simeq F_d(x+P)$

The conclusion from this is that, in the case of small contributions from new sources of electricity to an existing grid, and where the output of the new source is uncorrelated with the existing demand, the new source behaves as base load plant reducing net demand by a constant amount at all load factors. This is illustrated in fig 4.3.3.

This is equivalent to the result derived by Rockingham, though with slightly wider implications for the operation of the electricity system other than at peak.

The demonstration of the result is as follows. The probability density function of the residual electricity demand after the addition of a new source of electricity is given by equation 4.3.1 with the limits of the integration being $0<x'<W$ (where W is the peak power output of the wind system). If I substitute $x'=y+P$, I get:

4. $p_T = \int p_d(x+P+y)\ p_w(y+P)\ dy$

I can now substitute a Taylor series for p_d :

5. $p_d(x+P+y) = p_d(x+P) + y\,d/dx\; p_d + (y^2/2)\; d^2/dx^2\; p_d \ldots$

The function $p_T(x)$ is now equal to the sum of a series of integrals:

6. $$p_T(x) = \sum_{n=0} (1/n!) \quad (d^n/dx^n) \;\; p_d(x+P) \int y^n p_w(y+P)\,dy$$

The integrals are simply the moments about the mean of the probability density function $p_w(x)$. The first three of these are:

$m_0 = 1$

$m_1 = 0$

$m_2 = \sigma_w{}^2$

To second order:

7. $$p_T(x) \simeq p_d(x+P) + (\sigma_w{}^2/2)\; d^2/dx^2 \;\; p_d(x+P)$$

This proves the assertion that, to a first approximation, in the limit of small penetration, sources of electricity uncorrelated with existing demand, substitute as base load plant.

Now strictly we cannot apply the convolution formula to the probability density functions for wind and electricity demand, as these variables are correlated - both peak in the winter in the UK and in many other mid latitude countries with substantial electric space heating. However we can look at the probability distributions of wind and demand when demand is near its peak. If we assume that the tail of the distribution p_d can be approximated by a gaussian:

8. $p_d = [1/\sigma_d'(2\pi)^{1/2}] \exp\{-(x-L')^2/2\sigma_d'^2\}$

for $x > \sigma_d'$, where σ_d' and L' are the effective mean and standard deviation of the tail of the distribution

then I can evaluate $d^2/dx^2\ p_d$:

9. $d^2/dx^2\ p_d = -(1/\sigma_d'^2)\ p_d + (x-L')^2/(\sigma_d'^4)\ p_d$

Now the reliability criterion for the utility may be expressed as:

10. $$\int_z^{\infty} p_d(x)\ dx = A$$

Assuming that the gaussian approximation of equation 4.3.8 is adequate, z may be rewritten $z = \partial\sigma_d' + L'$. If windpower is added to the system, then equation 4.3.10 can be rewritten

11. $$\int_{z'}^{\infty} p_T(x)\ dx = A$$

Substituting equation 4.3.7 into 4.3.11, we get

12. $$\int_{z'}^{\infty}\{1 + \sigma_w'^2/2[(x+P'-L')^2/\sigma_d'^4 - 1/\sigma_d'^2]\}p_d(x+P')\ dx = A$$

Replacing x by $y = x + P' - L'$, I get

13. $$\int_{z'+P'-L'}^{\infty}\{1 + \sigma_w'^2/2\sigma_d'^2[y^2/\sigma_d'^2 - 1]\}\exp(-y^2/2\sigma_d'^2)\, dy$$

$$= (2\pi)^{1/2}\sigma_d' A$$

Integrating by parts and substituting $y'=z'+P'-L'$ I get

14. $$(1-\sigma_w'^2/2\sigma_d'^2) \int_{y'}^{\infty} \exp(-y^2/2\sigma_d'^2)\, dy$$

$$+ \sigma_w'^2/2\sigma_d'^2 \int_{y'}^{\infty} \exp(-y^2/2\sigma_d'^2)\,dy$$

$$- \sigma_w'^2/2\sigma_d'^2\, y' \exp(-y'^2/2\sigma_d'^2) = (2\pi)^{1/2}\sigma_d' A$$

Cancelling and substituting for A in the above I get

15. $$\int_{z'-L'+P'}^{z-L'} \exp(-y^2/2\sigma_d'^2)\, dy$$

$$= -\sigma_w'^2/2\sigma_d'^2\, y'\exp(-y'^2/2\sigma_d'^2)$$

which can be simplified to give

16. $(z' + P' - z) = \sigma_w'^2/2\sigma_d'^2\,(z' - L' + P')$

But $z'+P'-L' = z-L' = \theta\sigma_d'$

Hence, capacity credit Z (given by z-z') is simply

17. $Z = P' - \theta\sigma_w'^2/2\sigma_d'^2$

This is precisely the approximation to capacity credit which was obtained in the previous section, following Rockingham. What I have demonstrated here is that this approximation is not affected by the form of the probability distribution function for wind power output, depending only on the variance of this distribution. The method above could be extended to higher order approximations, by including more terms of the Taylor expansion of the function p_T (see equations 4.3.5 and 4.3.6 above). Rather than do this I have used numerical methods to investigate the importance of the assumption that p_w is gaussian. This work is described in the next section.

The probability distribution of the sum of two independent random variables.

I wish to find the probability density function $p_T(x)$ of the sum of two random variables $p_1(x)$ and $p_2(x)$. The prior probability of finding variable 1 in the range $x < y < x+dx$ and variable 2 in the range $x' < y < x'+dx'$ where:

18. $z = x + x'$

is given by:

19. $p_{1,2}(x,x')dxdx' = p_1(x)p(x')dxdx'$

since the two variables are independent. The total probability of variables 1 and 2 satisfying equation 4.3.18, is the sum of all terms of the form of equation 4.3.19. If I substitute from equation 4.3.18 into equation 4.3.19, and

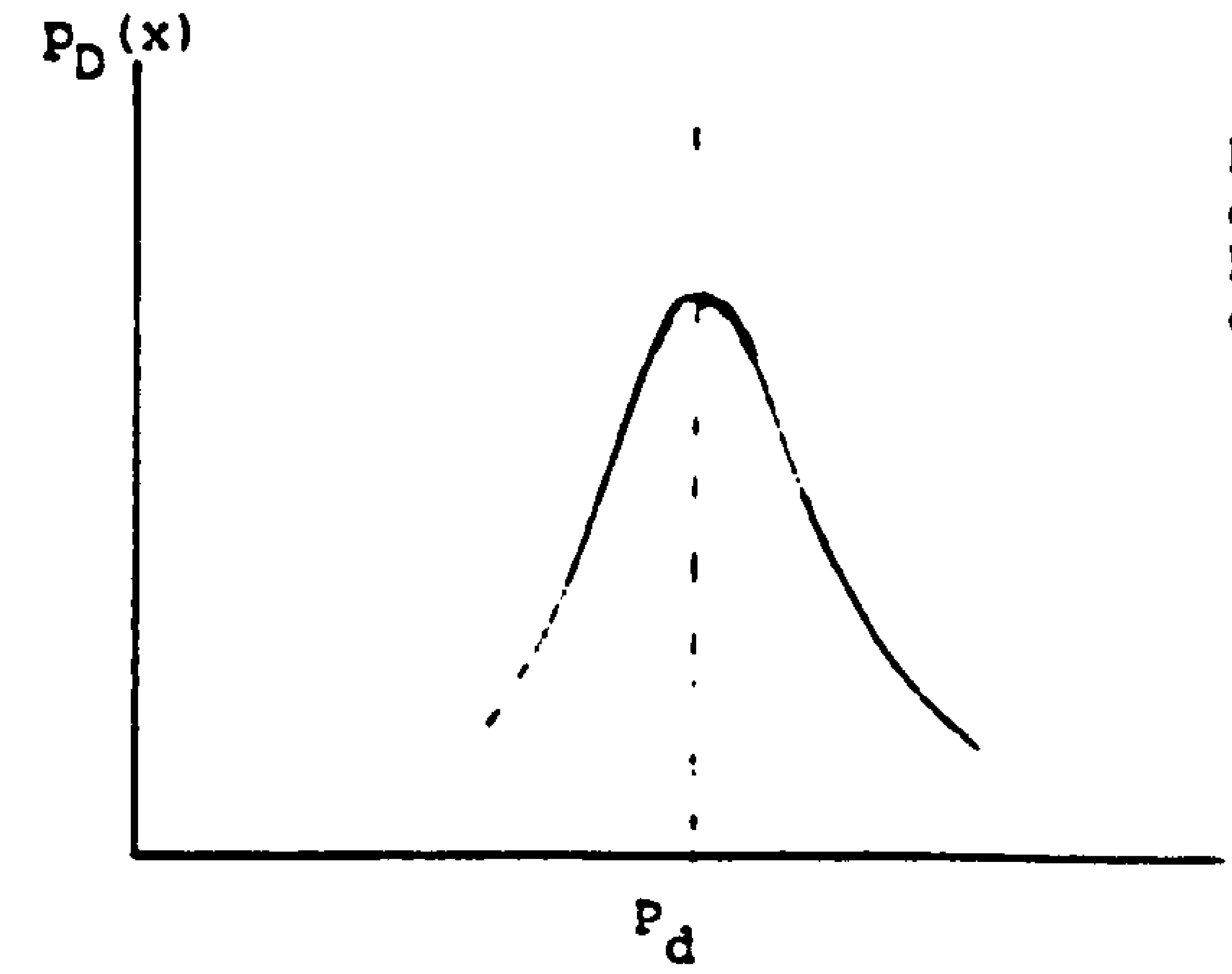


Fig 4.3.1 Probability density for demand before the addition of wind generation.

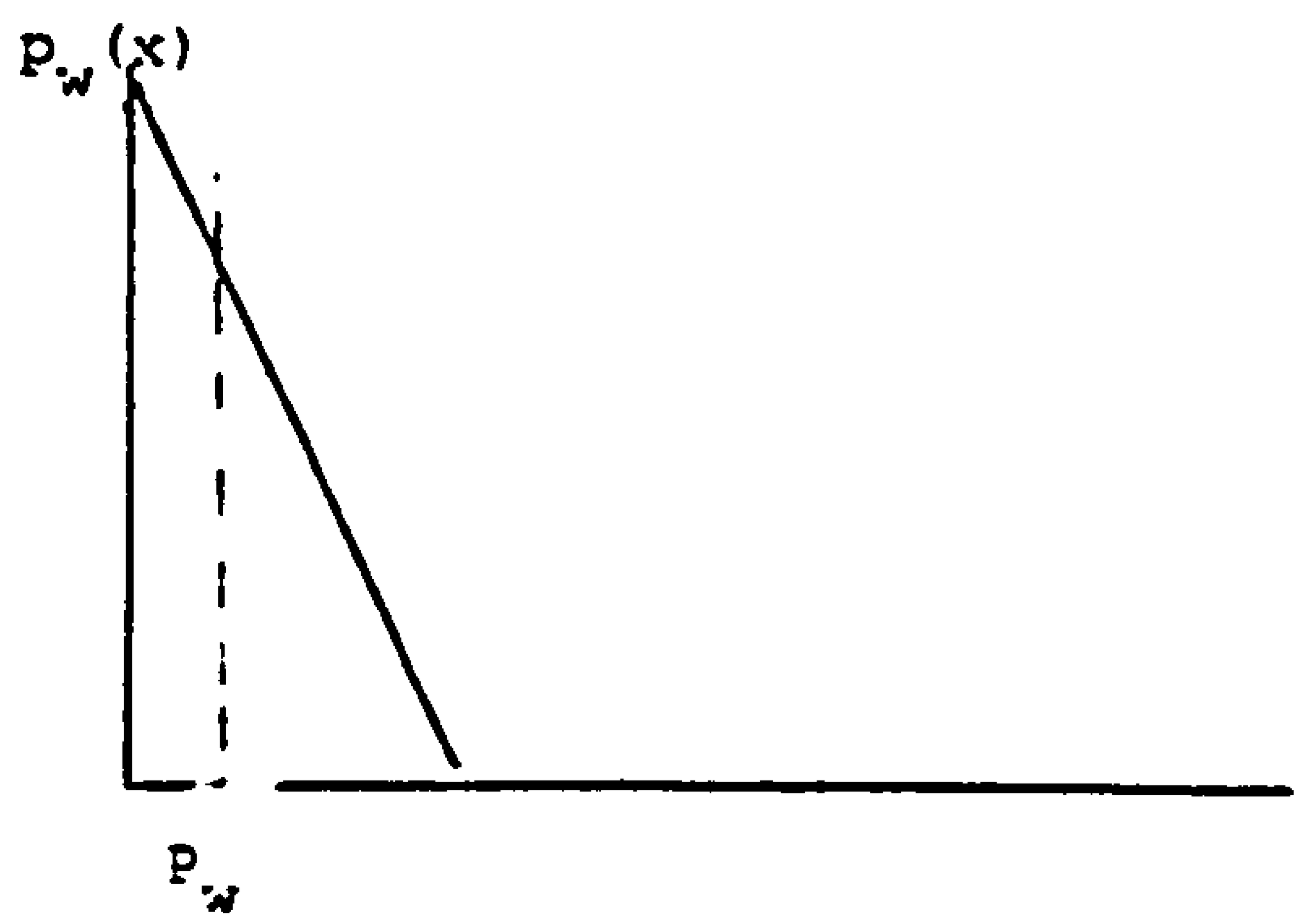


Fig 4.3.2 Probability density for wind power.

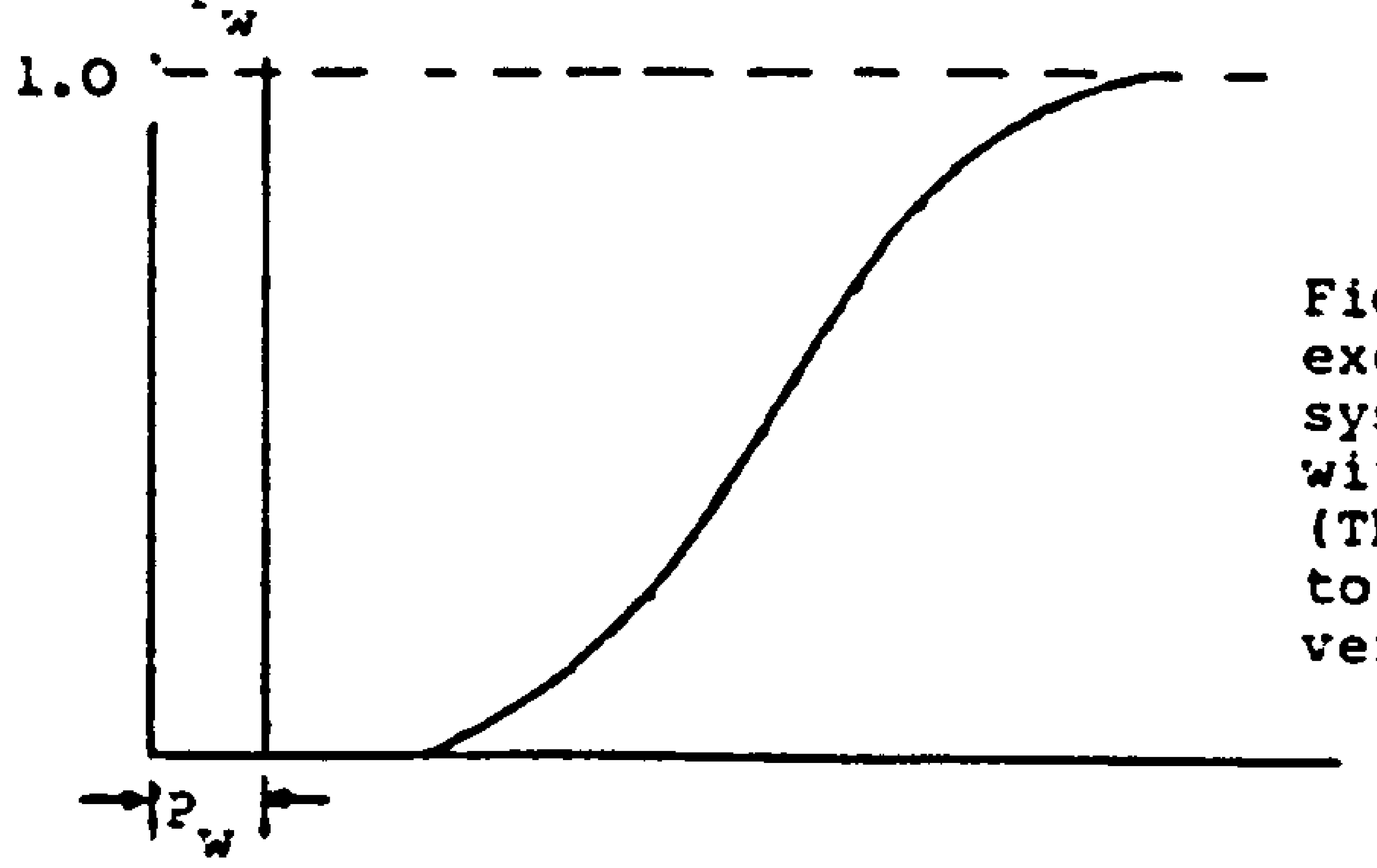


Fig 4.3.3 Probability exceedence curve for system with and without wind power. (The former is relative to the displaced vertical axis).

re-interpret the sum as an integral, then the total probability of finding the sum of the two variables in the range $z < y < z+dx$ is given by:

$$20. \quad p_T(z)dx = [\int p_1(z-x')p_2(x')dx'] \, dx$$

The right hand side of this equation is the convolution of p_1 with p_2.

4.4 THE EFFECT OF THE SHAPE OF THE PROBABILITY DISTRIBUTION FUNCTION OF WIND ENERGY OUTPUT ON CAPACITY CREDIT.

I have shown that the probability distribution function of wind power output is highly non gaussian. In the previous section I was able to show that based on a simple loss of load probability criterion definition of capacity credit, and with the assumption that the probability distribution function of wind power output is gaussian, that capacity credit for wind power can be expanded in terms of the moments of the probability distribution function of wind power output. Therefore for small penetrations of wind power into a grid, the non gaussian nature of this distribution affects capacity credit only through moments of order higher than the variance.

This section describes a numerical exercise to examine the variation of capacity credit for a system containing an arbitrary quantity of installed wind power. The assumptions that I have used are

1. The probability distribution function for net demand

on the system without wind power, is gaussian.

2. The probability distribution function for the output of the wind system is linear between zero output and installed capacity, and zero elsewhere. As can be seen from figs 4.2.5, 4.2.6 and 4.2.7 , this is a much better approximation to reality than a normal distribution.

The probability distribution function for the system uncertainty without wind energy can be written

1. $p_x(x) = (1/2\pi)^{1/2} (1/\sigma) \exp(-x^2/2\sigma^2)$

This function is sketched in fig 4.2.3 above. The probability distribution function for wind power output is given by

2. $p_w(w) = (bw + a)/(b\hat{W}^2/2 + a\hat{W})$

in the range $0 < w < \hat{W}$, and zero elsewhere. This function is sketched in fig 4.4.1. The probability distribution function of the system including wind power is given by the convolution of p_w and p_x:

3. $$p_T(x) = \int_0^{\hat{W}} p_w(w)\, p_x(x-w)\, dw$$

This formulation involves the evaluation of two sets of integrals

4. $p_T(x) = (bI_1 + aI_0)/\{(2\pi)^{1/2}\sigma\}$

I_0 in the above is

5. $$I_0 = \int_0^W \exp(-(x-w)^2/2\sigma^2) \; dw$$

which can be rewritten

6. $$I_0 = \int_x^{x-W} \exp(-y^2/2\sigma^2) \; dy$$

I_1 in equation 4.4.4 is

7. $$I_1 = \int_0^W w \; \exp(-(w-x)^2/2\sigma^2 \; dw$$

and hence

$$I_1 = x\int_0^W \exp(-(w-x)^2/2\sigma^2) \; dw + \int_0^W (w-x) \; \exp(-(w-x)^2/2\sigma^2) \; dw$$

which simplifies to

$$I_1 = xI_0 + \int_{-x}^{W-x} y \; \exp(-y^2/2\sigma^2) \; dy$$

The second half of this expression can be evaluated as follows

$$\int_{-x}^{W-x} y \exp(-y^2/2\sigma^2)\, dy = 1/2 \int_{x^2}^{(W-x)^2} \exp(-u/2\sigma^2)\, du$$

$$= -[\sigma^2 \exp(-y^2/2\sigma^2)]_{x}^{W-x}$$

Hence equation 4.4.7 becomes

$$8. \quad I_1 = xI_0 - [\sigma^2 \exp(-y^2/2\sigma^2)]_{x}^{W-x}$$

To summarise, $p_T(x)$ is given by

$$9. \quad p_T(x) = \{(bx + a)I_0 + b\sigma^2[\exp(-y^2/2\sigma^2)]_{W-x}^{x}\}/\{(2\pi)^{1/2}\sigma\}$$

Equation 4.4.9 gives the probability distribution function for net available capacity on the system including wind power, in a form that can be relatively easily integrated numerically. I have done this, and solved the equation 4.3.11 above for total system firm capacity (by an iterative linear interpolation method). Capacity credit for wind power can then be estimated by subtracting total system firm capacity with zero wind power input. Capacity credit calculated in this manner can be compared with credit calculated from equation 4.2.5 above. This is done graphically in fig 4.4.2. The difference between the numerical result and the result based on the assumption of a gaussian wind distribution is of the order of 2% at an installed wind capacity corresponding to a penetration into the CEGB system of about 20%. The nature of the difference depends on the required level of reliability of the system. For a loss of load probability of 19% (the CEGB figure) the gaussian approximation overestimates capacity credit. For smaller LOLP's the gaussian approximation underestimates capacity credit.

The most important conclusion from this numerical work is that the insensitivity of capacity credit to shape of the probability distribution function of wind power output is confirmed. The gaussian approximation predicts the departure of the capacity credit from the first order (linear) approximation to within 5% at a penetration corresponding to roughly 20% in the CEGB system.

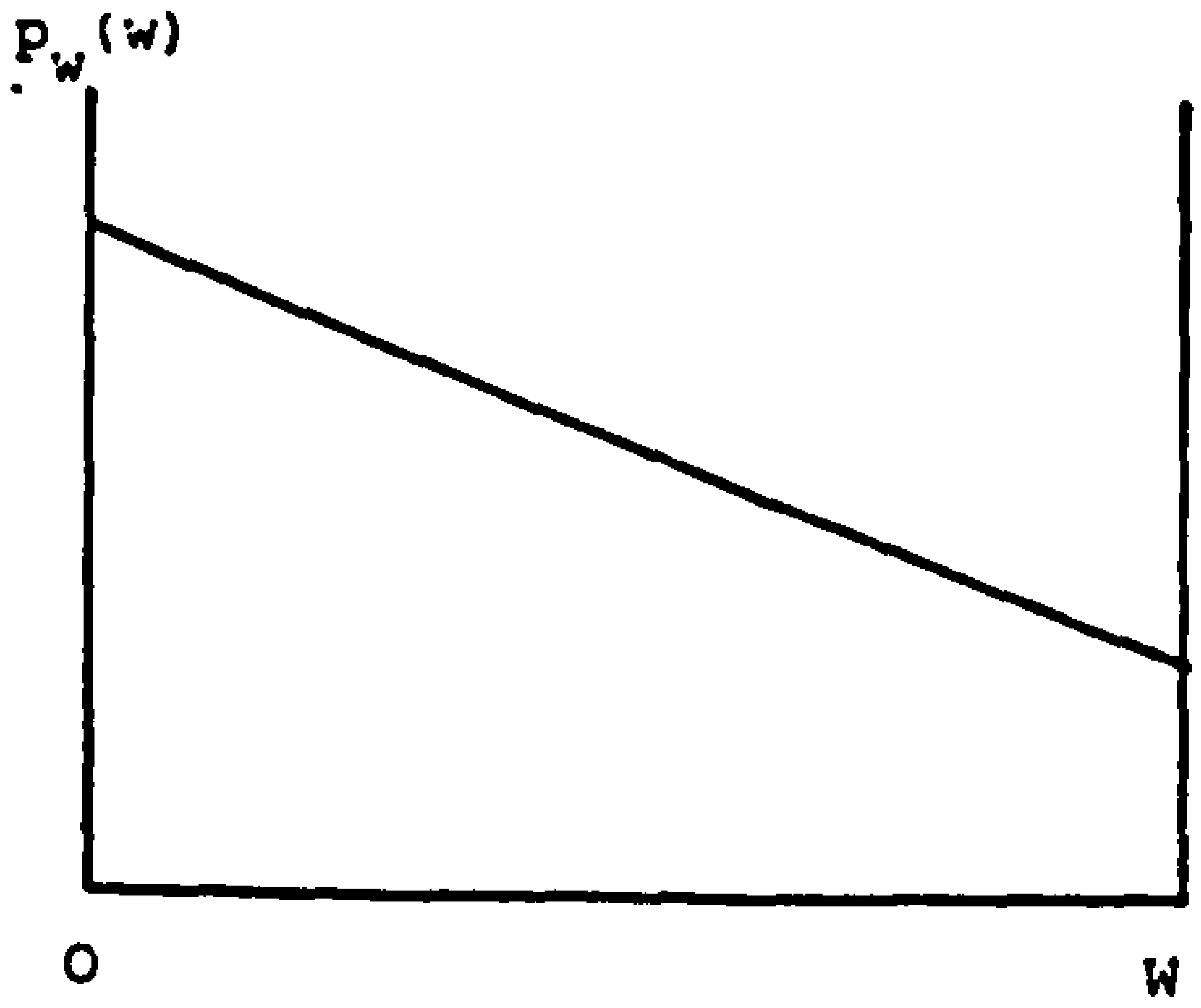


Fig 4.4.1 Linear approximation to probability density function for wind power. Installed capacity W.

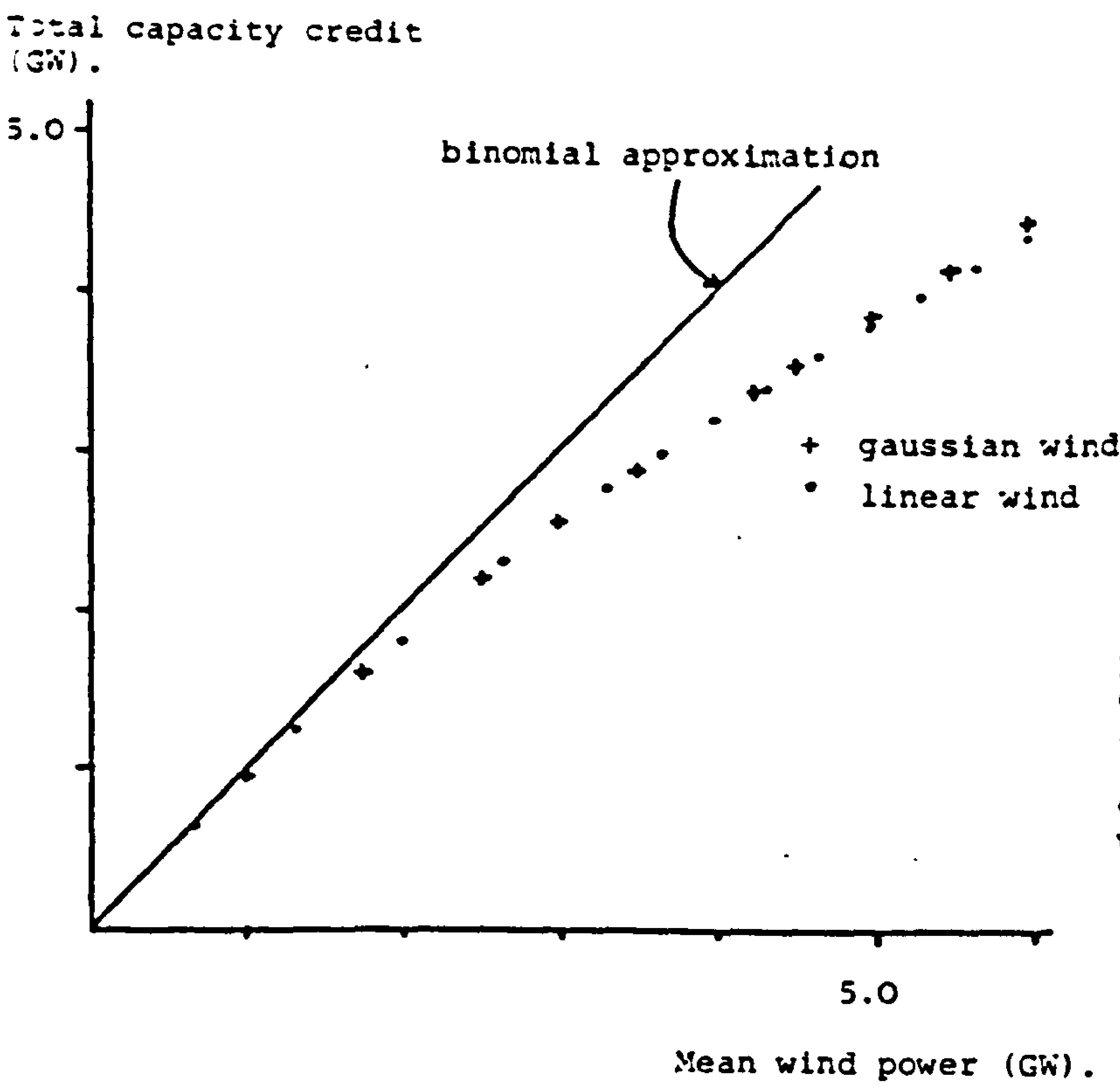


Fig 4.4.2 Wind capacity credit based on linear and Gaussian approximations to the probability density function for wind power.

4.5 CONCLUSIONS FOR WIND POWER CAPACITY CREDIT

In the previous sections I have reviewed and extended a theoretical attempt to clarify the problem of capacity credit for "non firm" sources of electricity. The conclusions from this work are:-

1) that non firm sources of electricity which are uncorrelated with electricity demand, substitute as base load at zero penetration. At small penetrations, the departure from base load depends on the variance of the output of the non firm source.

2) that the capacity credit of a non firm source of electricity for zero penetration, depends on the mean availability of the source at times of peak demand. The capacity credit at small penetrations is a function of the ratio of the variance of the new source at peak, to the effective variance of net available capacity at peak.

The most important factors in assessing the importance of these theoretical results for wind power are:

1) the extent to which wind power output is correlated with electricity demand at different frequencies.

2) the actual values of the mean and standard deviation of the output of wind turbines, at times of system peak demand.

3) the uncertainty in electricity demand and conventional plant output at times of system peak demand.

I shall deal with these factors in turn.

Correlation between wind power and electricity demand.

In the UK in the Winter, there is a positive correlation between wind speed and electricity demand, caused by the use of electricity for space heating, generally poor draught sealing in the housing stock, and the use of natural ventilation. Capacity credit for small amounts of wind capacity will depend on detailed statistical analysis of the sensitivity of peak electricity demand to wind speed, and the problem will have to be tackled numerically if at all. However, this sensitivity is itself likely to be sensitive to other decisions made on the energy demand side. Perhaps unexpectedly, weather stripping of houses could reduce the capacity credit for wind power. Removal of much of the electric space heat requirement by the introduction of combined heat and power would probably have a similar effect.

Cursory consideration leads to the conclusion that marginal capacity credit will be of the order of 0.8 (allowing for array efficiency of 0.84 and a machine availability of 0.95) until installed capacity is of the order of the rated wind speed of a typical wind turbine, multiplied by the sensitivity of peak demand to wind speed. Marginal capacity credit will then drop to the values predicted by the analyses presented in the preceeding sections. Leicester (private communication) has suggested a value for the coefficient of wind dependent demand of 3 $MW/(ms^{-1})$. Thus, in the UK, some 30 to 60 MW of wind turbines could potentially be installed at high capacity credits.

Diurnal windpower fluctuations.

The interpretation of the diurnal fluctuation is quite difficult, since the diurnal fluctuation is correlated with the annual, and because the diurnal fluctuation is itself a function of height. This second point is well illustrated by a series of wind speed spectra (figs 4.5.1 to 4.5.6) measured at Risø (Petersen 1975), and both points by a highly detailed series of monthly average diurnal windspeed graphs for 7 different heights, measured at the same site (the graphs for January and June are given in figs 4.5.7 and 8). These graphs may be better understood by looking at the mechanism which produces the daily fluctuation of wind speed in temperate latitudes. Basically, the induction of a temperature gradient in the earth's boundary layer by solar heating of the earth's surface, causes instability in the boundary layer, which both magnifies mechanically induced turbulence and gives rise to convection cells. Both of these latter effects increase the mixing of the boundary layer and increase the momentum diffusion coefficient. Momentum in the upper atmosphere (associated with the geostrophic wind) thus diffuses more rapidly downward into the last few 10's of metres of the atmosphere, resulting in higher wind speeds there, and correspondingly lower wind speeds at greater heights. The strength of these effects are clearly greatest when the solar influx on the horizontal is greatest, ie. during the day in summer, and lowest in the winter.

The problem of a priori estimation of the diurnal fluctuation of the output from real wind turbines is complicated by the fact that turbine discs for the latest generation of machines are up to 100 m in diameter. The top and bottom of such rotors will therefore frequently see opposite diurnal fluctuations (a MOD 2 rotor, for instance, operates between heights of 23 m and 115 m, with 60% of the rotor area between 46 and 92 metres). The problem is also complicated by the non-linear response of wind turbines to the power flux through their rotors. My conclusions on the subject of diurnal fluctuations in the power output from wind turbines are:

1) that the level of fluctuation will tend to be greatest in the summer (for sites in temperate or higher latitudes).

2) that for the larger machines the level of fluctuations will be reduced.

3) that in summer the fluctuation will tend to be advantageous, since it is correlated with electricity demand.

4) that at high wind speeds, (ie. over the operating range of the wind turbine) the diurnal fluctuations will be reduced, since the mechanical turbulence generated will dominate the effects due to the thermal stability of the boundary layer. With heat flux H from the ground upwards, the stability of the atmosphere depends on the ratio of z/L, where z is the vertical height, and L is the Monin Obukhov length, given by:

$$L = - (u_*^3/H) . (\rho \, c_p \, T/kg)$$

where u_* is the so called friction velocity (equal to $(J_o/\rho)^{1/2}$ where J_o is the horizontal shear stress, or rate of vertical transport of horizontal momentum), ρ is the density of the atmosphere, c_p is the heat capacity of the amosphere at constant pressure, T is the absolute temperature, and k is a dimensionless constant (Pasquill 1978). For large positive values of H/u_*^3, $z/L << 0$, corresponding to an unstable atmosphere. The friction velocity u_*, to a first approximation, is proportional to mean wind speed, hence at high wind speeds H/u_*^3 is small and $z/L \simeq 0$ for points in the boundary layer, corresponding to a stable atmosphere, regardless of the sign of H. Hence my conclusion that mean diurnal wind speed fluctuations will tend to over estimate mean diurnal fluctuations in wind turbine output.

There are relatively few sites at which data of the quality necessary to determine conclusively the nature of the diurnal fluctuation of wind turbine output, is available. The only one that I am aware of in the UK, is the Belmont tower. Without more data, it is difficult to go further than the tentative conclusions advanced above.

Variation of single site power fluctuations.

The problem in analysing single site standard deviations is to find ways of non-dimensionalising the dependent and independent variables. In particular one needs a parameter which indicates the relationship of the turbine installed power to the windiness of the site. A number of possibilities exist. Perhaps the simplest to choose is the ratio of rated wind speed to mean wind speed at the site for a given machine type, V_r/V_m. Criticism of this non-dimensionalisation has been made by Dixon and Lowe (Dixon and Lowe 1981b). The main problem with it is the discrepancy between pitch controlled machines such as the Mod 2 and stall controlled machines such as the Gedser 200 kW, Taywood 60 m and WPG/ERA 30 m machines.

This discrepancy is illustrated in fig 4.5.9 for 2 machines, the stall controlled WPG/ERA 30 m design reported by Warne et al 1979, and the pitch controlled Aeolus (WTS 75) machine reported by Hermansson 1981. An alternative non-dimensionalisation, suggested by Dixon (op cit), is to divide turbine installed power by the power at the mean wind speed, to give the specific power p:

1. $p = P / (\rho S V_m^3/2)$

(where S is the rotor area). Rearrangement of this equation gives a wind speed, V_p, which can be used instead of V_r, to non-dimensionalise the sustained wind speed V in expressing the turbine characteristic. Note that V_p is a function both of the turbine and the site, unlike the rated wind speed. Fig 4.5.10 shows the power characteristics of the two machines shown in fig 4.5.9, plotted against V/V_p for $p = 1.0$.

Any non-dimensionalisation of this sort is a simplification of a complex problem. It is clear that V_m in equation 4.5.1 could have been replaced by a variety of other 'characterstic' wind speeds (median wind speed, characteristic wind speed, or root mean cube wind speed). The root mean cube wind speed may be preferable on physical grounds, but in practice may weight wind speeds above the turbine cut out speed too highly. In the following I will use the definition in equation 4.5.1.

I have chosen to non-dimensionalise standard deviations against mean power output rather than peak power output. The justification for this is that the mean power output is the quantity of primary significance in any economic analysis of wind power.

If we consider first the standard deviation of wind power output evaluated from continuous hourly records over several (nine) years, the best explanatory relationship I have found is between site standard deviation and p . A plot of these two variables for 27 UK sites is shown in fig 4.5.11. The super imposed line has been fitted by least squares regression and has a correlation coefficient of 0.92 . A number of points need to be made here. First, I have fitted a straight line only to data for sites with $p > 0.55$. The reasons for this are a) that the relationship becomes decreasingly linear for $p < 0.55$ [1] and b) very few land sites have high enough mean wind speeds to give $p < 0.55$ with modern wind turbine designs, so data for very high wind speed sites would give a misleading picture of the outcome of a large wind power programme in the UK.

Second, the practical range of site standard deviations for a machine of a given type is quite small. The data presented here was derived using a characteristic based on the Boeing Mod 2 design (specific power 1.0, for $V_m(10\ m) = 14$ mph , with a velocity height exponent of 0.19). As stated above, there are few sites available for which $p < 0.55$, and installation is unlikely for economic reasons if $p \geq 2.0$, $V_m < 5.0\ ms^{-1}$. Normalised standard deviations of power output for this type of machine in the UK are therefore all likely to lie in a band between 0.85 and 1.2.

---

[1] The curve starts to turn upwards at very low p, due to the increasing fraction of the time that the turbines spend above their cut out wind speed.

For assessing the effect on capacity credit in the UK the variation of power output in the middle of Winter is probably more important than the variation averaged over the whole year. In order to investigate this I have calculated the standard deviation of wind power output about the monthly mean, for each month. The monthly standard deviation for January has then been plotted against specific power p (fig 4.5.12). The standard deviation for January is slightly lower than the annual average (a factor of about 0.88), which was to be expected from the typically higher wind speeds in mid winter in the UK. The monthly standard deviation for June is correspondingly higher than the annual average (see fig 4.5.13) by a factor of about 1.14 . The correlation coefficients of the regression lines in these two cases are 0.86 and 0.87 respectively.

In addition to evaluating the variance of wind power output in mid winter, it is also necessary to look at the variation of capacity factor F over the year. In most UK sites, the average power output is higher in the winter than in the summer. To estimate the likely magnitude of the effect of this on the availability of wind power during system peaks, I have calculated F_{jan}/F , where F_{jan} is the average capacity factor in January, for 25 UK wind sites. Wind turbine outputs have been estimated in the way descibed in section 3.2. The values of this ratio have been plotted against the specific power p (see fig 4.5.14).

For sites in the range $0.4 < p < 2.01$ the mean value of F_{jan}/F is 1.24 ± 0.02 . For a system with mean specific power 1.0, the annual capacity factor is approximately 0.38, implying a mid winter capacity factor of approximately 0.47.

The plot of F_{jan}/F is not particularly sensitive to p in the range $p > 0.8$, but falls off in the range $p < 0.8$. This is due to the increased frequency of turbine shut downs in very high wind speed regimes. Bossanyi (1981) has shown that turbine shut downs are particularly sensitive to the Weibull shape factor of the wind probability distribution.

Geographical diversity of wind power outputs.

The authors who have most directly studied this area are Justus and Mikhail (see Justus and Mikhail 1978). The starting point for their work was a calculation of the variation of the cross correlation coefficient r_{ij} for wind speed for two sites i and j, with the distance d_{ij} between the two sites. The authors used the equations

2. $$\bar{r} = [1/n(n-1)] \sum_{i \neq j} r_{ij}$$

and

3. $$\delta_n^2 = \delta_0^2 \{1 + (n-1)\bar{r}\}$$

where δ_0 is the standard deviation of the wind speed at a single site. These equations have already been discussed in section 3.3.5. Variation in the standard deviation of wind speed from site to site leads to an increase in δ_n as predicted theoretically in section 3.3.5 above. For the sites that Justus and Mikhail examined in the USA, they were able to demonstrate that the following relationship was approximately valid

4. $$\delta_n \simeq 1.11\, \delta_0 \{1 + (n-1)\bar{r}\}^{1/2}$$

In this study it was assumed that sites would have equal installed capacities of wind plant. This probably explains the factor of 1.11 by which equation 4.5.3 underestimates the diversity that Justus and Mikhail found for US sites.

Knowledge of the mean and standard deviation of the array average wind speed allows the fitting of a Weibull distribution function. Justus and Mikhail then go on to show how an empirical relationship between array average power and array average wind speed can be used to estimate the probability distribution of array power output.

The general approach of this paper is limited by the concentration on wind speeds. Since the power output of wind turbines at a single geographical site is a non-linear function of the wind speed, the extraction of power statistics from wind speed statistics has to go back to the original time series data for wind speed at each site in order to be reliable. The introduction by Justus and Mikhail of the array power vs array wind speed characteristic $P(V_n)$ (which is modelled as a simple linear relationship) begs as many questions as it answers.

An additional failing of the work of Justus and Mikhail is that it is not guided by an awareness of the origins of capacity credit for non firm sources of generation in large grid systems. Other work in this area has frequently been clouded by the same incomplete understanding. The classic mistake is to assume that capacity credit is equal to the level of power output which is exceeded for more than a specified fraction of the time. Justus himself appears to subscribe to this view (Justus 1978). Work on the correlation coefficient of wind speeds at different sites has been done by Goh and Nathan (1979 and 1981). These authors did not attempt to analyse wind power output. Qualitatively their results agree with those presented here. They report a steady fall of correlation coefficient with site separation (see fig 4.5.15 taken from Goh and Nathan 1979). The slope of the data is considerably steeper than the slope of the data I have presented. This is to be expected from the fact that the graph of correlation coefficient versus site separation must have an intercept of 1.0 at zero separation. Goh and Nathan have fitted the curve

$$r = 1 - 0.1021\, d^{0.5}$$

to the data in fig 4.5.15. This simple power law clearly breaks down at values of d approaching 100 km. The values of the correlation coefficient for wind speeds, estimated by Goh and Nathan at a site separation of 50 km is less by a factor of over 2 than the correlation coefficients of power outputs that I have calculated. This may indicate that wind turbines magnify the correlation between wind speeds, or it may be due to the fact that Goh and Nathan's data is for the far east.

For the purpose of estimating diversity of wind power at sites in the UK, I have used equation 4.5.2. Cross correlation coefficients of power for pairs of UK sites have been estimated, and these results have been reported in section 3.5 above. As has been discussed in section 3.5, the mean separation of pairs of sites in the UK is likely to be of the order of 150 km. The linearity of fig 3.5.11 allows us to move directly to the conclusion that the mean cross correlation coefficient for UK wind sites is likely to be approximately 0.55.

Conclusions

I am now able to make estimates of capacity credit for wind power in the UK, and to discuss the major parameters affecting it, for small penetrations of wind power into the CEGB grid.

Geographical diversity.

First, to illustrate the affects of geographical dispersion, I will calculate capacity credit for

1) wind turbines installed in arrays of mean power output 800 MW (corresponding roughly to installed capacity of 2 GW per array).

2) wind turbines installed in 10 arrays, the total wind power being determined by the array power.

3) a system of wind turbines assumed to be installed at a single site, ie with a diversity factor of 1.0

The wind power systems described above will all be assumed to consist of turbines with an average specific power of 1.0. In addition the parameters of the wind systems will be assumed to be

b_0 (specific standard deviation of
peak wind output for a single site) = 0.88

F (annual capacity factor) = 0.38

F_{jan}/F (ratio of capacity factor at
winter peak to annual capacity factor) = 1.24

r (mean cross correlation coefficient
for output from well separated wind arrays) = 0.55

The system into which they will be integrated has the following parameters

L (mean demand) = 25 GW

L' (mean peak demand) = 44 GW

a' (specific standard deviation of peak demand) = 0.105

A (loss of load probability LOLP) = 0.19

Marginal capacity credits based on the above parameters are shown graphed against wind penetration I in figs 4.5.16, 4.5.17 and 4.5.18. The important points to notice in these figures are

1) the precise arrangement of wind capacity does not make much difference to capacity credit for systems consisting of more than 5 arrays. This is really just restating conclusions drawn above.

2) the way in which wind power is developed initially may make as much as 10% difference to the capacity credits assigned to it.

3) capacity credit at very high penetrations is strongly affected by site diversity. Neglecting site diversity at a penetration of 0.5 leads to an underestimation of capacity credit by a factor of 0.6 .

4) while a straight line approximation to capacity credit for small penetrations is a useful simplification for systems involving a constant geographical diversity, this is not the case if wind power is built up from arrays of a constant mean power output.

Conventional system uncertainty.

To illustrate the effect of changes in the parameter a (uncertainty of system without wind power) I have examined the effect of the CEGB's adopting a policy of meeting uncertainties in peak demand by delaying the decommissioning of old power stations (this possibility was mentioned at the beginning of chapter 4). According to the Monopolies and Mergers Commission, the adoption of this revised planning basis would lead to a reduction of the CEGB planning margin from the present 28% to 22%. The resulting reduction in a' would be from 0.105 to 0.042. I have calculated marginal capacity credit using the parameters for case 2 above (constant geographical diversity, etc.) with this one alteration. Marginal capacity credit versus penetration has been plotted in fig 4.5.19 for the two values of a'.

Two observations can be made. First, reducing the conventional system uncertainty has a large effect on marginal capacity credit for intermediate values of I. A glance at the first order approximation to dZ/dP in equation 4.2.7 will show why - the initial slope of the curve of dZ/dP is inversely proportional to a'. Second, at very high penetrations, the difference between the two cases decreases. Inspection of equation 4.2.6 will show why - for large values of P

5. $dZ/dP \simeq (P'/P)[1 - \theta b]$

and with the parameters I am using,

$$\lim_{P \to} dZ/dP = 0.50$$

Figure 4.5.19 demonstrates another point, which is that although with the base case parameter values the high penetration limit of equation 4.2.6 is not reached until penetration is improbably high, a change of one of the parameters may make this limit much more interesting.

Specific power.

The next effect that I will investigate is the variation of specific power. In this section I have shown that dispersion of wind power output is approximately a linear function of specific power p. The mean specific standard deviation of peak power output is likely to rise from about 0.88 for p=1.0 to about 1.06 for p=2.0 . The annual capacity factor of turbines is likely to be approximately 0.26 for p=2.0 (see section 4.1.4). The other parameter values do not change. The effect of this change is to increase the value of the specific standard deviation of wind power output by a factor of about 1.2 . Fig 4.5.20 shows marginal capacity credit calculated for p=2.0 and p=1.0 with the other parameter values as in case 2 above.

Two observations may be made here. First the increase of specific power has the not unexpected effect of steepening the initial slope of the curve of marginal capacity credit. The initial slope of this curve is proportional to b^2. The second observation is that the behaviour of capacity credit at very high values of penetration is rather different between the two cases. The capacity credit approaches a limit of about 0.5 in the p=1.0 case, and a limit of 0.13 in the p=2.0 case. It is necessary to point out that equation 4.5.5 clearly breaks down for sufficiently high b, where it predicts negative marginal capacity credits.

Summary and comparison with other work.

Fig 4.5.21 shows a comparison of the results presented above with the results of a study by Jarass et al 1979 for West Germany. These results are based on a numerical study for the German coastal grid and are in broad agreement with the results I have presented. Unfortunately it has not been possible to work backward through this study to estimate the system uncertainty. It is therefore not possible to check that the initial slopes of the capacity credit lines agree with the theory set out above.

Fig 4.5.22 is taken from a paper by Taylor and Rockingham (1980), and shows the wide range of capacity credit for wind power which is possible in different utilities. The analytical work presented here and by others emphasises that there are a wide range of factors which affect capacity credit for wind turbines, and the results of work in this area have to be taken in context or they become meaningless.

Both analytical work, along the lines presented above, and numerical studies are necessary for the understanding of capacity credit. The analytical work emphasises the similarity between capacity credit and wind operating reserve requirements. The work on the latter in chapter 5 follows the analysis presented above quite closely. The difference is basically one of time scale.

To complete this section I need to put capacity credit into economic perspective. In order to do this, I first need to discuss briefly the value of capacity credit. In chapter 2 of this thesis it was shown that in an optimised electricity system without spot pricing (or significant storage), that the value of capacity credit is equal to the cost of providing peaking plant capacity. In the UK gas turbine plant is normally used for this role. The capital cost of this type of plant has been given as approximately 250 £ (1979)/kW (Department of Energy 1979), or approximately 270 £(1980)/kW. The corresponding annual capital charge is $\phi_g \simeq 17.6$ £/kW . If the first order fuel saving value of wind generated electricity is assumed to be the cost of coal generated base load electricity, γ_c, then the annual first order value of wind generated electricity may be written:

$$P\gamma_c = I\gamma_c L$$

The value of the capacity credit is approximately $Z\phi_g$. Hence the fractional value of capacity credit, R, is given approximately by:

$$R \simeq Z \phi_g / P \gamma_c$$

which in the limit of small penetrations may be approximated by:

$$R \simeq (P'/P) (\phi_g/\gamma_c) \simeq 1.24 \; 17.6 / 131 \simeq 0.15$$

A better approximation, valid for small penetrations, is given by substituting equation 4.2.7 into the above:

$$R \simeq (\phi_g/\gamma_c) [(P'/P) - \theta b'^2 (P'/P) (L/L') / 2a]$$

Fig 4.5.23 shows the above evaluated with the values for L' and L already given, with a specific power of 1.0, and an array penetration of 0.006.

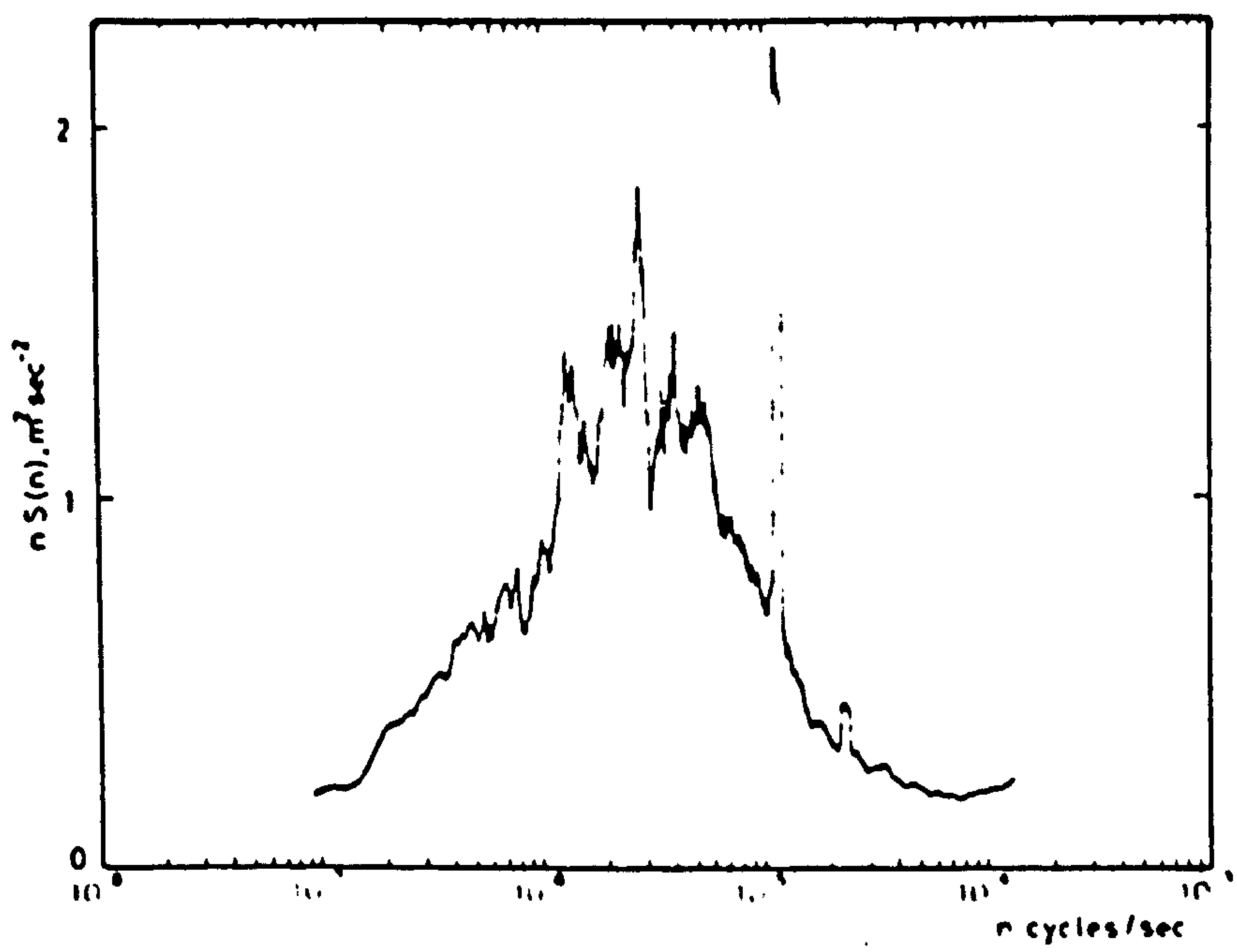


Fig 4.5.1 Spectrum of the horizontal wind speed. Measured at a height of 23 m. From Petersen 1975.

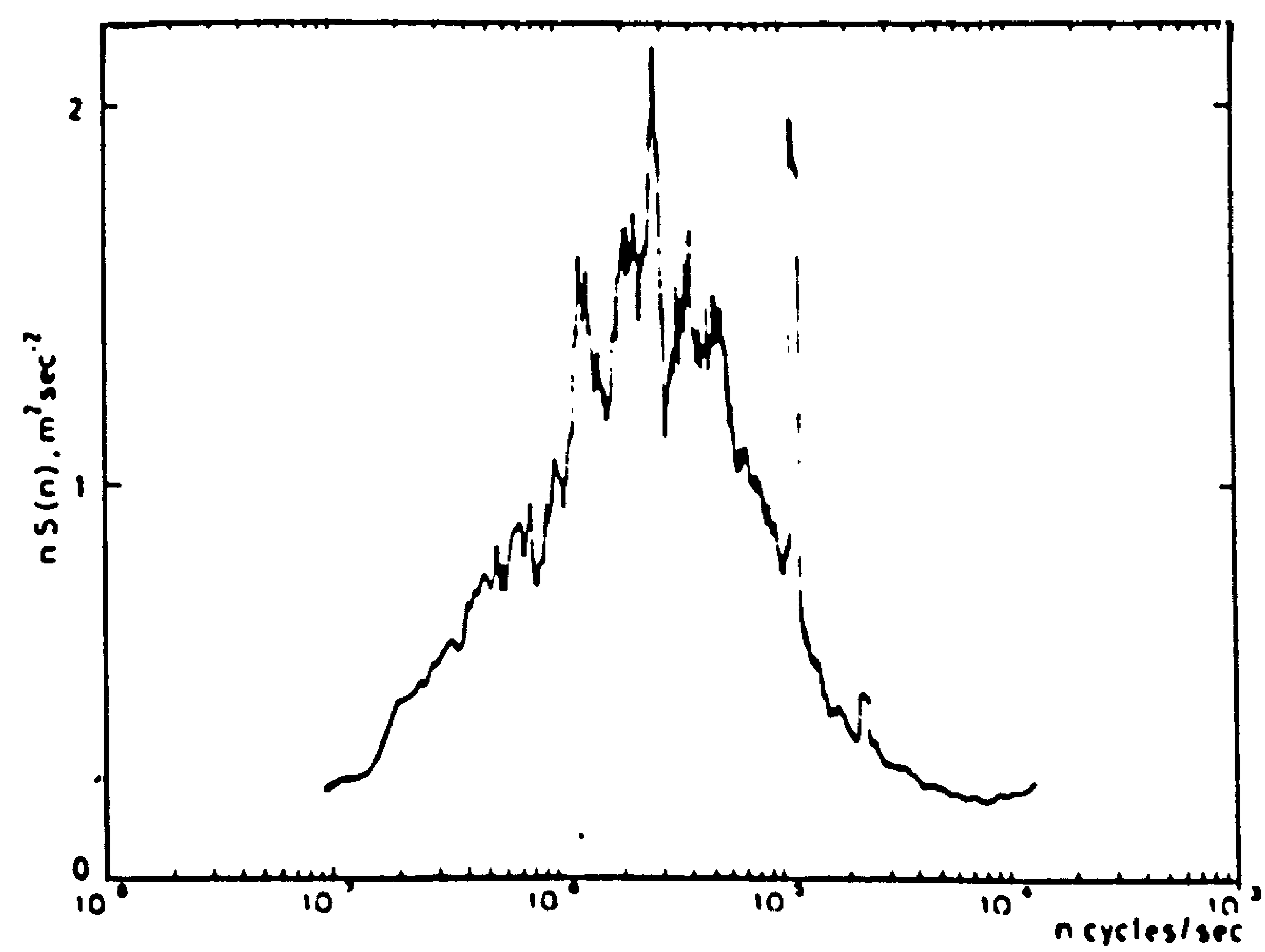


Fig 4.5.2 Spectrum of the horizontal wind speed. Measured at a height of 39 m. From Petersen 1975.

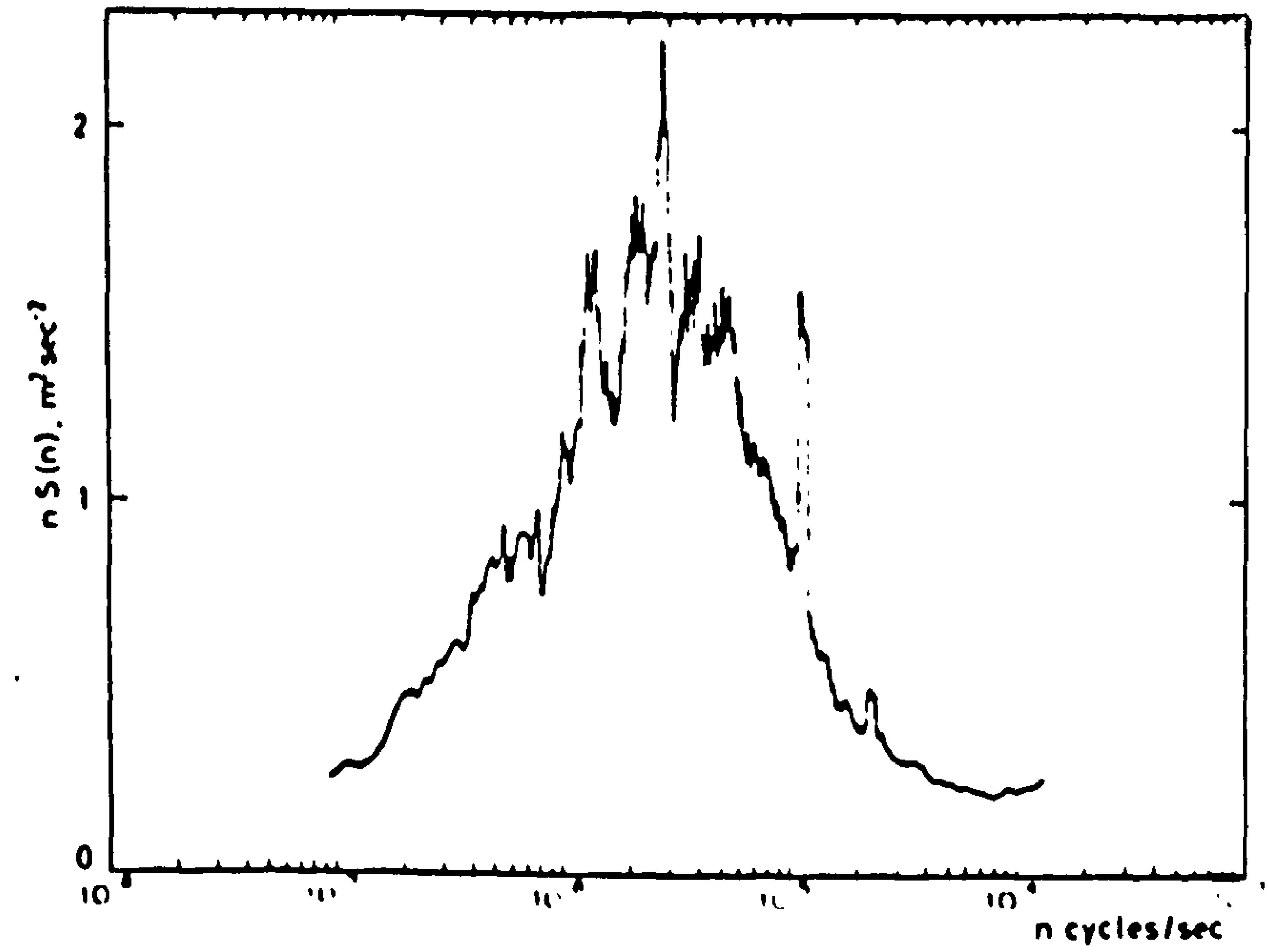


Fig 4.5.3 Spectrum of the horizontal wind speed. Measured at a height of 56 m. From Petersen 1975.

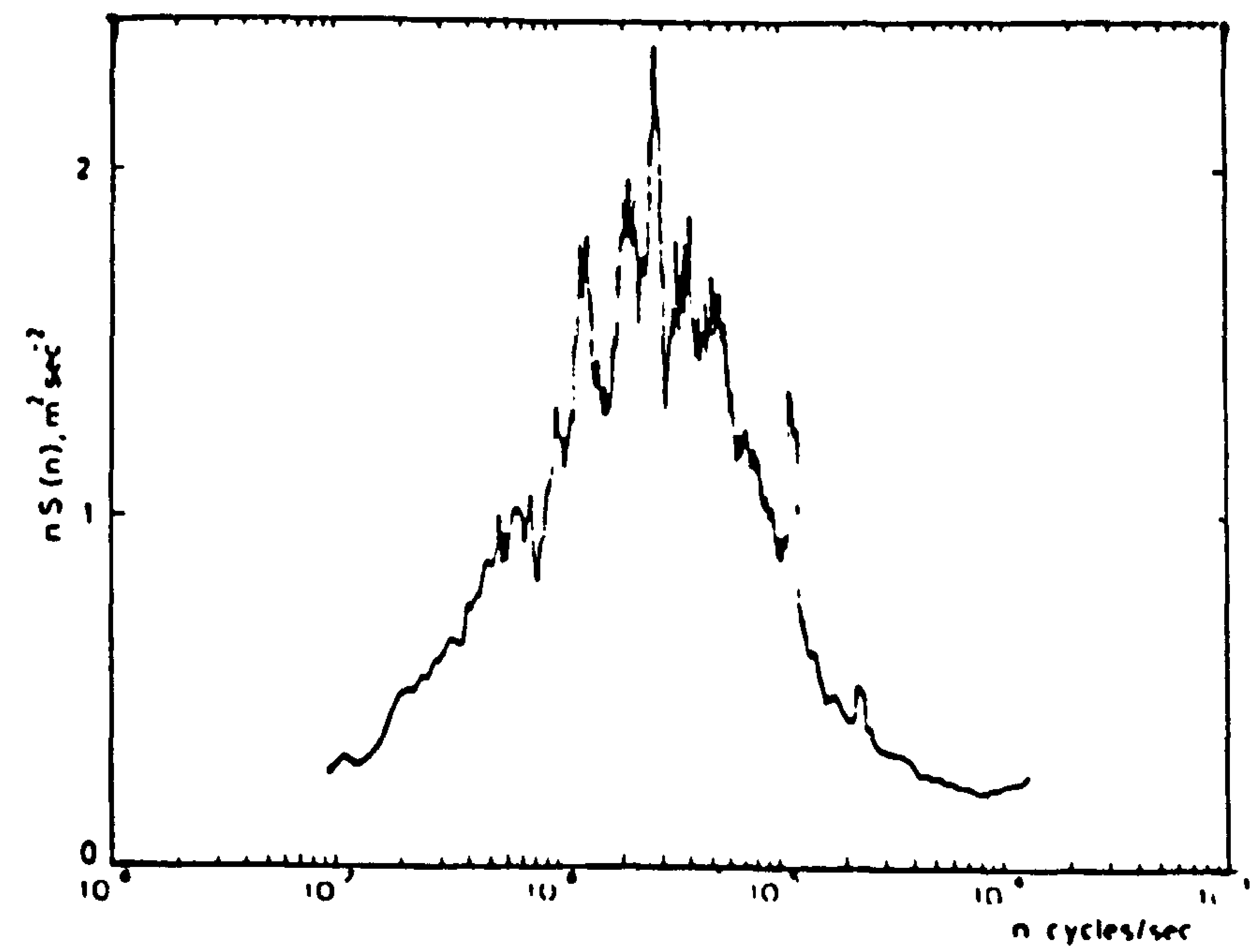


Fig 4.5.4 Spectrum of the horizontal wind speed. Measured at a height of 72 m. From Petersen 1975.

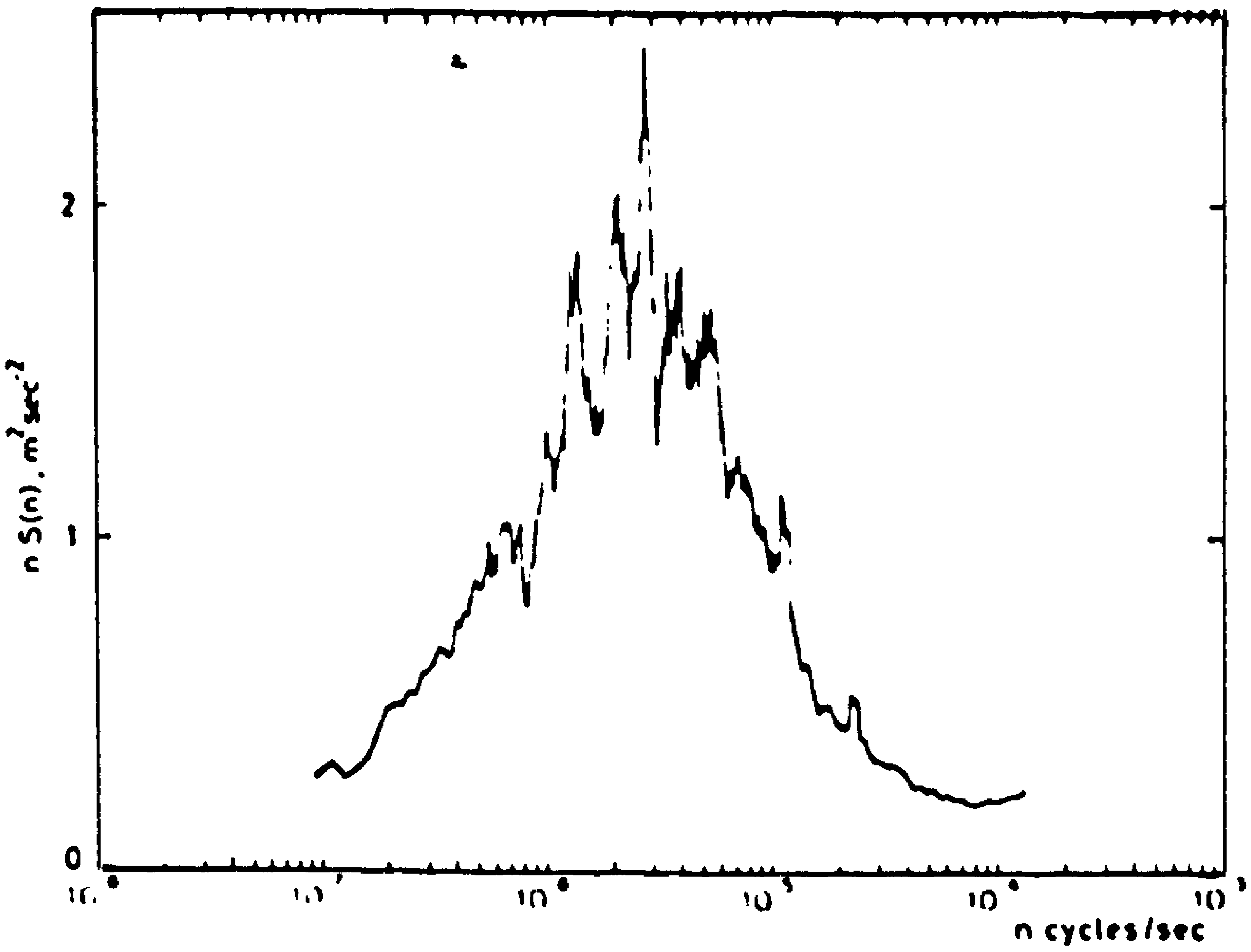


Fig 4.5.5 Spectrum of the horizontal wind speed. Measured at a height of 96 m. From Petersen 1975.

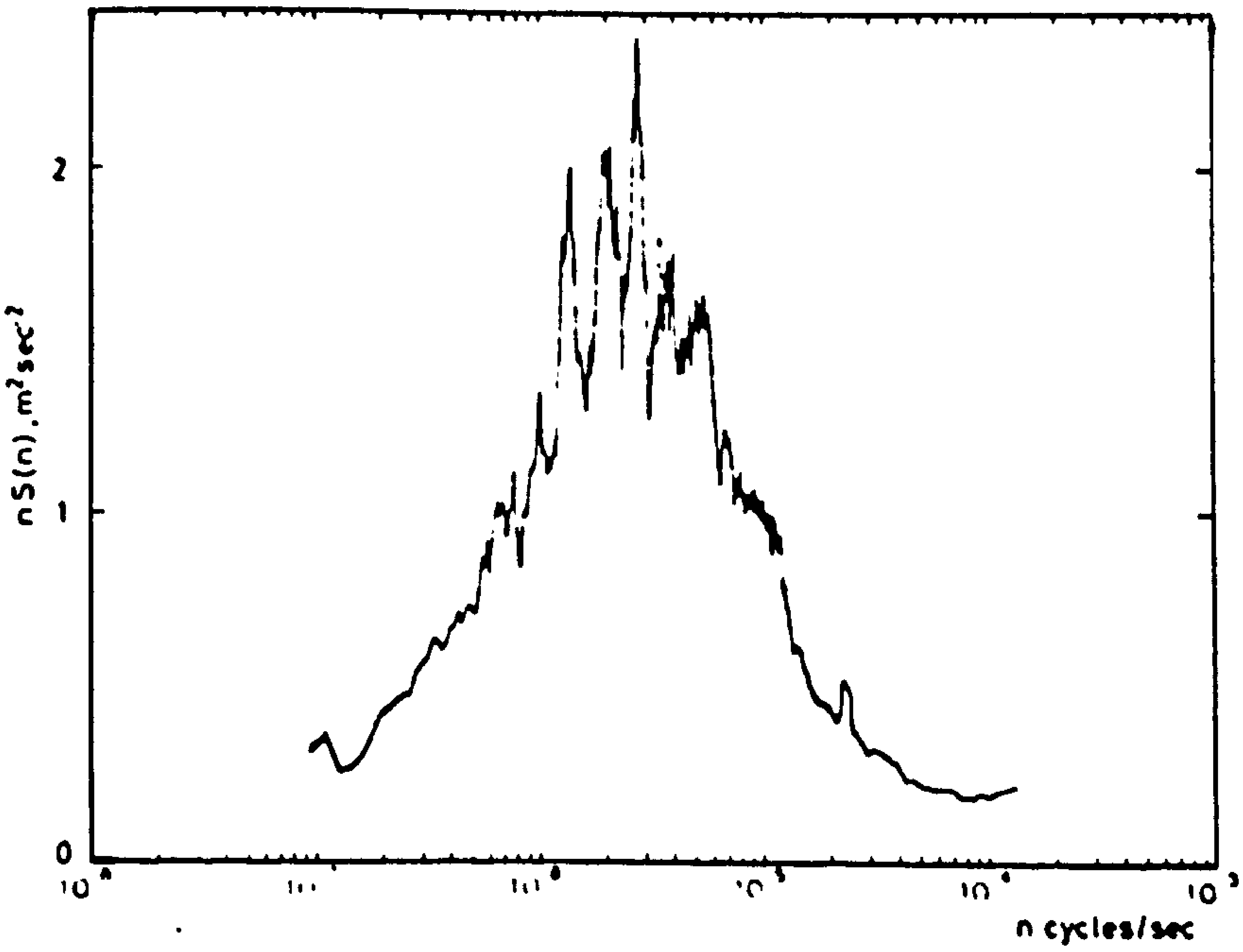
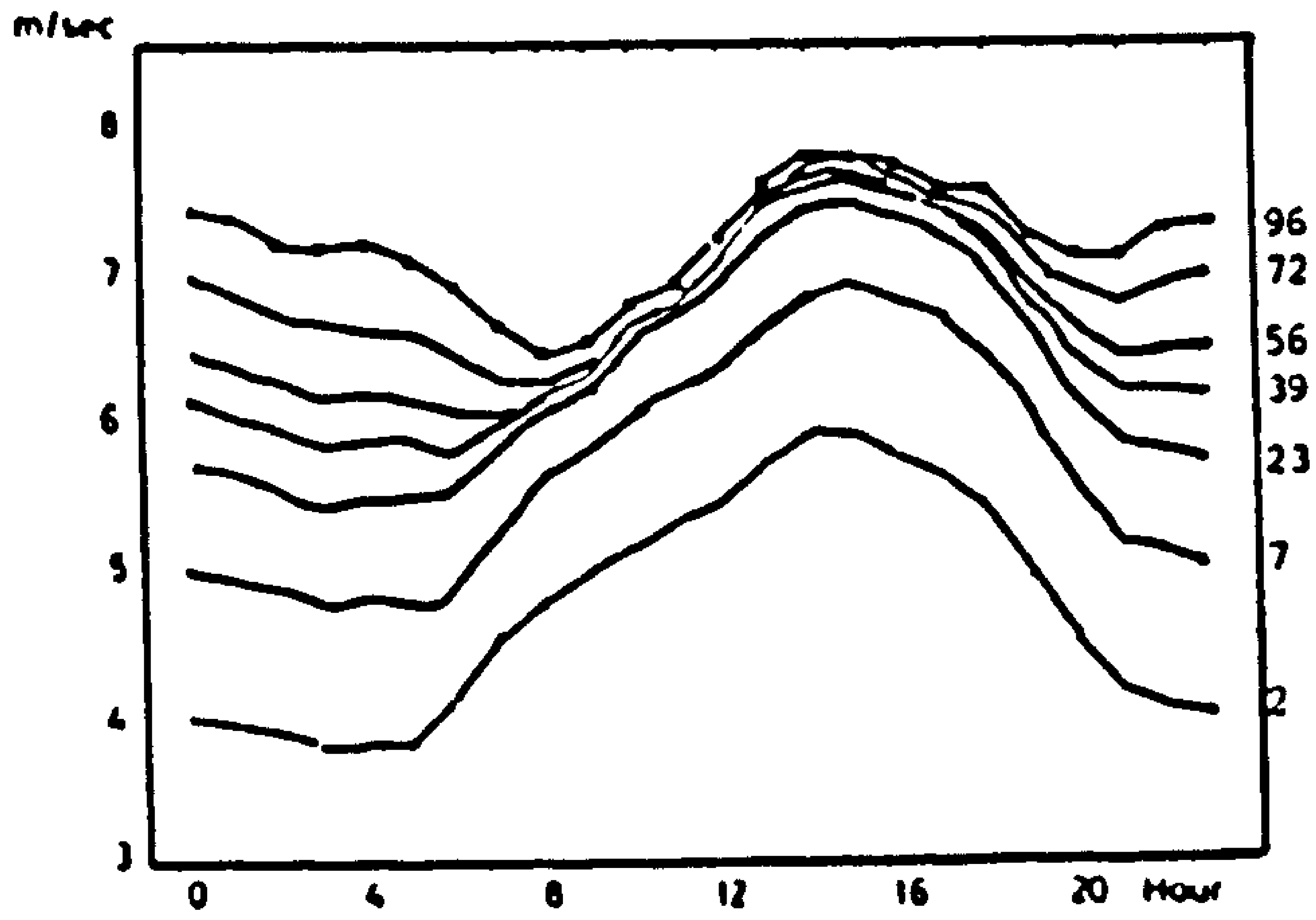
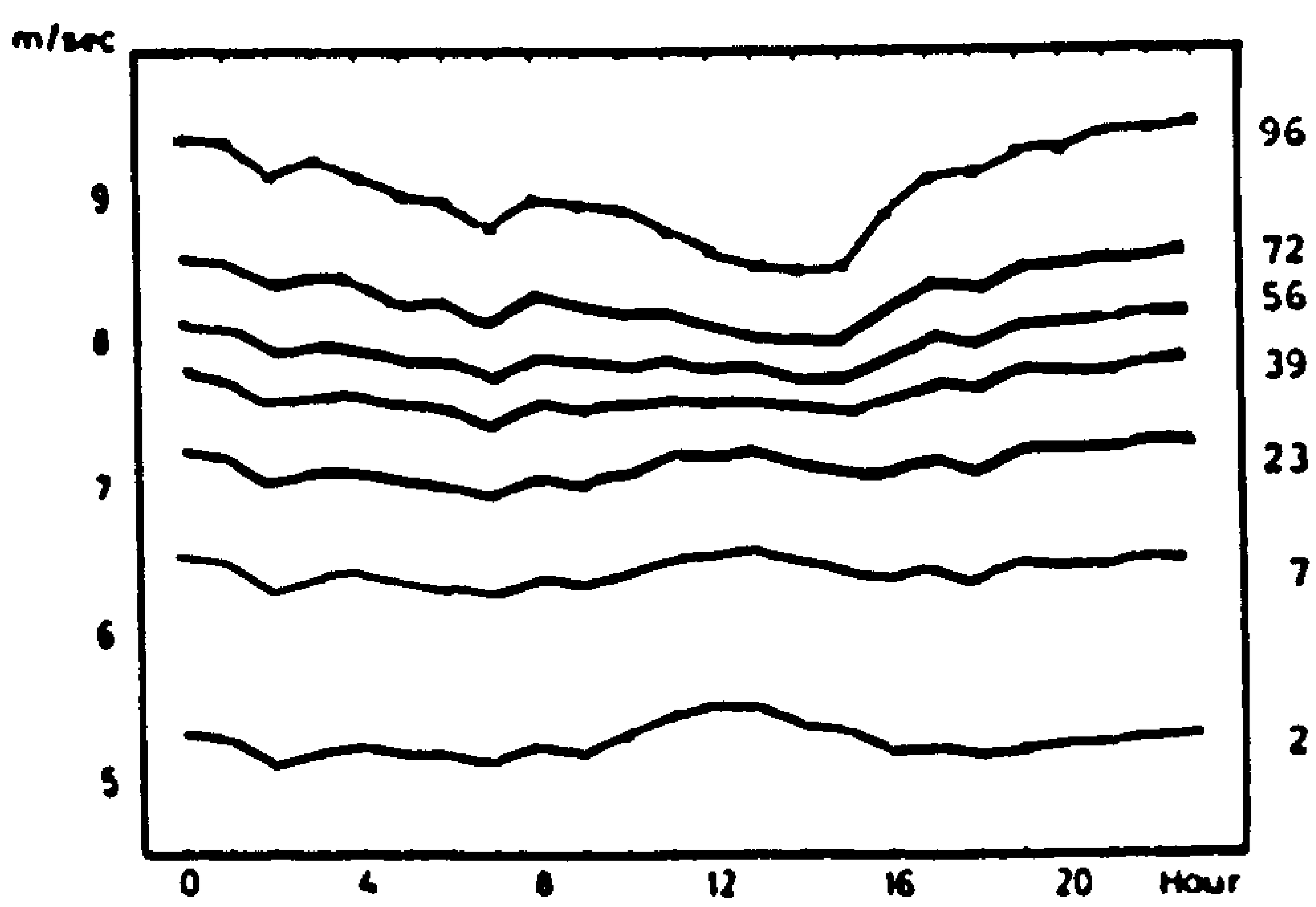


Fig 4.5.6 Spectrum of the horizontal wind speed. Measured at a height of 123 m. From Petersen 1975.



June



January

Figs 4.5.7 and 4.5.8 Diurnal variation in monthly mean wind speed as a function of measurement height. June and January. From Petersen 1975.

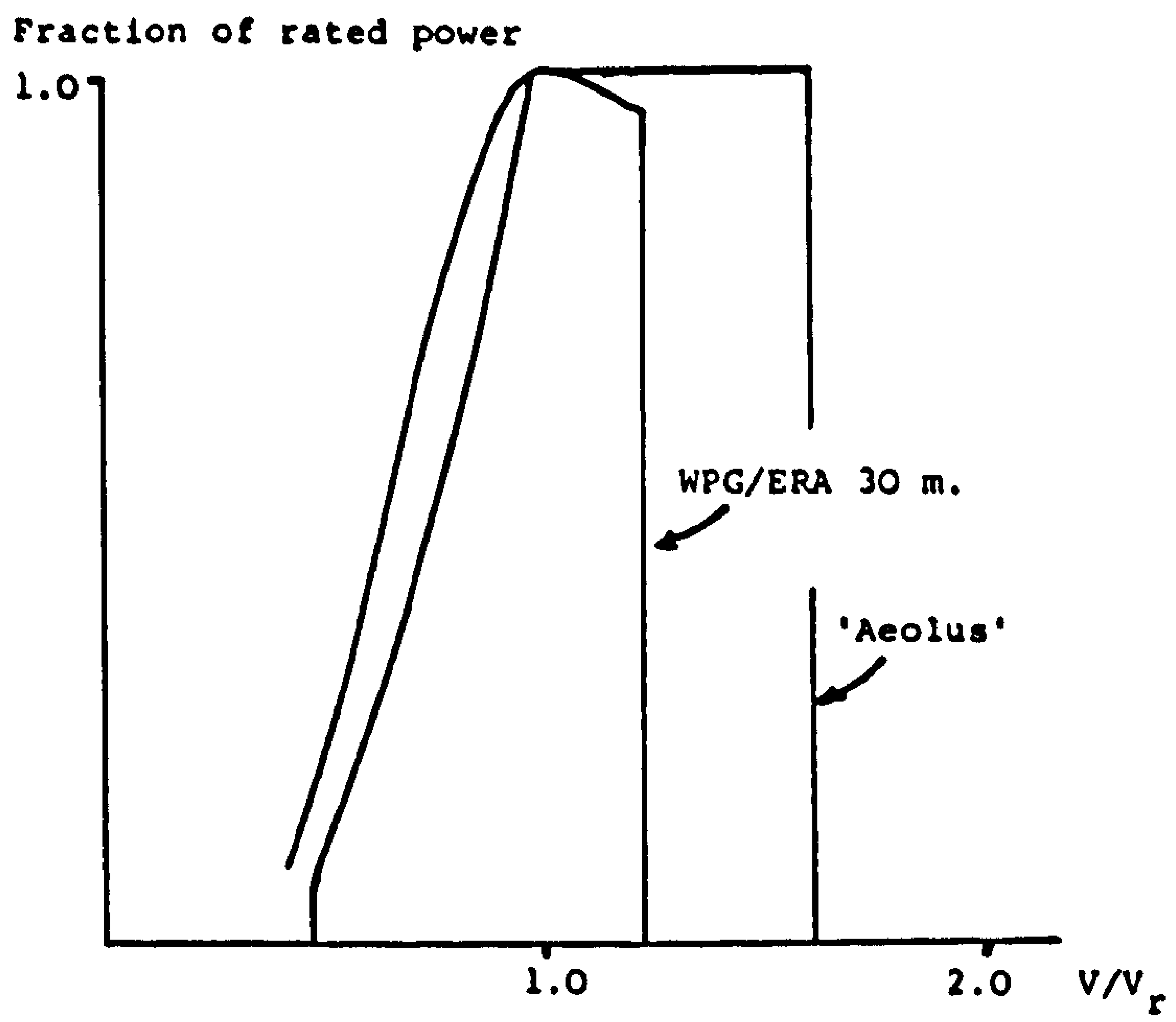


Fig 4.5.9 Turbine power characteristics plotted against V/V_r .

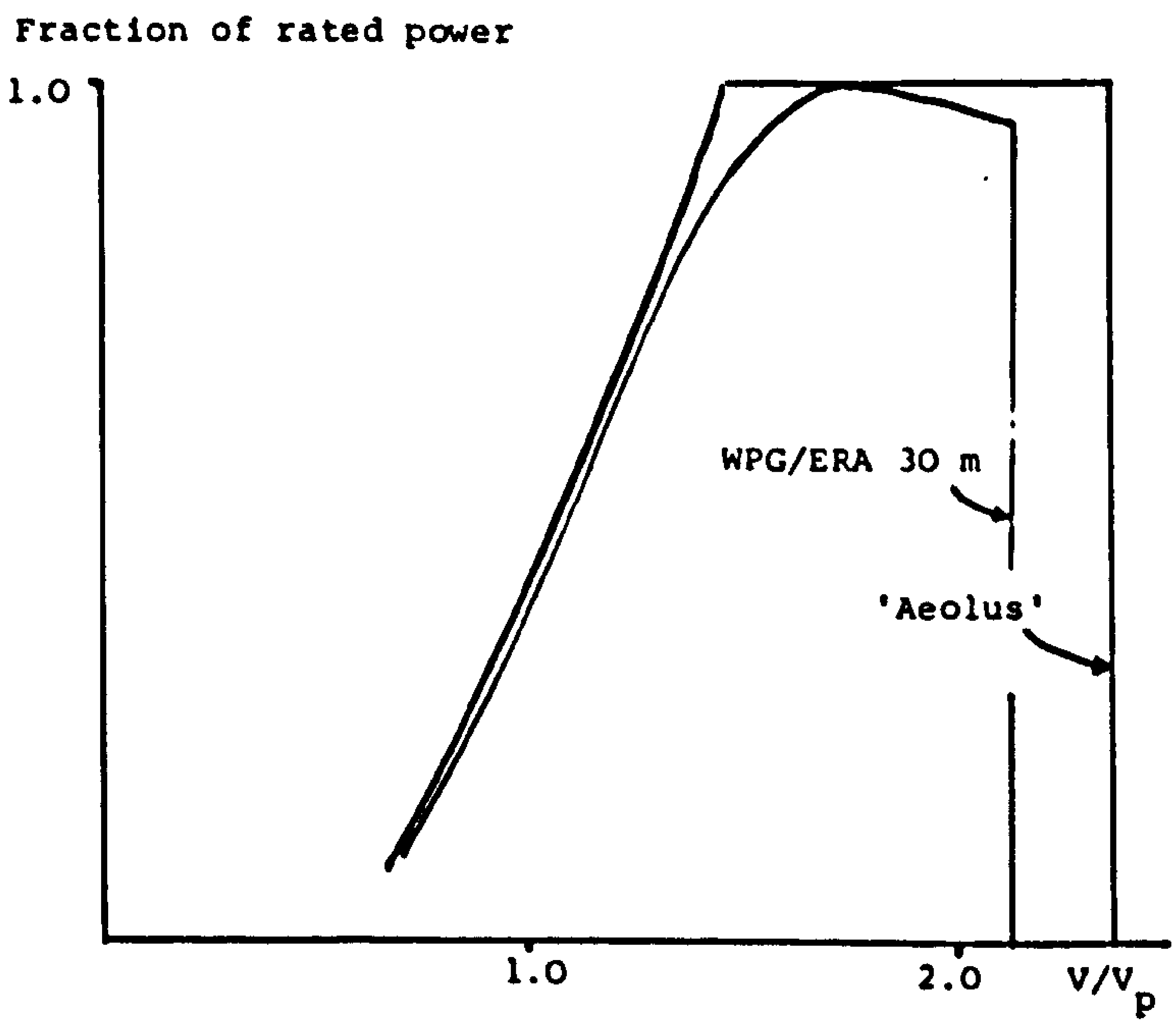


Fig 4.5.10 Turbine power characteristics plotted against V/V_p .

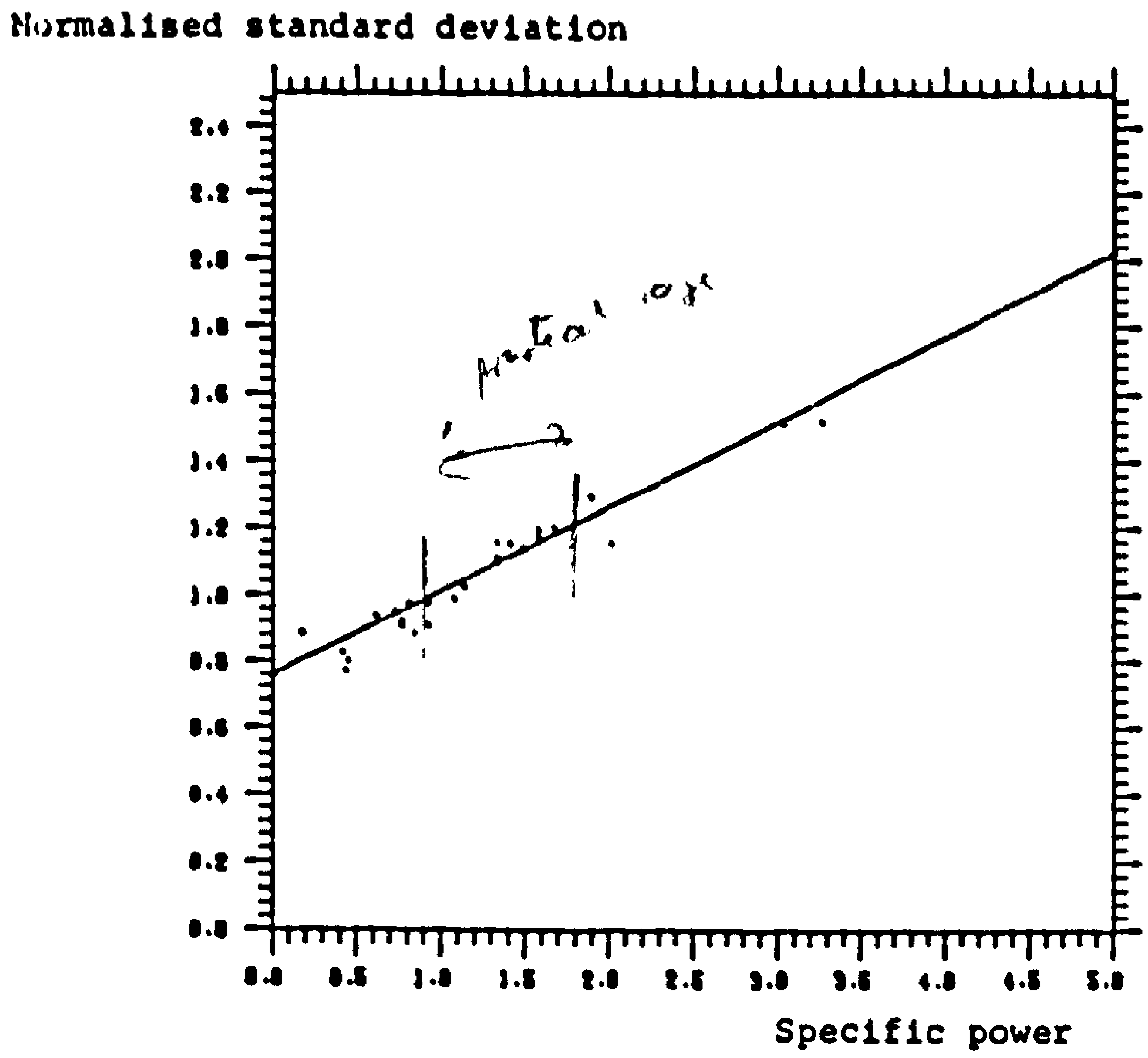


Fig 4.5.11 Normalised standard deviation of wind power vs. specific power of turbines: annual average.

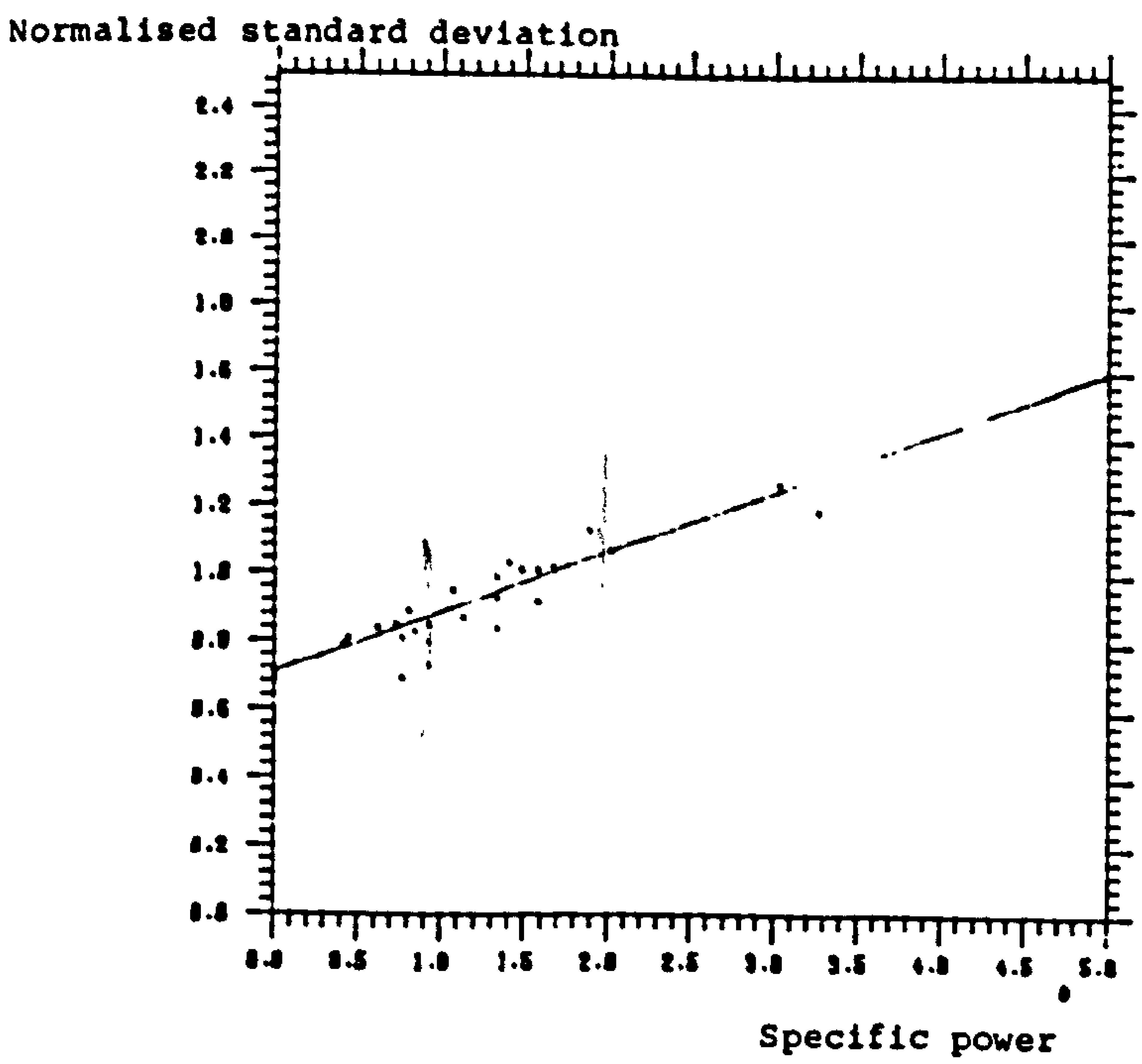


Fig 4.5.12 Normalised standard deviation of wind power vs. specific power of turbines: average for January.

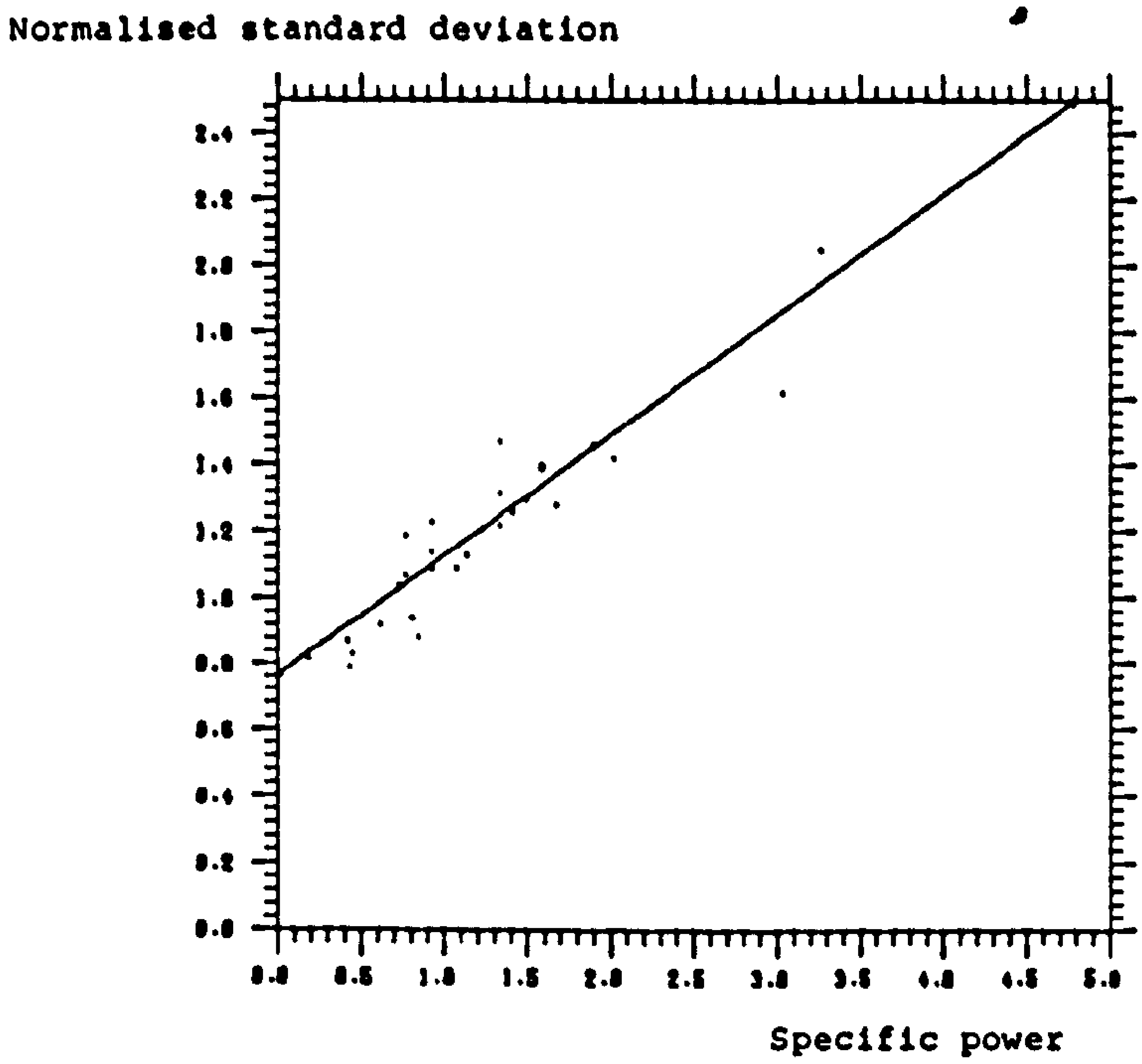


Fig 4.5.13 Normalised standard deviation of wind power vs. specific power of turbines: average for June.

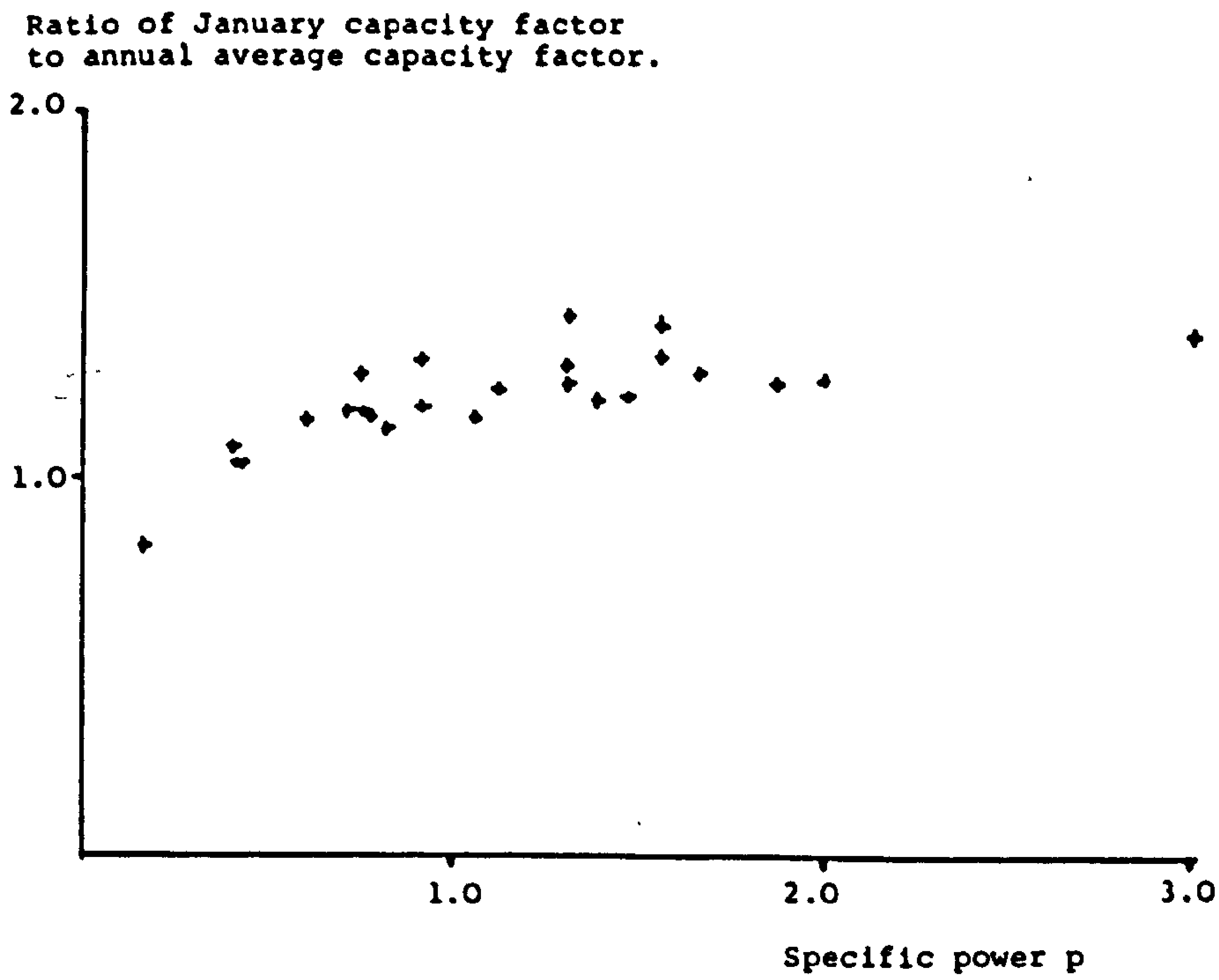


Fig 4.5.14 Ratio of January capacity factor to annual average capacity factor vs. specific power.

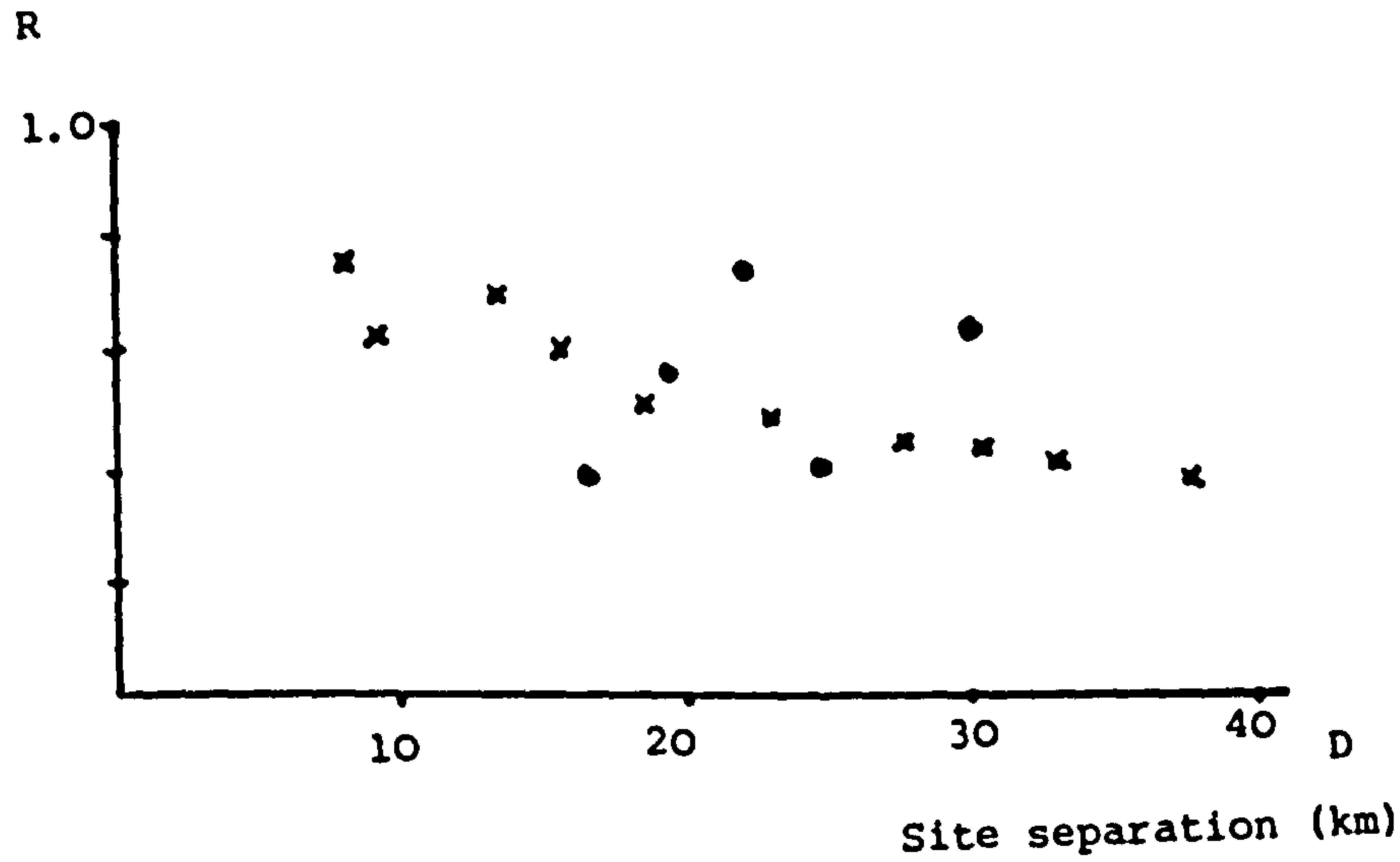


Fig 4.5.15 Cross correlation of wind speeds vs. site separation. From Goh and Nathan, 1979.

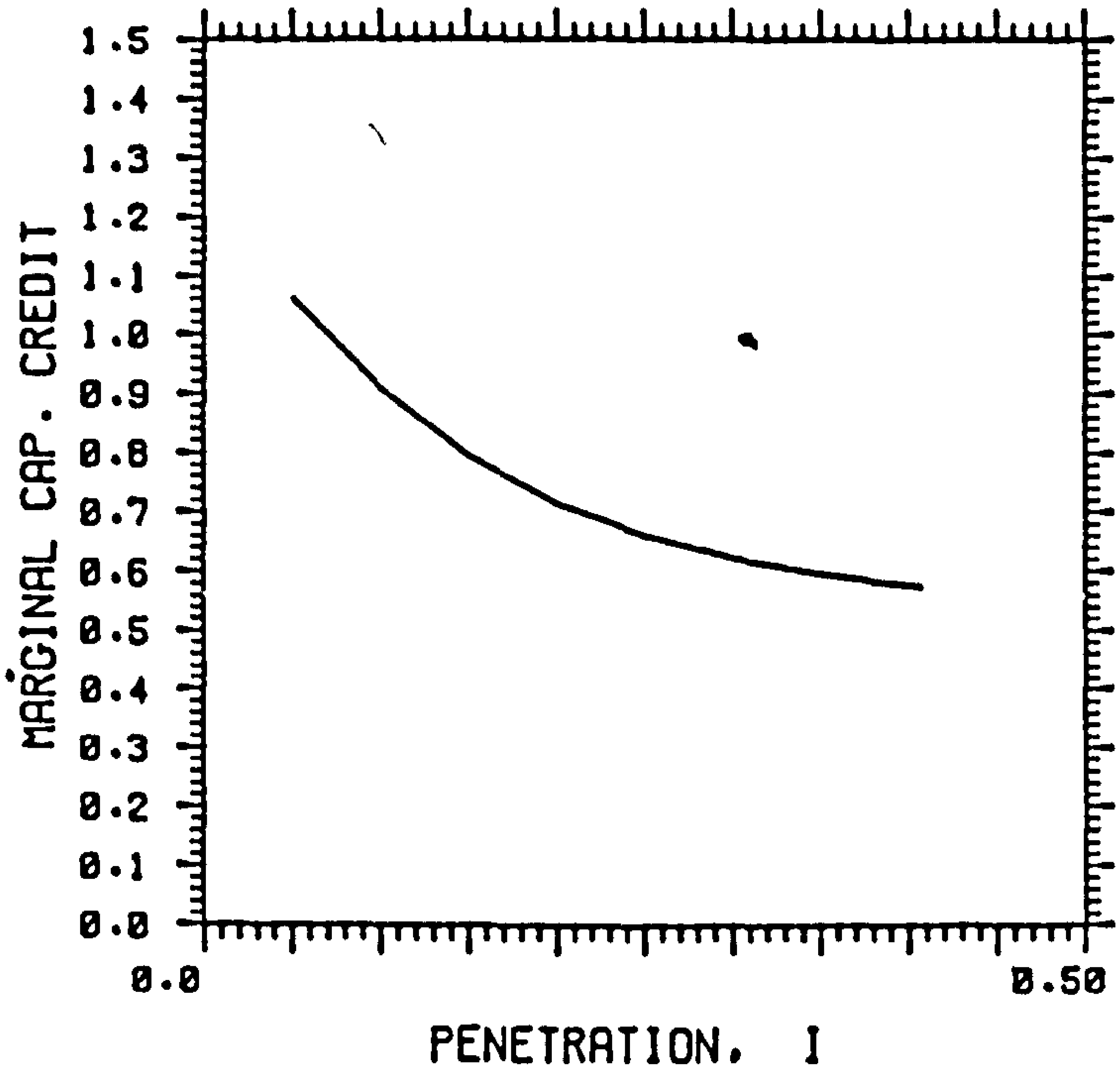


Fig4.5.16 Marginal capacity credit for wind. Constant geographical diversity corresponding to 10 well separated wind turbine arrays.

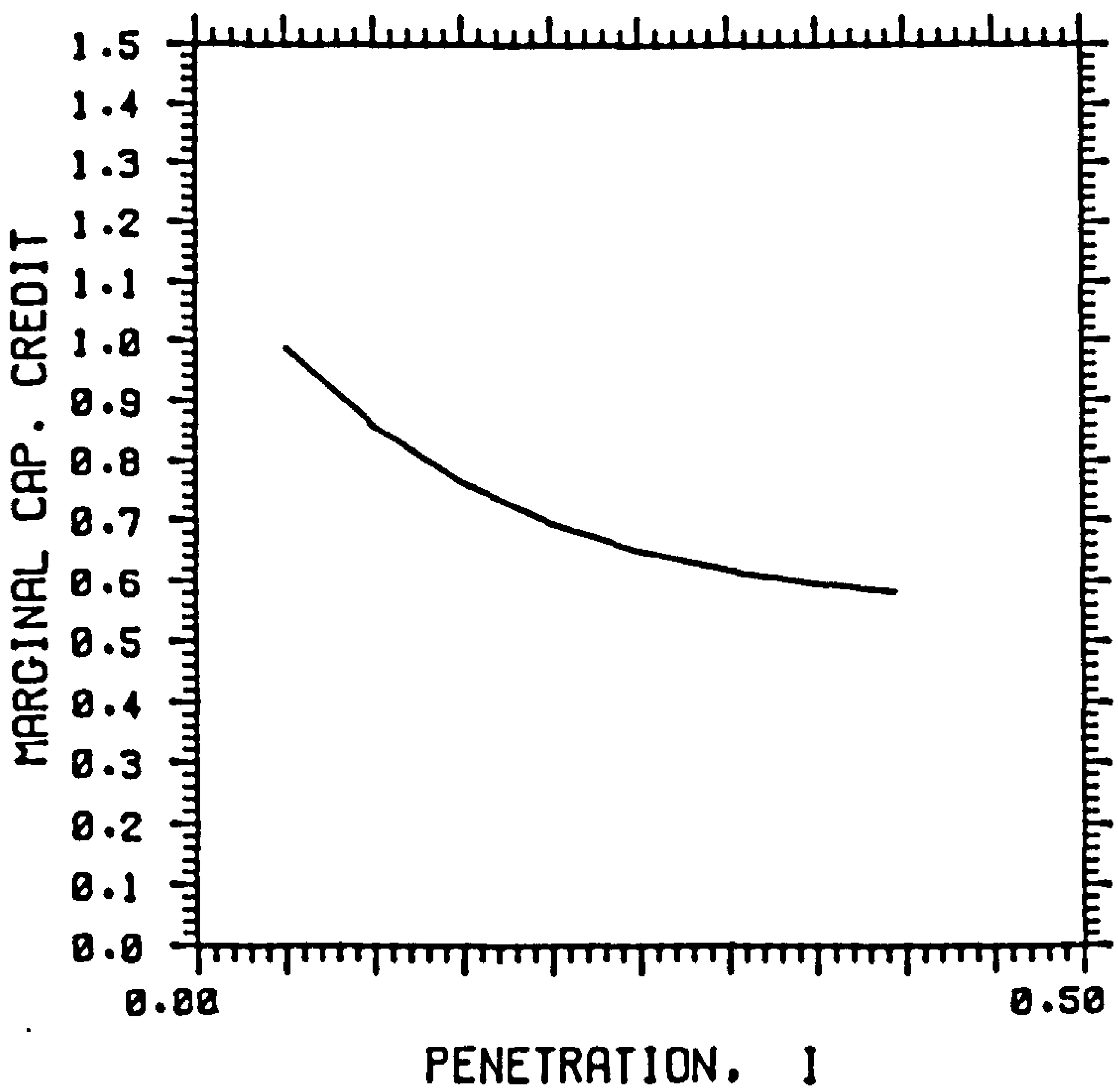


Fig 4.5.17 Marginal capacity credit for wind. Geographical diversity corresponding to an array penetration of 0.032.

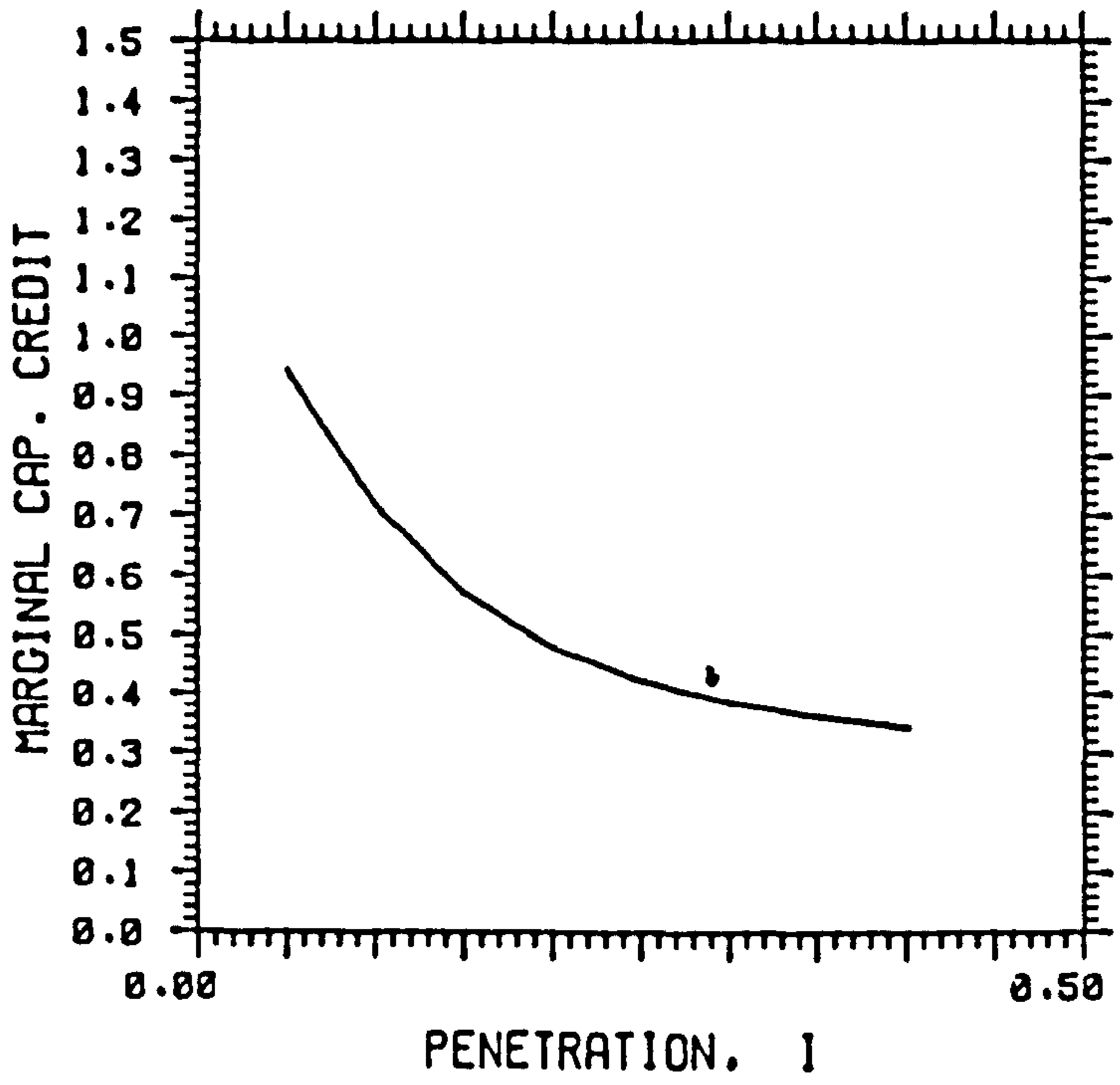


Fig 4.5.18 Marginal capacity credit for wind.
No geographical diversity.

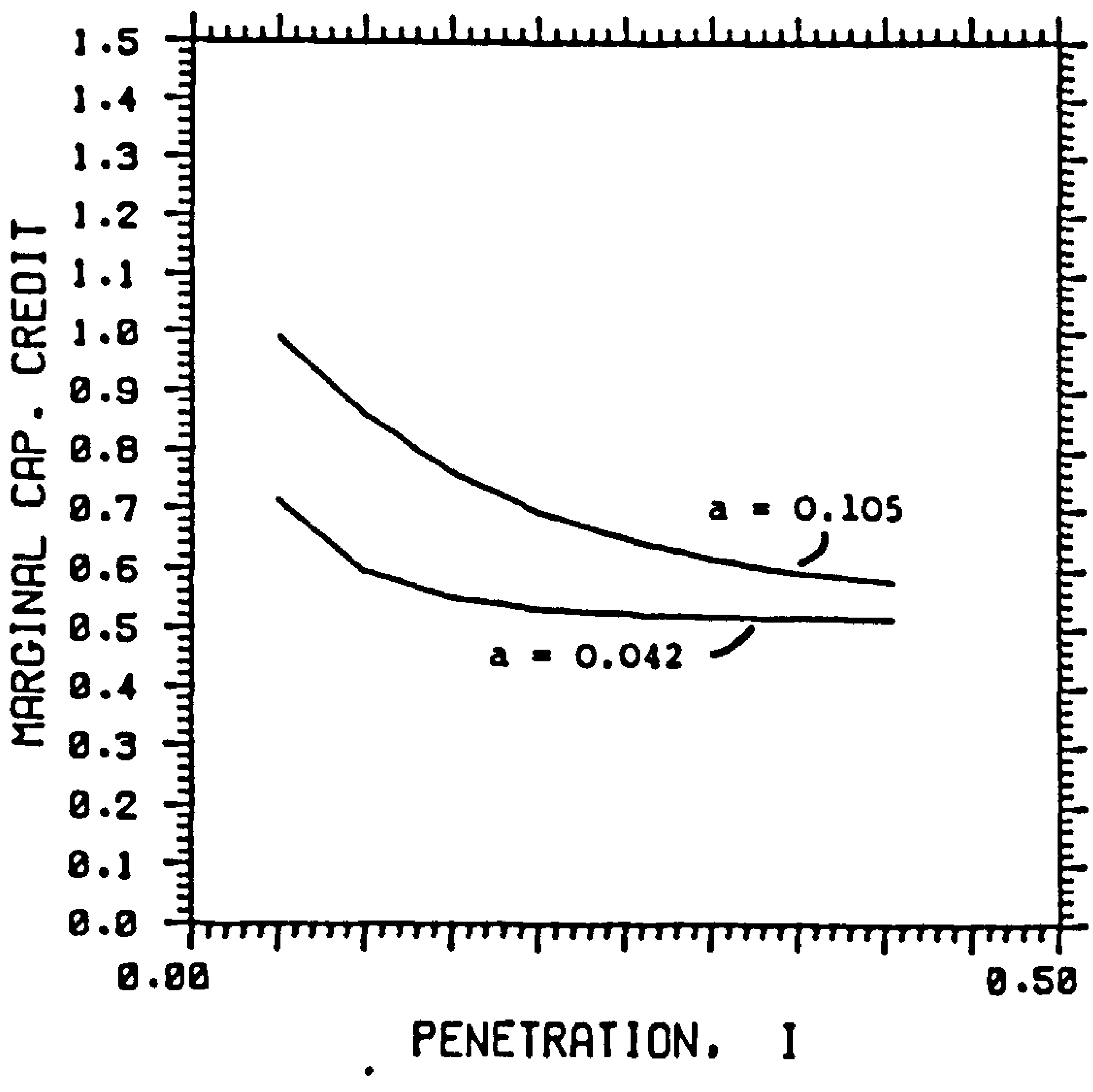


Fig 4.5.19 Marginal capacity credit for wind. Effect of variation in conventional system uncertainty, a. Based on an array penetration of 0.032.

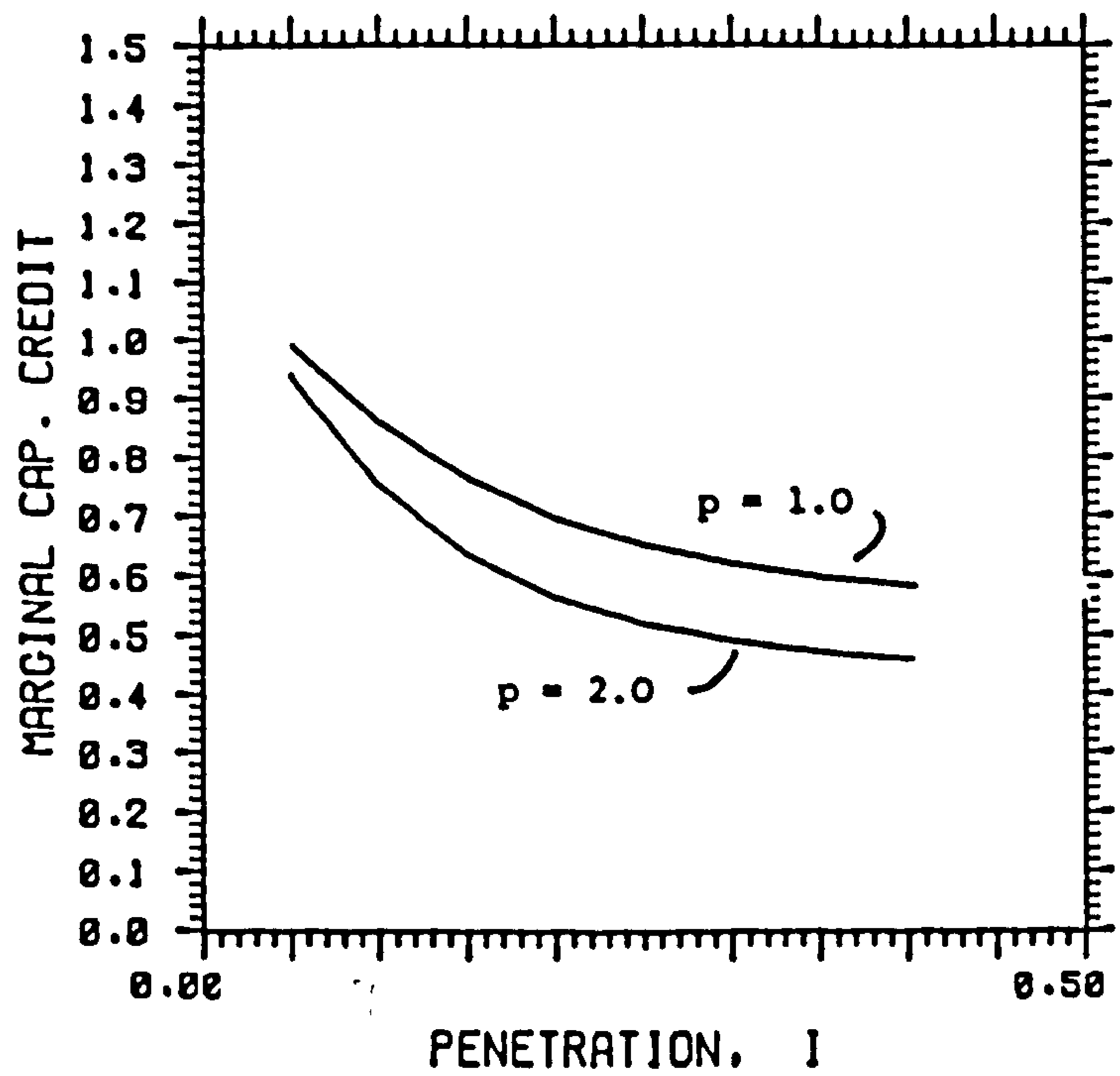


Fig 4.5.20 Marginal capacity credit for wind. Effect of variation of specific power of wind turbines. Based on an array penetration of 0.032.

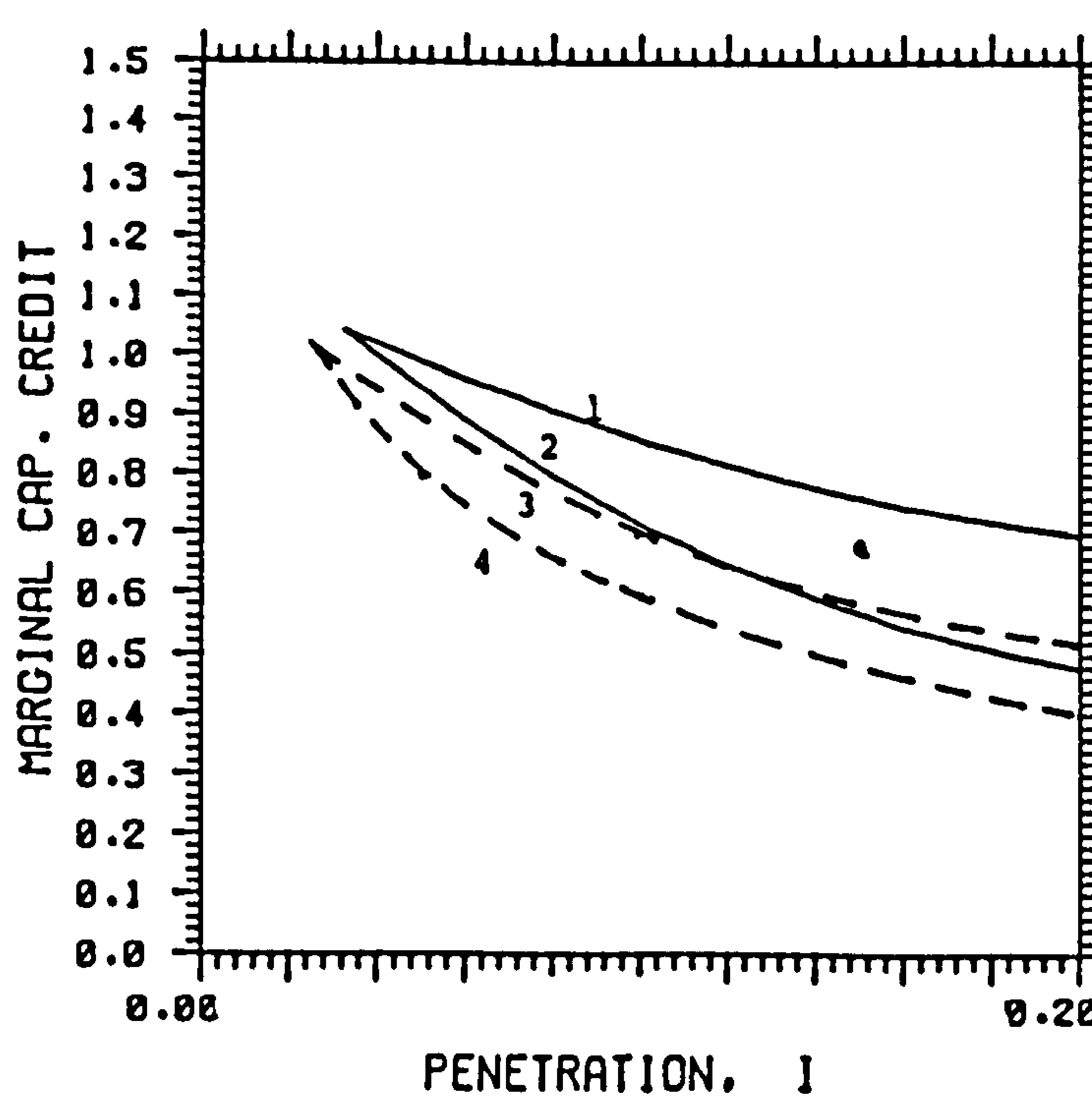


Fig 4.5.21 Marginal capacity credit for wind. Comparison with results reported by Jarass et al.

1. Lowe, array penetration 0.032.
2. Lowe, no geographical diversity.
3. Jarass et al, geographical diversity.
4. Jarass et al, no geographical diversity.

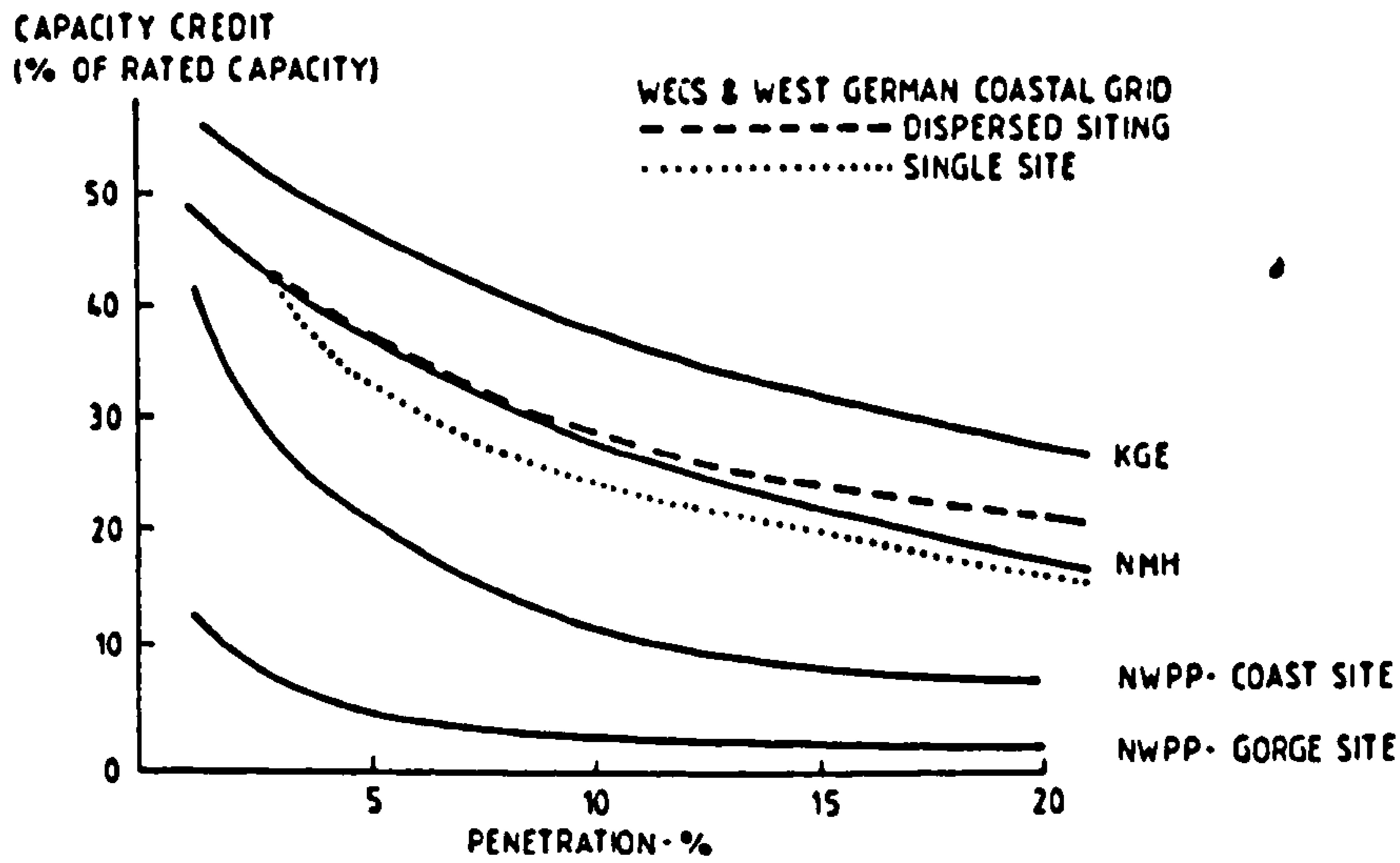


Fig 4.5.22 Marginal capacity credit for wind. Results reported by Rockingham and Taylor, 1980. Note that definitions of capacity credit and penetration differ from those used in the present work.

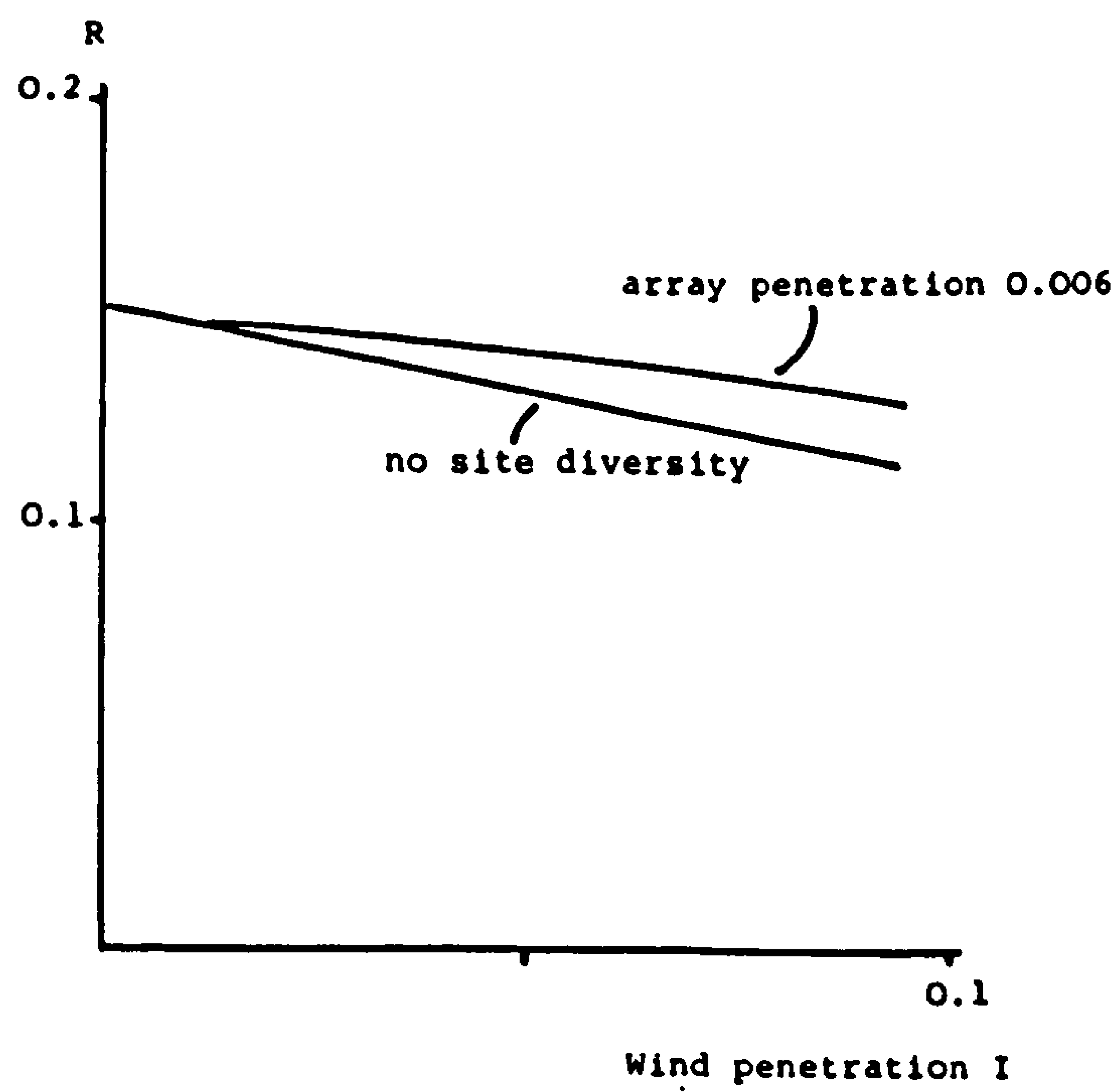


Fig 4.5.23 Fractional value of wind capacity credit.

4.6 LIMITATIONS OF THE CONCEPT OF CAPACITY CREDIT

From within the welfare economic paradigm, the definition of capacity credit which has been used hitherto is quite seriously at fault. This is because it depends on a judgement on the part of the utility, of the value to the public as a whole, of a given level of security of supply. This formulation is inadequate in the first place because it lumps together consumers who would ascribe different values to security of supply, and on a priori grounds leads to a security of supply standard greater than most consumers would opt for, if faced with the marginal costs of achieving that standard. A second inadequacy in the formulation is that the organisation which makes the judgement as to the level of security of supply, is likely to have an interest in the result of the judgement. This second difficulty is not easily avoided - given any theoretical framework, the technical competence of the utility will be a powerful factor in decision making.

In order to illustrate the deficiencies in the security of supply based definition of capacity credit used above, I will outline a theoretical economic approach to electricity tariff structure and system planning based on the assumptions:

1)that the utility is able to vary the price of electricity depending on its own costs at any time, and

2)that consumers are able to respond to varying electricity prices by adjusting their demand in line with their own ordering of priorities. I will refer to such a system as a "spot pricing system". The analysis which follows owes much to work by Berrie (Berrie, 1981a, 1981b, 1981c and 1981d).

Given a spot pricing system, what is the best way to design the electricity tariff? We can approach the problem from a simple welfare economics position, following eg. Williamson, 1966. The first postulate that we need is that electricity consumers behave in a classical economic fashion - that at any point in time the marginal utility of consuming electricity is a monotonically decreasing function V, of power supplied. So, given a spot pricing system, consumers will increase demand until marginal utility of consumption equals the spot price.

Within the welfare economics framework we wish to maximise the benefit to the consumers plus the benefit to the utility of producing and consuming electricity. Consideration of fig 4.6.1 will show that a) the price of electricity should never be less than its marginal production cost, and b) that if the demand at this marginal cost does not exceed the available capacity Q' of the system, then the price should equal the marginal cost of production. I will call periods for which the above is true, off-peak periods.

To simplify the mathematics in the following I will assume that the costs of supplying electricity can be linearised, as set out in chapter 2, into a fuel based running cost Y, and a capital charge Ø.

The next question is, what does the utility do if demand at the marginal cost exceeds installed capacity? This case is illustrated in fig 4.6.2. The utility has two courses of action open to it in this case. It can raise the price of electricty above the marginal cost Y, to the level P, at which point demand equals available capacity Q'. Or it can turn consumers off until the residual demand curve gives a demand Q' at the price Y. The first point to notice here is that the welfare function (consumer surplus plus utility surplus) is indifferent to changes in price which do not affect anything else (such changes being known as rent). If the electricity utility can arrange to turn off loads in order of marginal utility of consumption, then it could set the price of electricity anywhere between Y and P and shut off excess demand without affecting the welfare function. (In practice utilities do pay some attention to which loads are shut off, in the event of power shortages.) However, if we can assume that such problems as income distribution and the marginal utility of money are not important (!), the electricity utility can leave decisions as to what to turn off or down, up to consumers, by taking the first of the two courses of action. Arguably this is easier than trying to do the job for them at some other price. Therefore I will assume that in periods described by fig 4.6.2 (which I will call peak periods) this is what the utility does. Note however, that in a system in which most loads do not interact with spot price in real time, the utility may have to adopt some mixed system.

The next question is how does the utility decide what value of Q' to use? It is at this point that the discussion starts to intrude into the area of capacity credit.

Assume that there n peak periods, with durations $x_1, x_2, \ldots x_n$ as a fraction of total time. The case where n=2 is illustrated in fig 4.6.3. The sum of consumer plus utility surplus is:

1. $$S = \sum_{i=1}^{n} x_i \int_0^{Q'} v_i(Q)\,dQ \quad - \quad x_i P_i Q'$$

$$- \emptyset Q' \quad + \quad \sum_{i=1}^{n} x_i P_i Q' \quad - \quad x_i Q' Y$$

Differentiating this with respect to Q' gives:

2. $$dS/dQ' \quad = \quad - \emptyset \; + \; \sum_{i=1}^{n} \; x_i V_i(Q') \quad - \quad x_i Y$$

Hence for S to be a maximum:

3. $$\sum_{i=1}^{n} x_i V_i(Q') \quad = \quad \emptyset \; + \; Y \sum_{i=1}^{n} x_i$$

The conclusion from equation 4.6.3 is that for the combined welfare of the utility and the consumers to be maximised by a system of spot pricing, the utility should install plant until the total revenue received in peak periods ($\sum x_i V_i(Q')$) is equal to the running costs of meeting demand in peak periods ($Y\sum x_i$) plus all the capital charges which the utility pays on its plant ($\emptyset$). In other words:

1) in off-peak periods the utility should charge all consumers the fuel cost of producing electricity.

2) in peak periods the utility should charge high enough prices to keep demand equal to available capacity.

3) if the system is optimised, the utility will find

that the rent collected in peak periods (which I will call peaking charges) will cover its capital charges.

4) if the system is not optimised, the utility will either make a profit, in which case it should build more plant, or it will make a loss, which should be taken as a messsage that the installed capacity of the system is too great.

This type of analysis can be extended to include the case where the utility operates a variety of types of plant with different fuel and fixed costs Y_i and $\emptyset_i$ respectively. I do not wish to go into the detail of the general operation of such a system with spot pricing, but the basic ideas can be illustrated using a theorem derived by Vimukta (Vimukta et al, 1978). This demonstrated that the total annual costs of operating an electricity system, given by the following in the linear approximation:

$$4. \qquad a = \sum_i \{ Y_i \int_{x_i}^{x_{i-1}} (f(x)-f(x_{i-1})\, dx \;+\; x_i(f(x_i)-f(x_{i-1})) + \emptyset_i(f(x_i)-f(x_{i-1})) \}$$

can be rewritten:

$$5. \qquad a = \emptyset_1 f(0) \;+\; \sum_{i=1} Y_i \int_{\bar{x}_{i-1}}^{\bar{x}_i} f(x)dx$$

when the plant mix is optimised (this equation has been referred to previously as equation 2.2.8). Diagramatically, the implication of equation 4.6.5 is that the area under the load duration curve can be split into vertical strips, as shown in fig 4.6.4. In this figure the area of region 3 is equal to the energy which is supplied when the power demand is less than the installed capacity of base load plant, the area of region 2 is the equal to the energy which is supplied when the marginal plant is mid range, and the area of region 1 is equal to the energy which is supplied when the marginal plant is peaking plant. The total cost of operating the system is given by the sum of each of these quantities of energy multiplied by the respective fuel cost, plus the installed capacity multiplied by the unit cost of the peaking plant. This is in contrast to the implication of equation 4.6.4, which is that the total cost of operating the system can be found from a horizontal division of the area under the load duration curve. Equation 4.6.5 implies that, for a system in which the plant mix is optimised, the decision whether to install additional capacity should be made on the basis of whether or not the peaking charges collected in such periods cover the capital costs of the installed capacity priced at the cost of peaking plant.

It should be noted that the dimension of time has been excluded from the above analysis in that all decisions are assumed to be based on flows of money in the current year. A more complete analysis would replace the single annual revenues in the above by present values. A spot pricing system does not remove the need to forecast demand.

Conclusions

The first point to make about the forgoing is that it presupposed that it is possible to introduce a spot pricing system for electricity. It is probably true that up to recently this would have been technically difficult and expensive. It now seems that such a system may be both technically possible and cheap enough to install in the domestic sector (Cogle, 1981). It is interesting to speculate on the magnitude of the effects of the introduction of such a system in the UK. Pryke, (Pryke, 1981) states that peak load control might shave an additional 3 - 6 GW off the peak demand of the combined CEGB and Scottish systems. Berrie (Berrie, 1981a op cit) suggests savings from spot pricing combined with interactive load control of the order of 1/7 of the total annual costs of operating the CEGB system.

As a corollary of the reduction in peak load resulting from a system of spot pricing, the mix of demand would probably be affected. Those loads with a low annual load factor, which contribute strongly to peak demand, would become more expensive. On peak electric space heating is a good example. Loads which have high annual load factors and which are not correlated with system peaks would become cheaper - in fact there would be a reduction in cross subsidisation which would favour the latter class of consumers.

It should be noted that the ability of all consumers to respond interactively in real time to changes in spot price is not a necessary condition for the introduction of such a system to be a good thing. It is also not a necessary condition for spot pricing to affect the mix of appliances. It would rapidly become general knowledge that certain devices were more expensive to run under a spot pricing system (unless the consumer were careful only to use them at times of low spot price). These devices would simply become uneconomic (or rather would be seen to be uneconomic) under a spot pricing system, and would be phased out. The same devices might make a come-back if converted to intelligent control - depending of course on the cost of such control.

The objective of this section has been to counter balance the attention that I have paid to one particular method of calculating capacity credit, by demonstrating the ad hoc nature of the definition of peak demand that it is based on. Given this definition, capacity credit for wind generators is very much a second order quantity in the economics (perhaps of the order of 10 - 20% of the total value to the grid of wind generators). Changing the basis of tariffs and system planning in the ways outlined above would probably make much of the analysis and discussion of the rest of this chapter irrelevant.

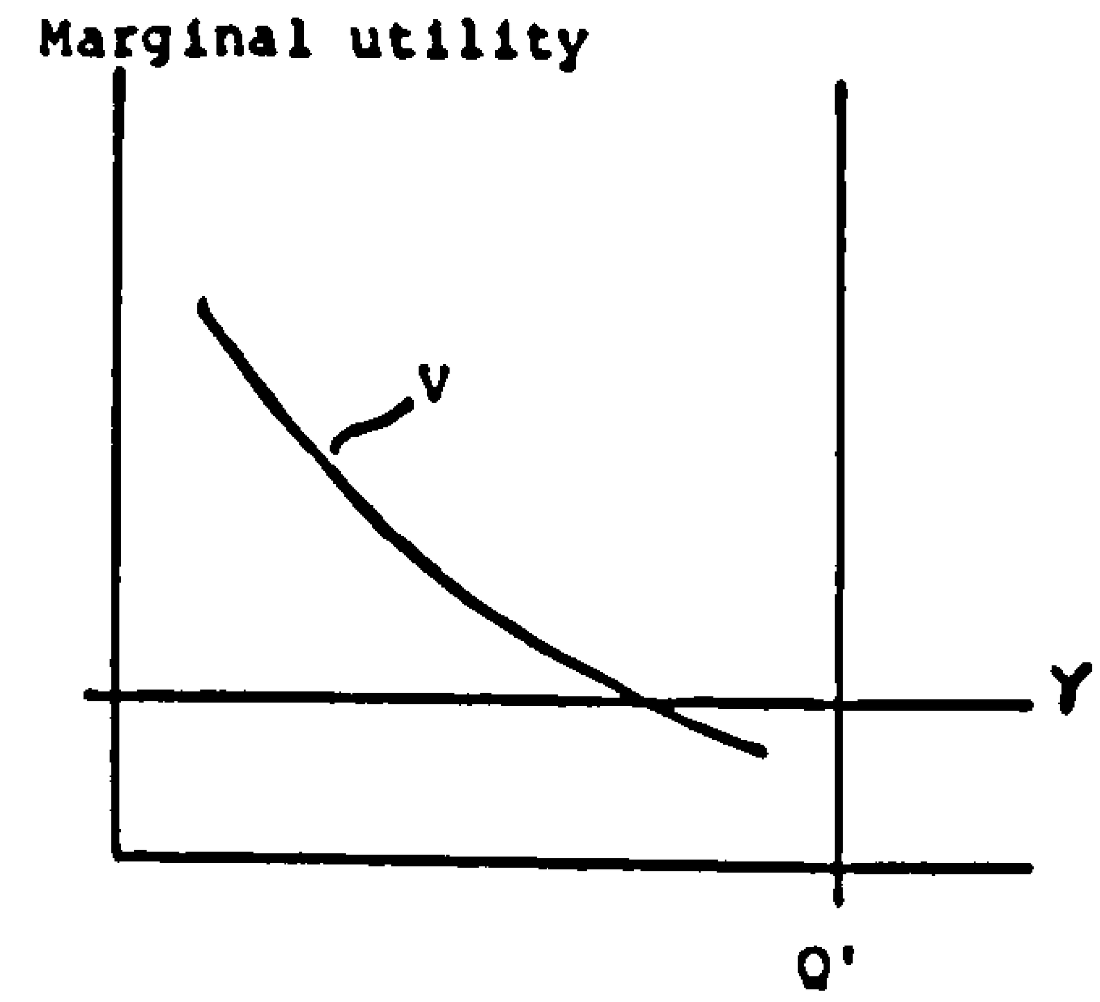


Fig 4.6.1 Off peak periods.

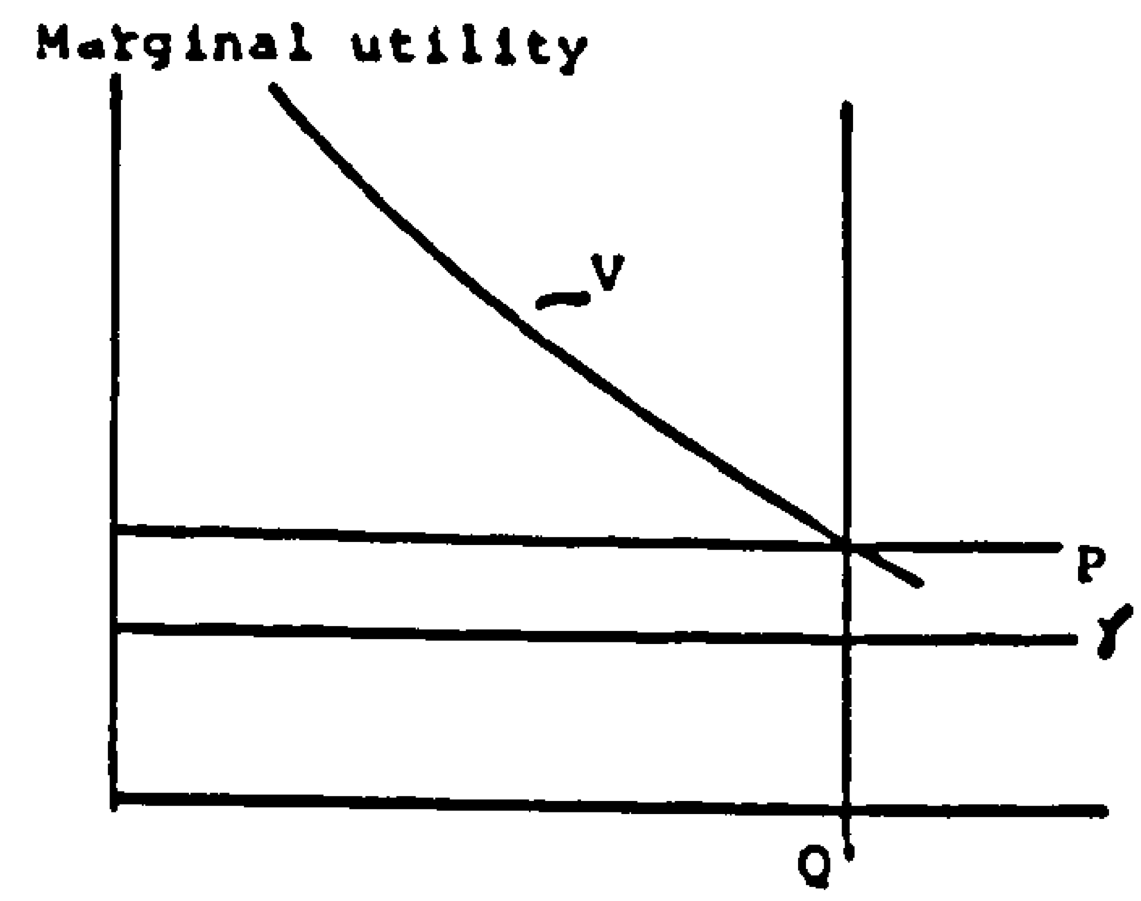


Fig 4.6.2 Peak periods.

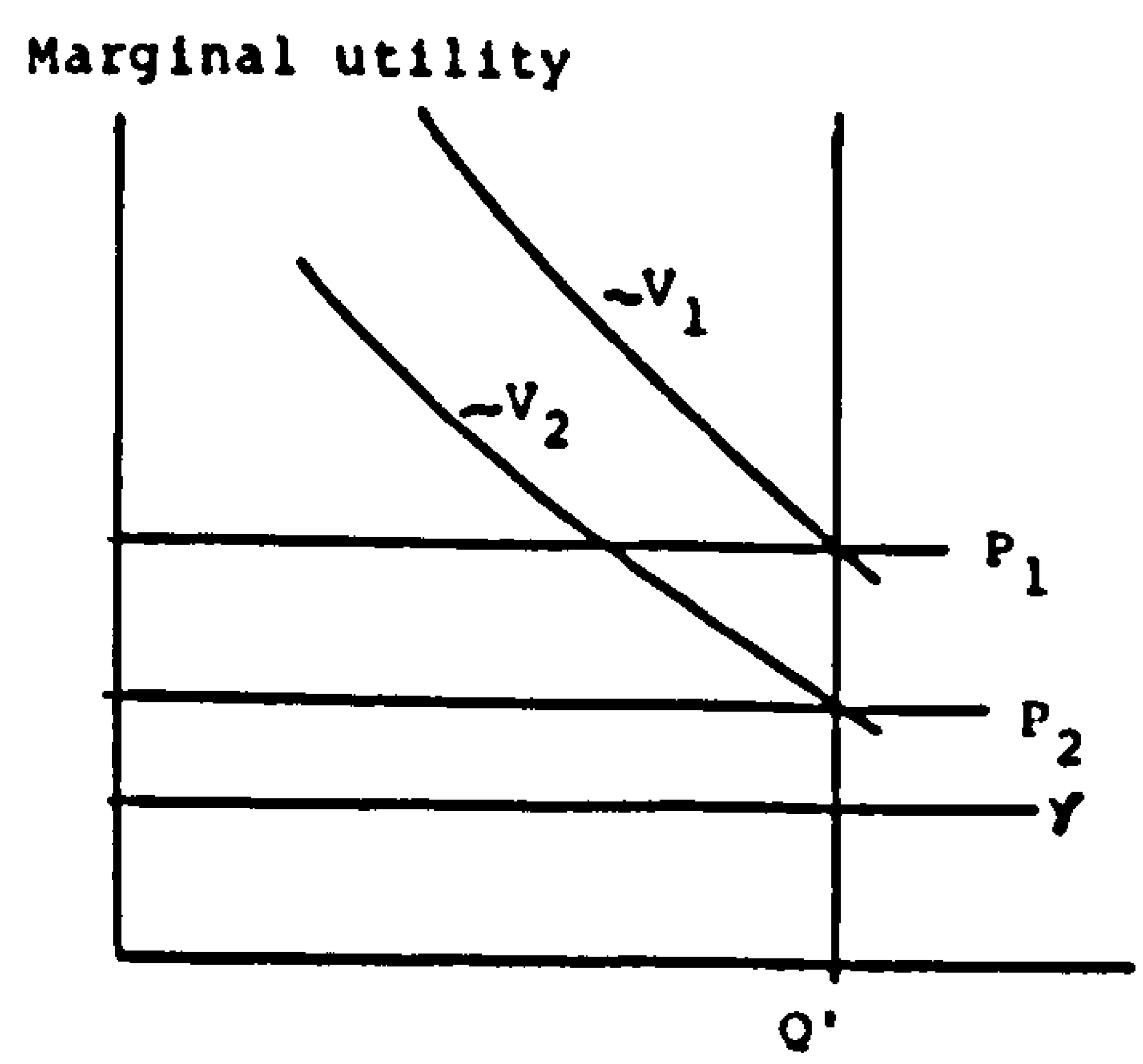


Fig 4.6.3 Optimisation of installed capacity.

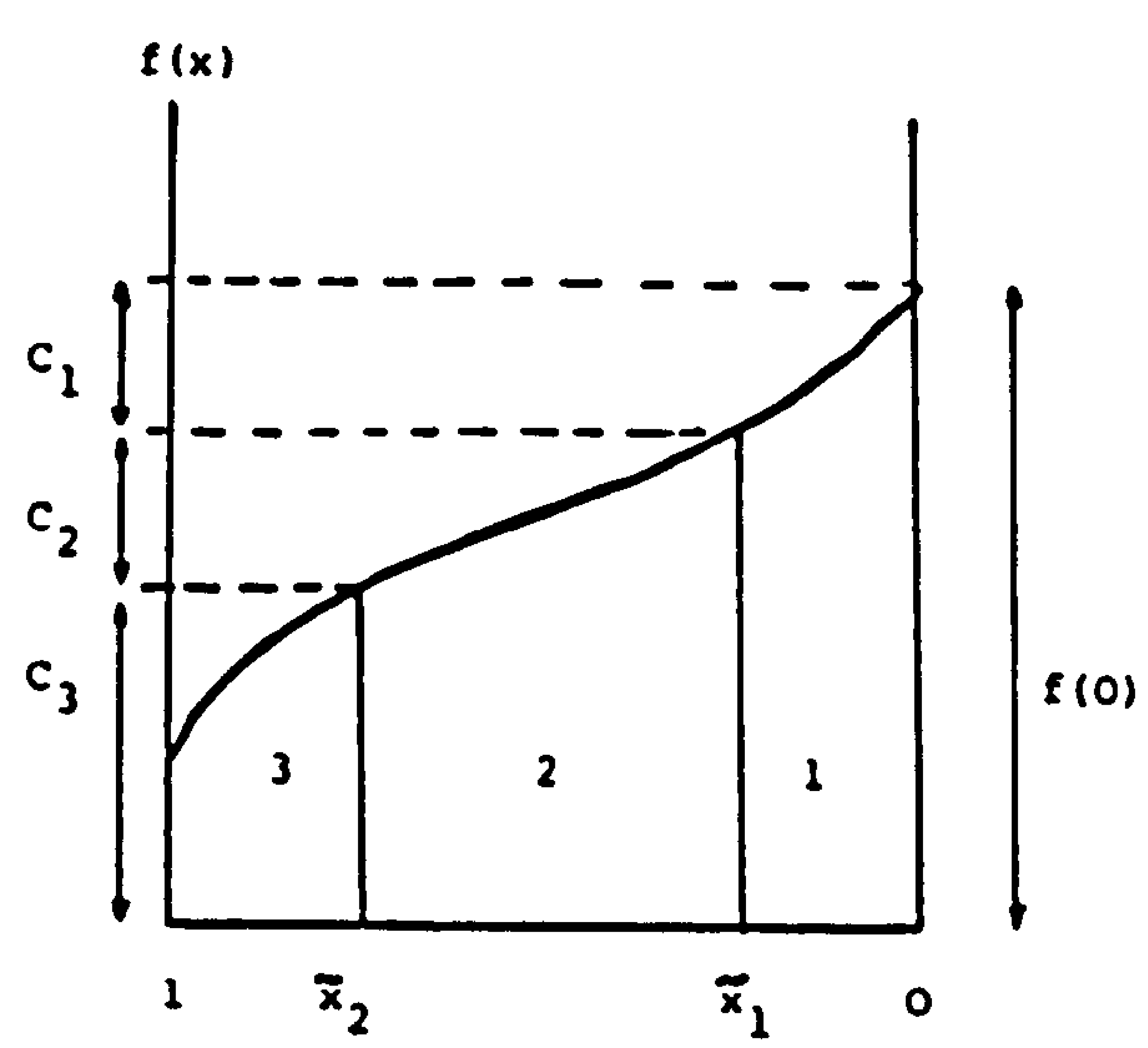


Fig 4.6.4 Annual costs of optimised mix electricity system.

5. SHORT TERM FLUCTUATIONS IN WIND POWER OUTPUT AND WIND OPERATING RESERVE REQUIREMENTS

5.1 INTRODUCTION

Tne operating reserve requirement of wind power arises because wind power output is unpredictable over time spans less than the time needed to start conventional electricity generating plant. The result of this is that in order to supply electricity to consumers with a given level of reliability, the electricity system must make available rapid response capacity additional to what is required to supply the net demand for electricity (net electricity demand is the demand from consumers minus the current wind power output). So wind operating reserve arises from a complex interaction between wind power fluctuations and the time constants associated with conventional plant. In this section I will primarilly be concerned with the effect of adding wind power to a predominantly steam based system. The approach taken, as far as possible, is a parametric one, which was introduced by Dixon and Lowe (Dixon and Lowe, 1983). Much of the notation of that paper is retained in this chapter.

5.2 FAST RESPONSE PLANT

There are three main types of fast response plant available to electricity systems. These are steam spinning reserve, gas turbines and hydro plant (in the case of the CEGB this last category consists mainly of pumped storage plant). The optimisation of the mix of rapid response plant depends on the interaction between the probability density function of system uncertainty, and the fixed and marginal costs of the main types of fast response plant. This area of analysis has been described in some detail by Farmer (Farmer 1980), but basically it boils down to determining "break even load factors" for the main plant types. One of the faults with this analysis is that it does not contain an estimate of the cost to the system of increasing the fluctuations of the output of a pumped storage facility - the only costs which are assigned are the direct fuel costs of pumped storage, gas turbine plant and steam spinning reserve. Evaluation of the "opportunity cost of pumped storage regulation" requires theoretical optimisation of electricity system operation at a level of complexity which is likely to prove intractable. The additional costs probably arise from the trade off of pumped storage plant availability for peak lopping and availability for fast response reserve.

The simplest assumption which it is possible to make in determining the cost of wind operating reserve is that it will be covered by steam spinning reserve. This assumption will be conservative in most cases.

There are a variety of time constants associated with conventional steam plant and it is worth discussing these here. Much of the data and material presented here are derived from the operation of the CEGB system. This is due in part to the availability of information, and partly to my own interest in the possible introduction of wind generation on a large scale into the UK.

The limitations on the rate of change of the power output of a power station (consisting nominally of a boiler and fuel supply system, a series of turbines and a condensing system) arise chiefly because of the thermal stresses which are caused by non uniform temperature rises as the various parts of the station are brought to their operating temperatures. The CEGB text, "Modern Power Station Practice" (CEGB, 1971) states that temperature rise rates on high pressure turbines are limited to about 220 ^{o}C per hour, and on boilers to about 55 ^{o}C per hour. Since the operating temperature of the inlet of many modern sets is nominally 560 ^{o}C turbines require of the order of 2 - 3 hours up to bring to operating temperature from cold, and boilers require of the order of 10 hours. These time constants are reduced on smaller sets, partly because of the smaller size of components and partly because of the lower operating temperatures.

Once a generator is at operating temperature, achievment of full output may take a further 30 minutes

Alteration of the output of a loaded generator can take place in a number of ways:-

1) The power station can be instructed by a central control facility to alter the output of the set. This involves altering the rate at which fuel is fed to the boilers of the set, adjusting the rate at which steam is fed to the turbines, and adjusting the electrical operating conditions of the generator. These operations may be wholly or partly automatic depending on the set in question.

2) The set output may be determined by a governor, which responds to the frequency of the grid. This may be either a centrifugal or electronic device. The governor operates directly on the turbine stop valve (essentially a throttle), and increases or reduces the flow of steam to the turbine.

Initially the change in power output causes the pressure of the boiler to change - the boiler is used as a short term energy store. If a prolonged rise or fall in output is required then action has to be taken to adjust the power input to the set's boilers accordingly. This action may either be automatic or manual.

Altering the firing rates of pulverised fuel boilers may take several minutes. Thus although increases of output of up to 20% may be achieved within seconds, a slight drop in output may occur after about 1 minute. Very rapid increase in set ouput may result in the carry over of water in the steam drum into the super heater tubes and the high pressure turbine because of the rapid drop in boiler pressure.

Summary of time constants.

We have seen that it takes of the order of 1 hour to load a steam generating set from hot standby, and of the order of 2 to 3 hours to load a generating set from a state in which the boilers are at operating temperature but the turbine is not. The first of these time constants is the most important for determining the amount of spinning reserve that needs to be available to the grid. The spinning reserve has to be sufficient to cover fluctuations in demand and power output that are likely to occur over a period of 30 minutes. Fluctuations on a longer time scale can be met by loading plant on hot standby. For ease of calculation, and in order to be conservative, I will assume in the following that spinning reserve meets fluctuations which occur at a lead time of 1 hour.

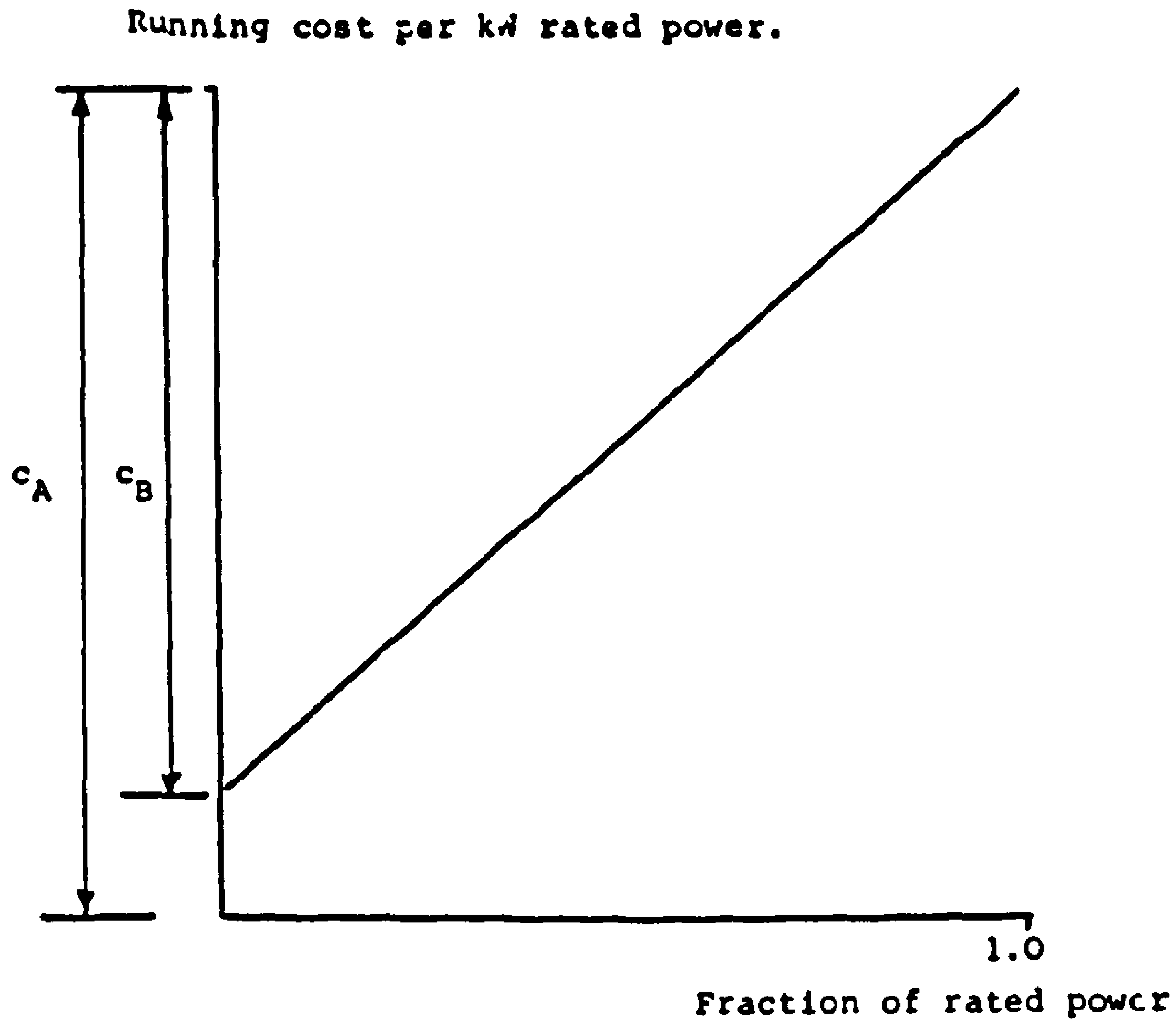


Fig 5.2.1 The Willan's line sketched for a steam turbogenerator.

Cost of spinning reserve.

The fuel cost of steam spinning reserve can be estimated as follows. The fuel cost of running a steam turbo generator is an approximately linear function of the loading. The graph of this function is known as the Willan's line for the set. The common convention is to call the total fuel cost of generating at rated output c_A and the marginal fuel cost c_B. A Willan's line is sketched in fig 5.2.1. Clearly the fuel cost per hour of providing a capacity of T kW and a demand of D kW is given by $T(c_A - c_B) + Dc_B$ regardless of how the spinning reserve T - D is allocated between generators. The only assumption needed for this to be true is the linearity of the Willan's line. Thus the fuel cost of providing the spinning reserve T-D is simply $(T - D)(c_A - c_B)$. For the CEGB, $c_B \simeq 0.85c_A$ (Farmer 1980).

5.3 WIND OPERATING RESERVE: A PARAMETRIC APPROACH.

The simplest approach in assessing the magnitude of wind operating reserve, is to assume that wind operating reserve is set equal to instantaneous wind power output at all times. The resultant annual fuel cost, normalised by the value of wind energy output, is approximately equal to

$$c \simeq (c_A - c_B)/c_A$$

The reduction in fuel saving value is therefore about 15% . This simple view of wind operating reserve requirement is similar to the simple view of capacity credit for wind power which was discussed in chapter 4 of this thesis, and it suffers from very similar limitations. The chief of these is that in order to maintain a given level of reliability, wind operating reserve must be related to the increase in the total uncertainty on the system caused by the introduction of wind power. In any electricity system the main sources of uncertainty in addition to wind power (or other "non firm" sources of power) are uncertainties in conventional plant output and demand uncertainty. These uncertainties are not well correlated with changes in wind power output, and will therefore tend to mask it.

If we can make the assumption that the probability density functions of system uncertainty and wind power uncertainty are gaussian, then the only quantities we need to determine operating the reserve requirement are the standard deviations of these variables, δ_L and δ_W. Making the conservative assumption that system uncertainty and wind uncertainty are not correlated

1. $\delta_T{}^2(t) = \delta_L{}^2 + \delta_W{}^2$

where e_T is the standard deviation of the system including wind power. Note that all of these quantities are functions of the lead time t.

It is worth saying something about the definitions of the quantities introduced above. Not all changes in demand or wind power output are unpredictable, and it is mainly the unpredictable changes which give rise to an operating reserve requirement. However what is classed as an unpredictable change in any variable depends on how well the behaviour of that variable is understood, and how sophisticated the forecasting proceedure which is used. Both of these factors are to some extent arbitrary, and the values asigned to δ_L and δ_W are therefore also arbitrary. In the case of wind power output, present meteorological methods do not seem to be able to do much better than the so-called persistence method of forecasting (see Bossanyi et al 1980). I have therefore assumed that wind power output prediction will be by the persistence method, at least over periods pertinent to the operating reserve problem. It seems almost certain that considerably better forecasing methods will be developed in the future with a resulting reduction in wind operating reserve.

Bearing in mind the above points, wind operating reserve will be proportional to $\sigma_T - \sigma_L$. An approximation to this quantity in the limit of small σ_W is given by the first two terms of the binomial expansion of equation 5.3.1:

2. $\sigma_T = \sigma_L \{1 + (\sigma_W/\sigma_L)^2 / 2 \ldots\}$

The increase in standard deviation caused by the introduction of wind power is given by

3. $\triangle\sigma \simeq (\sigma_W{}^2/\sigma_L) / 2$

If we introduce two further quantities, the specific standard deviations of the electricity system and the wind power output, a and b given by

$a = \sigma_L/L$ (where L is the average electricity demand)

$b = \sigma_W/P$ (where P is the average wind power output)

then equation 5.3.3 becomes

4. $\triangle\sigma \simeq 1/2 \ (b^2/a) \ I \ P$

where I is the penetration of wind power into the system P/L. Defined in this manner I is also equal to the ratio of wind generated electricity to electricity generated by conventional plant.

If the specific cost of providing operating reserve is c, then the fractional reduction in fuel saving value of wind power due to the operating reserve requirement, is given by R

5. $R \doteq 1/2\ \theta c\ (b^2/a)\ I$

where θ is the ratio of the planned fast response reserve divided by the standard deviation of system uncertainty. For gaussian uncertainty, and for loss of load probabilities of the order of 10^{-4} per hour, θ is of the order of 4. θ is not sensitive to changes in the reliability requirement.

Note that the binomial approximation of equation 5.3.3 overestimates the increase in total system standard deviation due to wind power. For very large wind penetrations, equation 5.3.2 gives

6. $R \doteq \theta\ c\ \ b$

The small penetration approximation of equation 5.3.4 reaches this limit at a pentration $I = 2a/b$. At this point the small penetration approximation for R overestimates it by a factor of $2/(5^{1/2} - 1)$, roughly 1.6. Figure 5.3.2 summarises the above comments on the small penetration approximation.

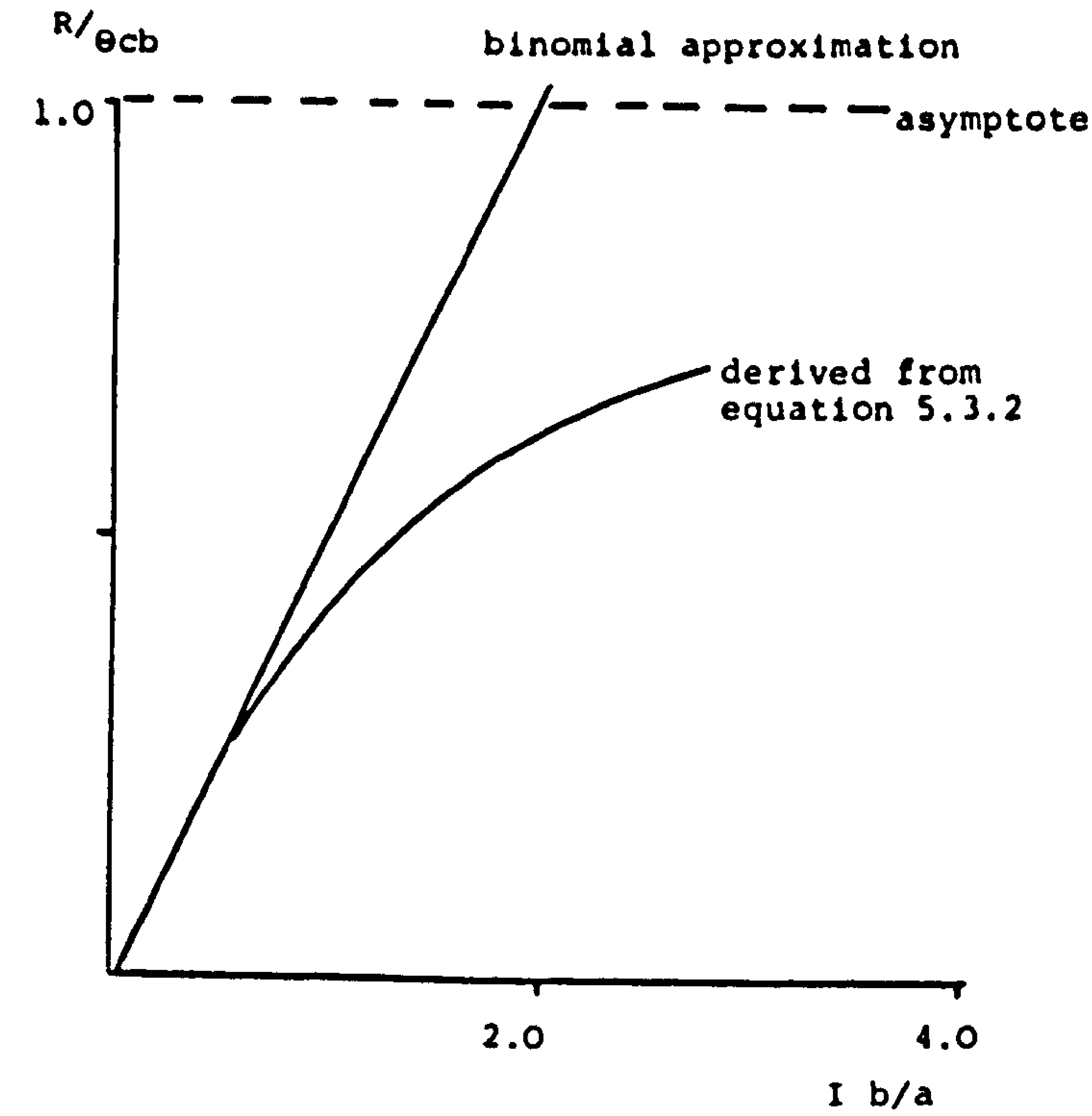


Fig 5.3.1 The effect of the binomial approximation for wind operating reserve costs. The axes have been non-dimensionalised for generality. The vertical axis is proportional to the fractional cost of wind operating reserve, and the horizontal axis is proportional to the penetration of wind into the system.

It is important to remember that a and b in the above are functions both of lead time, and of absolute time. In addition, b is a function of the geographical configuration of the wind power plant connected to the system. With these caveats, equation 5.3.5 illustrates that the reduction in fuel saving value for wind turbines is small in the limit of small penetrations of wind power into the grid, and that under certain circumstances may be a simple linear function of penetration.

5.4 ESTIMATING WIND VARIANCE AND DIVERSITY.

There is a major difficulty in estimating wind power variance at a lead time of the order of 1 hour. This is that the wind data that is available is in the form of hourly average wind speeds. This data therefore smoothes out variations in wind speed due to turbulence. In section 3.2 I have presented calculations of the variance of a single wind turbine due to turbulence, and have discussed the diversity of these fluctuations. In section 3.5 I have presented estimates, based on hourly mean wind speed data, of variance of wind power output due to the high frequency tail of the spectrum of synoptic wind speed variations. Section 3.5 also discusses the correlations between wind power at geographically separate sites. In this section I will summarise these results and estimate the total variance of wind power due to the combined effects of the turbulence spectrum and the tail of the synoptic spectrum, for abitrary numbers of turbines and arrays. These variance estimates will then be used to estimate the cost of providing spinning reserve for wind, R.

Wind power variance due to turbulence.

The analysis presented in section 3.2 led to the result that for a wind turbine with a specific power of 1.0, in a wind of 20% turbulence (high, but possibly representative of conditions in an array), the fractional variance of wind power due to turbulence is 0.27. The diversity of this variance is rather hard to estimate. Clearly

1. $D^2 > 1/n_T$

where n_T is the number of turbines in operation. Using analysis due to Lipman et al (1979), with values of length scales for turbulence chosen conservatively from the literature (see section 3.2) and a turbine separation of 1 km, a reasonable approximation to this diversity might be:

2. $D^2 \simeq 2/n_T$

Thus if we write the fractional standard deviation of wind power due to turbulence as b_{turb},

3. $b_{turb}^2 \simeq 0.27 \cdot 2/n_T = 0.55/n_T$

Wind power variance based on hourly mean wind speeds.

The results of analysis of hourly mean wind speeds summarised in section 3.5 showed that in general:

4. $\langle \sigma^2_t \rangle \simeq \sigma^2 \{1 - \exp(-t/12)\}$

where $\langle \sigma^2_t \rangle$ is the variance of wind power, over periods of t, and σ^2 is the simple variance of wind power at the site.

The definitions of these quantities are discussed in detail in section 3.3.

For turbines with a specific power of 1.0, $\sigma \simeq P$. If we write the fractional variance of wind power evaluated from hourly records of mean wind speed as $b_s(t)^2$ then

5. $$b_s(t)^2 \simeq \{1 - \exp(-t/12)\}$$

I have presented the variance due to the synoptic part of the wind spectrum for completeness. In the discussion of spinning reserve I shall use a lead time of 1 hour.

The above equation suggests that a typical value of $b_s(1)^2$ is 0.08. The diversity factor of this variance may be estimated using an equation developed in section 3.3.5:

6. $$D^2 \simeq 1/n\ [e + (n-e)r]$$

where e depends on the variation in standard deviation of power output from site to site, and n is the number of discrete arrays of turbines. From the data on variance in appendix 1, $e \simeq 1.02$ for sites with specific power of 1.0. I therefore feel justified in setting $e = 1.0$. This approximation will be less good, but still useful if turbines are installed in arrays of equal mean power. It may underestimate the diversity by as much as 10% if turbines are installed in arrays of equal numbers of turbines with specific powers ranging from 1.0 to 2.0.

Note that the above makes the assumption that over the lead times we are talking of, turbines within an array are completely correlated, while between arrays there is a well defined mean cross correlation coefficient $\bar{r}$. This assumption deserves more comment.

The mean cross correlation coefficient can be estimated by an argument due to Justus and Mikhail, which has been discussed in chapter 4 in the context of capacity credit. If the cross correlation coefficient of output from pairs of sites is a linear function of site separation, then the mean cross correlation coefficient is simply the cross correlation coefficient at the mean site separation. The mean site separation $\bar{x}$, for turbine arrays spread uniformly over a given area can be estimated very roughly from the equation

$$\bar{x} \simeq R/3$$

where R the maximum site separation. This provides a useful rule of thumb method for estimating $\bar{r}$, if used intelligently.

Data on cross correlation coefficients are presented in section 3.5 and are summarised for a lead time of 1 hour in fig 5.4.1. From this figure, it will be clear that cross correlation coefficients are not a linear function of site separation, particularly at small separations. It is easy to show that where the graph of r(x) has a large second derivative (is convex toward the x axis) the value of $r(\bar{x})$ underestimates $\bar{r}$. In the case of turbines clustered in arrays with a spacing of the order of 1 km, with distances between arrays of the order of 10's or 100's of km, estimation of $\bar{r}$ from $\bar{x}$ is simply inappropriate, if one uses the individual turbine as the basic unit.

The approximation may however still be useful if one works in terms of arrays of turbines, with complete correlation assumed within arrays. For the range of between arrays separations which are feasible in the UK (50 - 400 km), r(x) appears to be linear. It must be remembered however that cross correlation coefficients for sites separated by more than about 200 km in the UK are dominated by the diurnal spike in the wind power spectrum. This point has been extensively discussed in section 3.5, but it is worth reiterating here. A graph of correlation coefficients estimated from wind speeds measured at 80 m height may well differ substantially from fig 5.4.1.

Equation 5.4.6 can then be approximated by

7. $D^2 \simeq 1/n \{1 + (n-1)\bar{r}\}$

This equation may be dominated by the first or second term. For small n and $\bar{r}$ (ie short lead times)

$$D^2 \simeq 1/n$$

while for large n and large $\bar{r}$,

$$D^2 \simeq \bar{r}$$

The first of these limits is of some use for $n \lesssim 5$, for a lead time of 1 hour in the UK. It is doubtful whether a sufficient number of sites is available in the UK to make the second limit interesting for a lead time of 1 hour. However, for longer lead times the value of r is high enough for it to be a useful rule of thumb (see section 3.5). The fact that (as discussed in this section and in section 3.5) cross correlation coefficients for large wind turbines at wind sites separated by more than about 200 km are not likely to be well estimated from wind speed data measured at 10 or 20 m may then become important, since wind operating reserve requirements are linearly dependent on r in this limit.

Based on the above, with a mean site separation x of 150 km, the mean cross correlation coefficient r is likely to be of the order of 0.05. The variance of wind power, including the effects of site diversity, over periods of 1 hour, can then be written:

8. $b_s(1)^2 \simeq 0.08/n\ \{1 + (n-1)0.05\}$

Total variance of wind power, including diversity.

The total diverse specific variance of wind power at a lead time of 1 hour is given by:

9. $b(1)^2 \simeq 0.55/n_T + 0.08/n\ \{1 + (n-1)0.05\}$

It is clear that the turbulence term in this equation is negligible for systems involving more than 100 turbines or so. This confirms the general point that was made in section 3.2, that fluctuations due to turbulence are unlikely to be important in practical systems. This conclusion is not sensitive to turbine rating, and likely variations in turbulence intensities are most likely to increase its validity. The effects of turbulence in this formulation do not increase with lead time, and so again, considerations of longer lead times increase the validity of the conclusion.

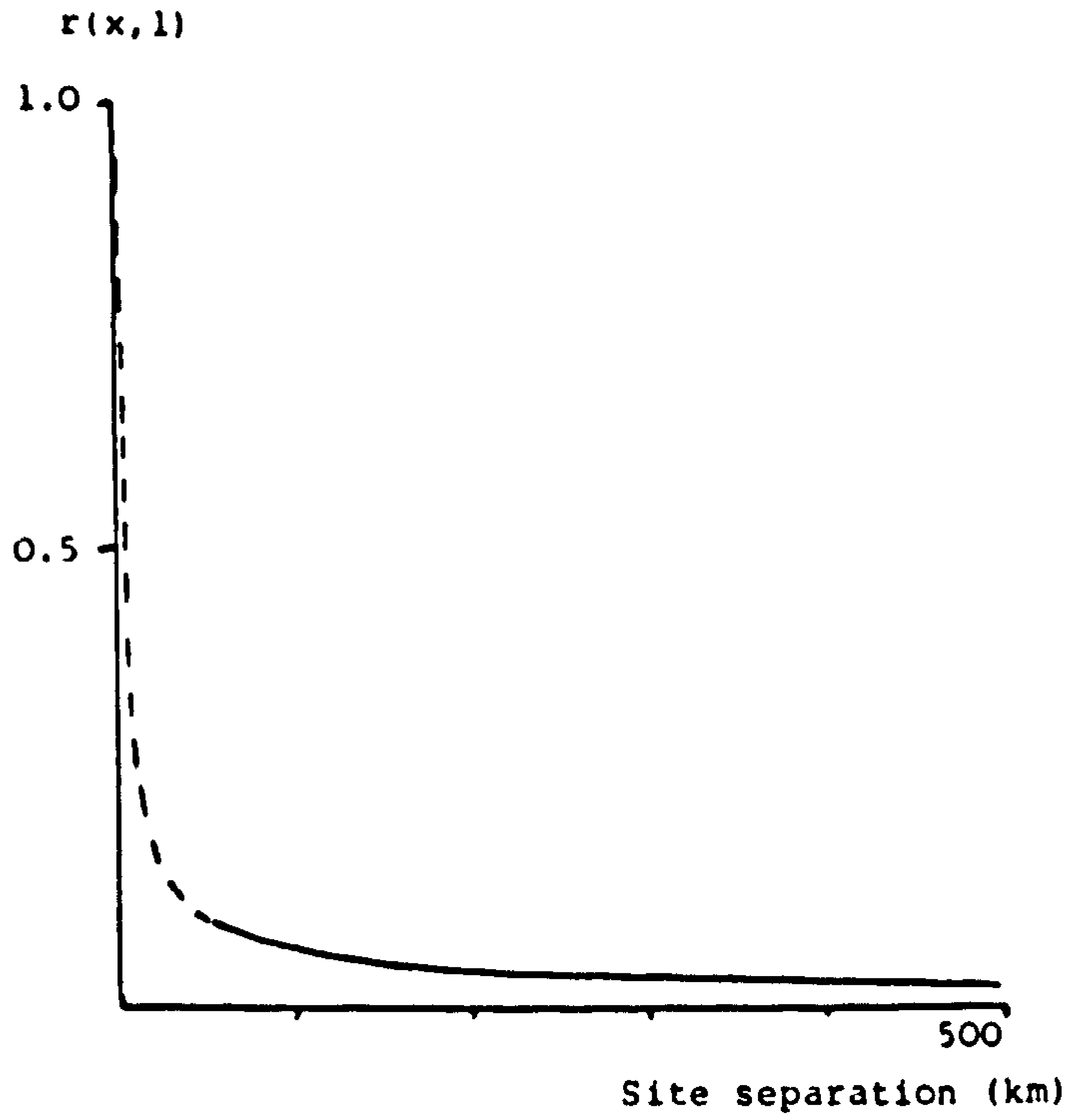


Fig 5.4.1 Non-linearity of cross correlation coefficients of wind power changes, illustrated with the smoothed plot of cross correlation for a lead time of 1 hour vs. site separation. This is a cross section through fig 3.5.11 with a dashed extrapolation to zero separation.

5.5 CONCLUSIONS FOR WIND OPERATING RESERVE REQUIREMENTS.

I will illustrate the conclusions on the short term reserve requirements for wind power with numerical examples based on the CEGB, with the following parameter values:

a	(specific system standard deviation)	0.01
L	(mean system demand power)	25 GW
θ	(reliability criterion)	4 std dev.
c	(normalised cost of spinning reserve)	0.15

An additional assumption will be that the mean power of single turbine is 1.5 MW.

There are perhaps two extreme siting policies for wind turbines:

1) Start construction of a number of arrays simultaneously; later additions to grid wind capacity come from increases in array installed capacity.

2) Install turbines in multiples of a "standard" array.

In the first case the diversity factor for the non turbulent part of the wind specific variance will be approximately constant, and the fractional reduction in fuel saving value of wind power R, will at first rise linearly with the penetration I, with a slope dependent on the number of sites. This behaviour is illustrated in fig 5.5.1

In the second case the situation is a little more difficult to explain. If the penetration of the "standard" array is I_a, and if turbulent fluctuations are initially ignored, then the equation determining R may be written:

1. $R \simeq \theta c/2a \ 1/n \{1 + (n-1)0.05\} \ n \ I_a$

which becomes

2. $R \simeq \theta c/2a \ \{1 + (n-1)0.05\} \ I_a$

In words, R rises from its initial value for a single array in a linear way, with a slope proportional to the mean cross correlation coefficient r. This behaviour, for $I_a = 0.006$, is also illustrated in fig 5.5.1 (dashed line). It should be noted that the behaviour of the approximation for R in this case is strictly discontinuous, only being defined for whole numbers of arrays. In particular, the dashed line in fig 5.5.1 cannot be extrapolated back towards the origin. Note that fig 5.5.1 includes the effects of turbulence.

It should also be noted that r is not necessarilly going to be a constant for this type of wind turbine installation policy. Arrays could be sited in such a way that it was, but this is not likely to be a very pressing constraint. However, unless the initial arrays are sited very close together, the effect of $\bar{r}$ on R is not very important until the number of arrays is of the order of 5 or more. By this stage the concept of a mean site separation and therefore a mean cross correlation coefficient, will be rather more meaningful. The broad conclusion for this installation policy, that R will increase slowly with the number of arrays, is a robust one.

The final point to make from this analysis is about the effects of different array sizes on operating reserve costs. Equation 5.5.2 shows that to a first approximation, R for a given penetration I, is proportional to array size. Fig 5.5.2 shows a family of curves similar to the dashed line in fig 5.5.1, for different array penetrations.

The binomial approximation and assumptions of normality.

I have already stated that the binomial approximation used in deriving equation 5.3.3 above overestimates the increase in total system variance due to the addition of wind power. The lines in figures 5.5.1 and 5.5.2 have been truncated at the point where the penetration I = a/b. At this point $\Delta\sigma$ is overestimated by a factor of about 1.2 (see section 5.3).

This is not to say that operating reserve costs which would be derived from a full analysis, would necessarily be overestimated by the same amount. Expressing the reliability criterion of the utility in the form of a number of standard deviations of fluctuation which will be covered by fast response plant, is only valid if the probability distribution function of short term fluctuations is unaltered except for its variance by the addition of wind plant. If this is not the case, then θ will not be a constant for a given level of required reliability. If θ is a strong function of penetration then the usefulness of the parametric analysis presented here is reduced. The best that can be said here is

1) that the above analysis is not designed to be extended into the region where the variance of wind power is more than the variance of the rest of the system.

2) at a lead time of 1 hour for a system consisting of 10 arrays of turbines, the specific standard deviation of wind power is of the order of 0.1. The distribution of power from such a system of wind turbines has a range of roughly 3

times the mean power. A confidence interval of $\pm$ 4 standard deviations therefore corresponds to about 1/4 of the total width of the distribution. A gaussian approximation to the probability distribution of wind power fluctuations may therefore not be a bad one for short lead times.

Comparison with other published work.

I am not aware of papers documenting this type of parametric approach to wind operating reserve costs, apart from Dixon and Lowe (1983). Some numerical work in this area has however been done, notably a study of operating reserve costs for a small US utility (North East Gas and Electric Association) (Johanson and Goldenblatt, 1979), and a study of the CEGB by Whittle (Whittle, 1981). Upper estimates of the cost of wind operating reserve were published by Rockingham and Taylor (1981). The conclusion of first order independence of R on I is in agreement with the results of the NEGEA study, although this is vague about the lead time that was considered. More detailed comments on this study may be found in Dixon and Lowe, 1983.

The numerical study reported by Whittle (op cit) is perhaps rather more interesting, since it deals (notionally) with the CEGB. Whittle's value for R without pumped storage, was 0.05. This value was calculated assuming a=0.015, and assumed a 2 GW (rated) wind installation. No geographical diversity was assumed. Putting I = 0.032, θ = 4, b = 0.08 and a = 0.015 into equation 5.3.5 above, gives R = 0.051. This close agreement is certainly fortuitous, but is nevertheless comforting.

Conclusions.

The above presents several important results with possible implications for utility policy on wind turbine demonstration and development. Essentially the above analysis says that there are not likely to be large operating reserve penalties for small numbers of wind turbines, and that operating reserve costs can be neglected in economic analysis of wind turbine demonstration projects. It is clearly important for a utility to be able to make some estimates of operating reserve costs of wind power at high penetrations, but the conclusions that can be drawn from such estimates, about the desirability of large wind power programmes, will depend on many rather basic factors which are not currently well known (costs of producing turbines, outage frequencies and O&M costs etc). The above analysis suggests that the operating reserve costs of discovering the answers to these major questions, are likely to be small.

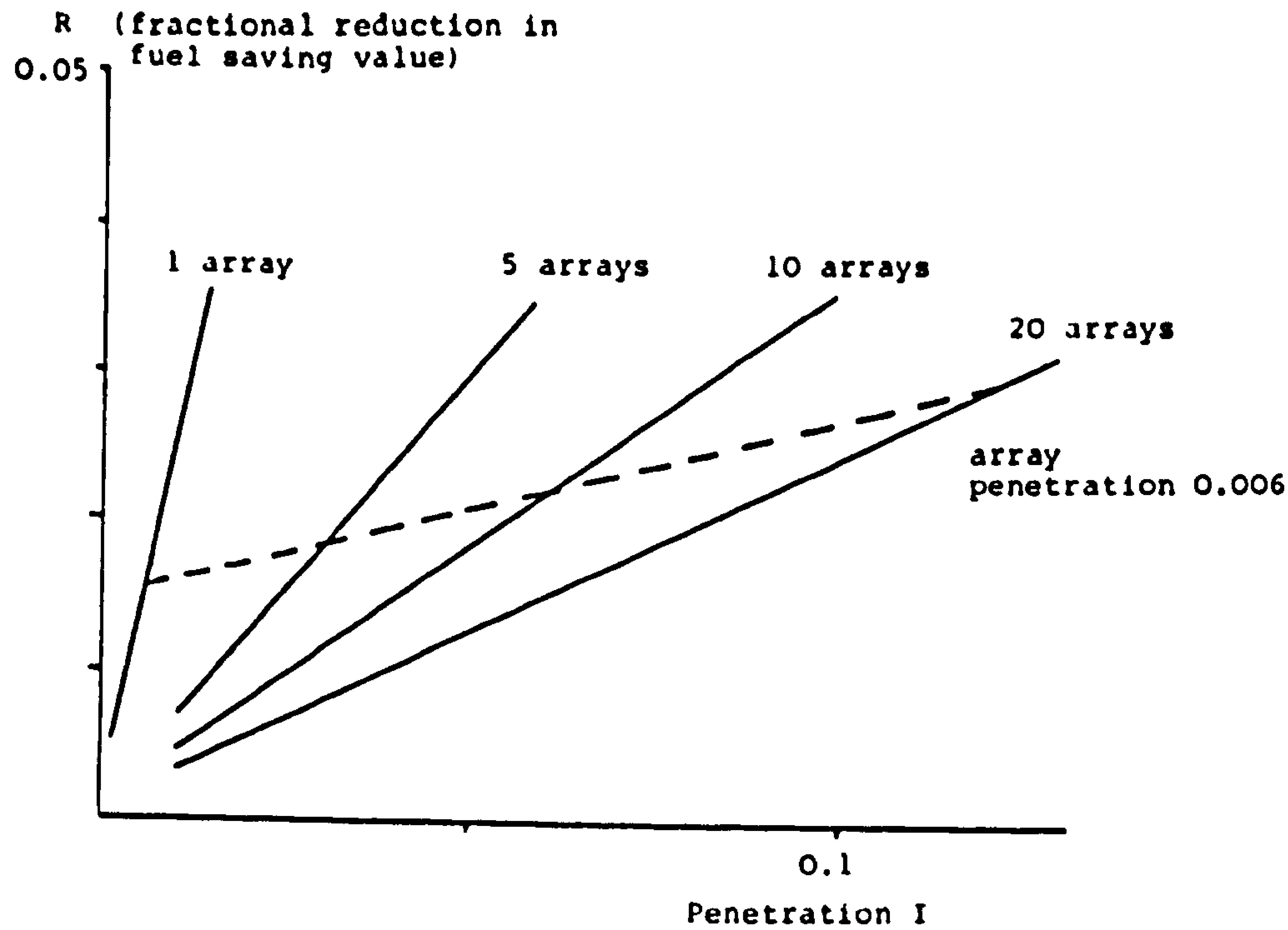


Fig 5.5.1 Wind operating reserve costs as a function of penetration and number of arrays.

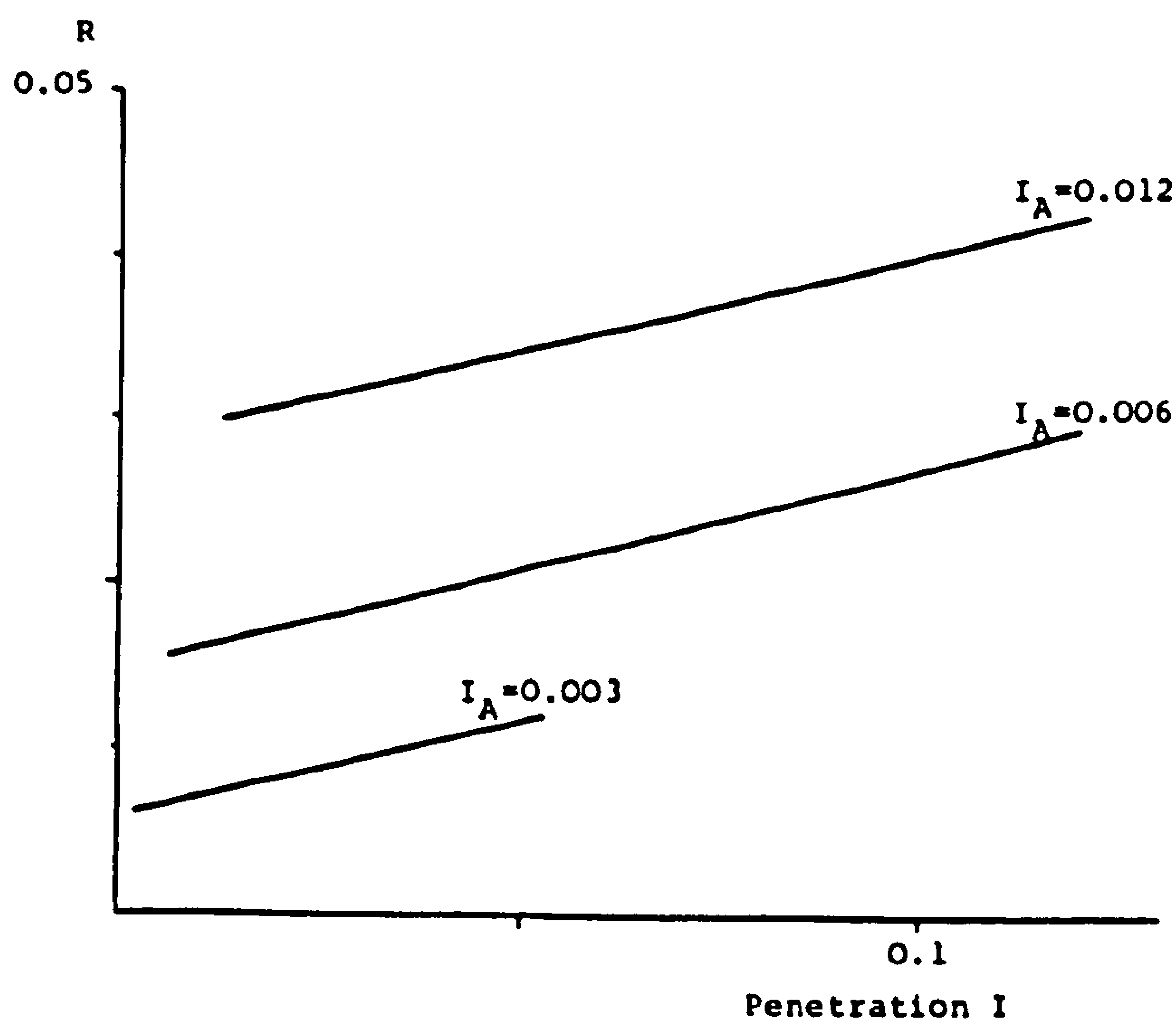


Fig 5.5.2 Wind operating reserve costs: effect of array size. Note: an array penetration of 0.006 corresponds roughly to a mean array output of 150 MW in the case of the CEGB.

6. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK.

This thesis has covered a number of areas of study connected with wind power, and it may be most useful to break down the discussion of conclusions and further work under headings corresponding to the preceeding chapters.

Chapter 2.

This chapter discusses a commonly used model of electricity system economics, and examines conditions which result in minimised costs of provision of electricity.

Incorporation of time into the normally static model yields a useful conclusion regarding the timing of new plant. If the savings from new plant can be assumed to increase annually, then the best time at which to plan to commission the new plant is in the year in which its fixed costs are equal to its net system savings.

Costs and economic value of wind turbines in the UK are discussed. The conclusions are that in the UK the economic value of wind power is presently heavily dependent on the future cost of coal. This conclusion is not in general true for other countries, in which the price of coal may be considerably lower than in the UK (for example the USA and Australia). CEGB estimates show coal prices rising in the future at a wide range of rates. The bottom end of this range is perhaps to be preferred on the grounds that energy demand growth rates are likely to be low or negative in the forseeable future.

The discussion of coal prices and future system running cost savings in this section, really only scratches the surface of the problem. There is an enormous amount of work to be done on elaborating the potential for wind generation in a wide range of possible future UK electricity grids. It is all too easy for the analyst concerned with wind to consider only a narrow range of possible policies for the rest of the electricity system, and in doing so they may be avoiding the most interesting questions. For example, I know of nobody looking in detail at the interactions between wind power and combined heat and power, or at the potential for wind power in a grid of reduced size. Yet both of these appear to be technically feasible and potentially desirable options for the development of the CEGB.

A number of papers dealing with costs of wind power using criteria appropriate to the UK are reviewed, with emphasis on papers by Dixon and Lowe, and Musgrove. The costs indicate that wind turbines are either already likely to be cheaper than coal, or soon will be, depending on the assumptions made regarding both wind turbine and coal costs. This conclusion is taken as a justification for the detailed work on second order costs in the following chapters of the thesis.

Chapter 3.

This chapter begins with a discussion of the data which will be used in the rest of the chapter. Of particular importance is the fact that the Meteorological Office data which is used consists of hourly averages. It is concluded that wind power time series based on this data may be taken as representing the power output from arrays of wind turbines, which will have the effect of smoothing power variations due to turbulence. The reduction in turbulent fluctuations due to arrays of turbines is estimated using a number of references.

The second part of this chapter presents a discussion of statistical analysis methods appropriate to wind power. Spectral analysis of random variables is introduced. The spinning reserve problem is discussed in terms of two statistical quantities. The conditions under which these may be appropriate indicators of spinning reserve requirements are discussed.

The coherence function is introduced and its estimation and statistical uncertainty are discussed. This is followed by a discussion of cross correlation coefficients, and their relationship with the diversity factor, which is the ratio of the standard deviation of the combined output of a set of wind turbines compared with the standard deviation which would result if they were all 100% correlated. The relationship of the cross correlation coefficient to the coherence function is also discussed. This is particularly important in connection with conclusions about the importance of diurnal fluctuations of wind speed at different heights.

This part of the chapter is finished with a discussion of fluctuations in the level of an energy store, over various time periods. This section is perhaps of most interest with respect to storage of wind energy output in the form of heat rather than electricity, and is included here for completeness.

The rest of this chapter is devoted to the discussion of analysis programs and results of analysis. It is shown (following Farmer) that the random walk characteristics of all sites fit closely to a curve of the form:

$$b(t)^2 = 2\,[1 - \exp(-t/12)]$$

for $t \leq 18$ hours. The coherence functions for two sets of wind sites show a scatter of results which can be roughly characterised by curves of the form

$$\gamma^2_{max} = \exp(-x/L)$$

However no simple non-dimensionalisation of the data appears to be possible. In particular there is frequently a large diurnal spike in the coherence function plots, the height of which varies from site pair to site pair, and as may be expected does not reduce greatly with site pair separation.

Cross correlation coefficients are at first sight rather more orderly than coherence function plots. Again as may be expected, cross correlation coefficients tend to decline with increased site pair separation, and increase with increased lead time. A general empirical surface was fitted for lead times up to 6 hours. The equation for this is

$$r(x,t) = [\, x/2.6t - 1.6 \,]^{-0.6}$$

Perhaps the most important conclusion to come out of this thesis is that this order is probably misleading. This is because the cross correlation coefficients of wind power output for pairs of sites separated by more than 200 km in the UK are dominated by the diurnal spike in the spectrum at each site. This effect is important as the diurnal fluctuations in wind speed depend critically on measurement height, and vary greatly with the time of year. The effect of this is that data gathered at a given height, say 10 metres, does not necessarily give a good guide even to the sign of diurnal fluctuations at the same site at different heights. Consequently, estimates of cross correlation coefficients made from data measured for example near 10 m, are quite unreliable predictors of cross correlation coefficients estimated from data measured at say 80 m, corresponding to the hub heights of the current generation of large wind turbines. It appears therefore that further work needs to be done on the collection of simultaneous wind speed data at widely separated sites at heights of potential interest for operators of wind turbines.

An important general lesson from the above is that it may sometimes be dangerous to calculate cross correlation coefficients for pairs of random variables, without also calculating coherence functions. The fact that correlation coefficients may be heavily dependent on a non random variation at a particular frequency, may be masked in the plot of a correlation coefficient, but should show up quite clearly in a coherence function plot.

Chapter 4.

This chapter discusses and attempts to estimate the capacity credit for wind turbines under a range of conditions. The first conclusion of this chapter is that capacity credit is fairly arbitrary and in particular may be rather sensitive to choice of system boundary.

Rockingham's work on capacity credit and some of the weaknesses of his formulation are discussed. A more general derivation of Rockingham's results is introduced. One result of this is that for small penetrations into an existing electricity grid, an uncorrelated new source behaves like baseload plant, displacing conventional plant by a constant amount at all loads.

The effect of the non-gaussian distribution of real wind power outputs is discussed and estimated by numerical calculation. The difference between the gaussian and more realistic linear wind distributions is about 5% at a penetration of 20%.

Conclusions for wind power capacity credit are that the marginal capacity credit declines from a little over 1 at zero penetrations (based on the mean power output of the turbines), to 0.5 to 0.8 at penetrations of the order of 0.2. The neglect of geographical separation of turbines can lead to an underestimation of capacity credit by a factor of the order of 0.6.

Wind power capacity credit is sensitive to a range of factors. Changing the uncertainties associated with conventional plant may have a large effect on wind capacity credit at higher penetrations. For example reducing conventional plant uncertainty by delaying the decomissioning of old plant to meet unexpected peaks of demand reduces capacity credit by a third at a penetration of 15%. Higher specific powers lead to lower capacity credits. These results are broadly consistent with other work in the field.

The final part of this chapter shows how the definition of capacity credit discussed in the forgoing is dependent on a rather arbitrary electricity supply system reliability standard. The classical economic basis for this standard is weak - it is basically a rule of thumb, which has been developed in the absence of technical means of introducing a market in electricity. An alternative spot pricing framework is discussed in sufficient detail to show that the nature of the electricity system may change dramatically if such a system of pricing were introduced. No attempt is made to estimate the effect of such changes on the capacity credit for wind power. Exercises such as this are essential if one is to avoid taking calculations such as appear in the earlier part of this chapter too seriously.

Chapter 5.

Chapter 5 presents a discussion of operating reserve costs of wind power. The analysis in this chapter follows the earlier analysis of capacity credit, and mathematically the two problems are very similar, differing chiefly in time scale (years on the one hand and minutes to hours on the other). Following an earlier paper by Dixon and Lowe, the analysis is presented as far as possible in non-dimensional form. While this may make the results rather more difficult to interpret, it increases their generality.

The main conclusion of this chapter is that operating reserve costs of wind power are approximately linearly related to penetration, at low penetrations. Of secondary importance is the conclusion, based on results and analysis presented earlier, that the turbulence term in the expression for operating reserve is likely to be small for systems involving more than 100 turbines.

The overall conclusion is that operating reserve costs of wind power are likely to be small at low penetrations. Their costs can therefore be neglected in demonstration projects, as can the second order reduction in capacity credit. For large penetrations, the operating reserve requirements will depend on factors which are presently not well understood, for example the cross correlation of diurnal fluctuations of wind power output at different sites.

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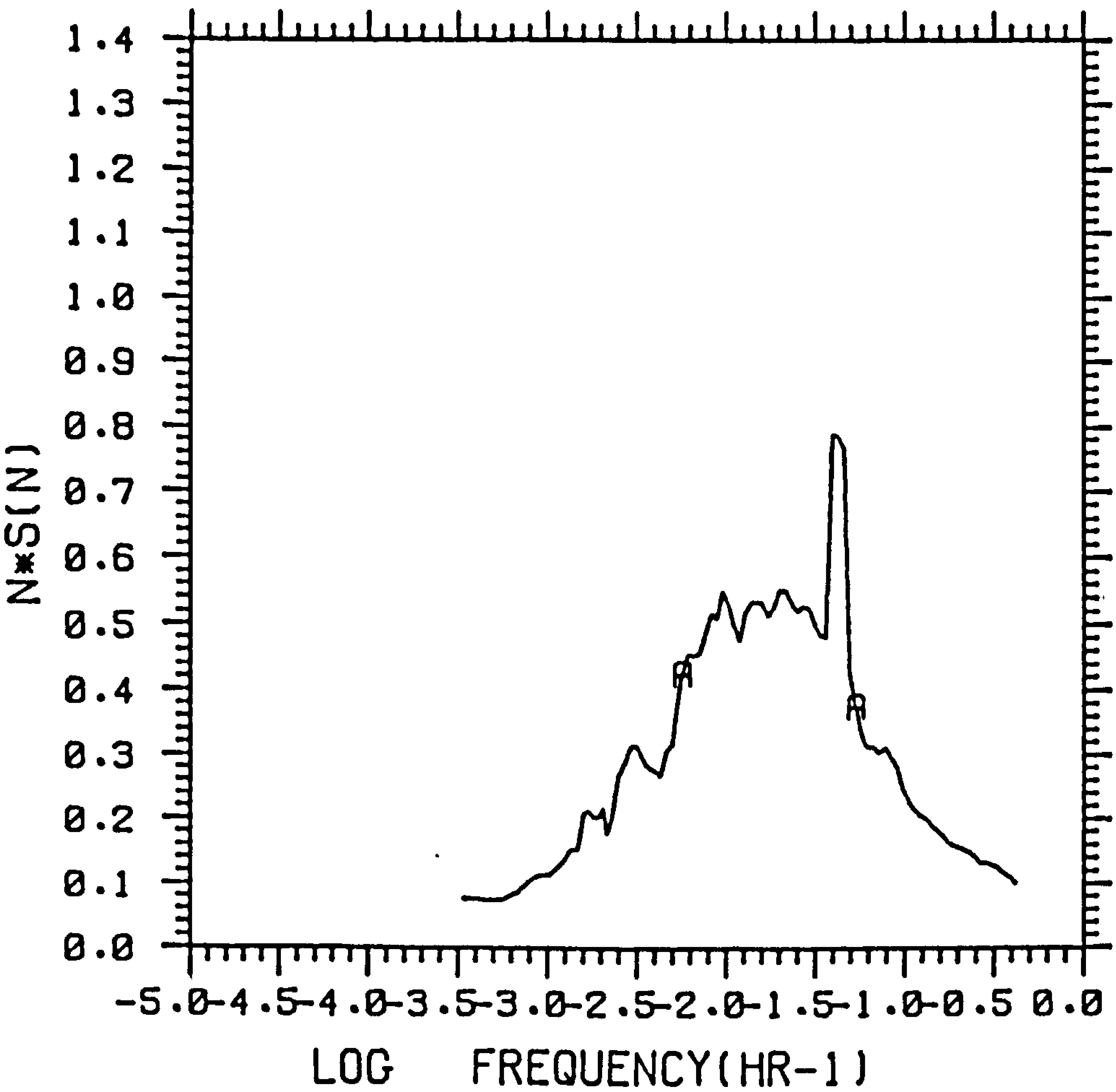
APPENDIX 1.

This appendix consists of 2 sections. The first presents a series of 30 spectra of wind power at selected UK sites, and the second presents coherence functions and cross correlation plots for 26 pairs of UK wind sites.

The most important point regarding the wind power spectra is that apart from the diurnal spike, the shape of the spectra differ little from site to site. The vertical scaling of the spectra depends largely on the specific power of the turbine site combination. The relationship between the variance of wind power at a site and the specific power is discussed in more detail in chapter 4.

The site pair plots are presented in two batches. The first is based on the common wind power site St. Mawgan, and the second is based on the common site Gorleston. There are several points to notice here. The first is that for large site pair separations, the cross correlation coefficient is not monotonic with lead time. Plots of this variable over periods of several days show a clear sinusoidal variation, superimposed on a monotonic trend. As discussed in chapter 3, this behaviour is to be expected when the spectra of the two sites contains a spike at the diurnal frequency, which is well correlated over great distances. The dominance of the diurnal spike is emphasised in the coherence function plots for these widely separated site pairs. The positions of the sites are shown in fig 3.2.1.

EFFECT OF SITE DISPERSAL. SITE STY MOD2

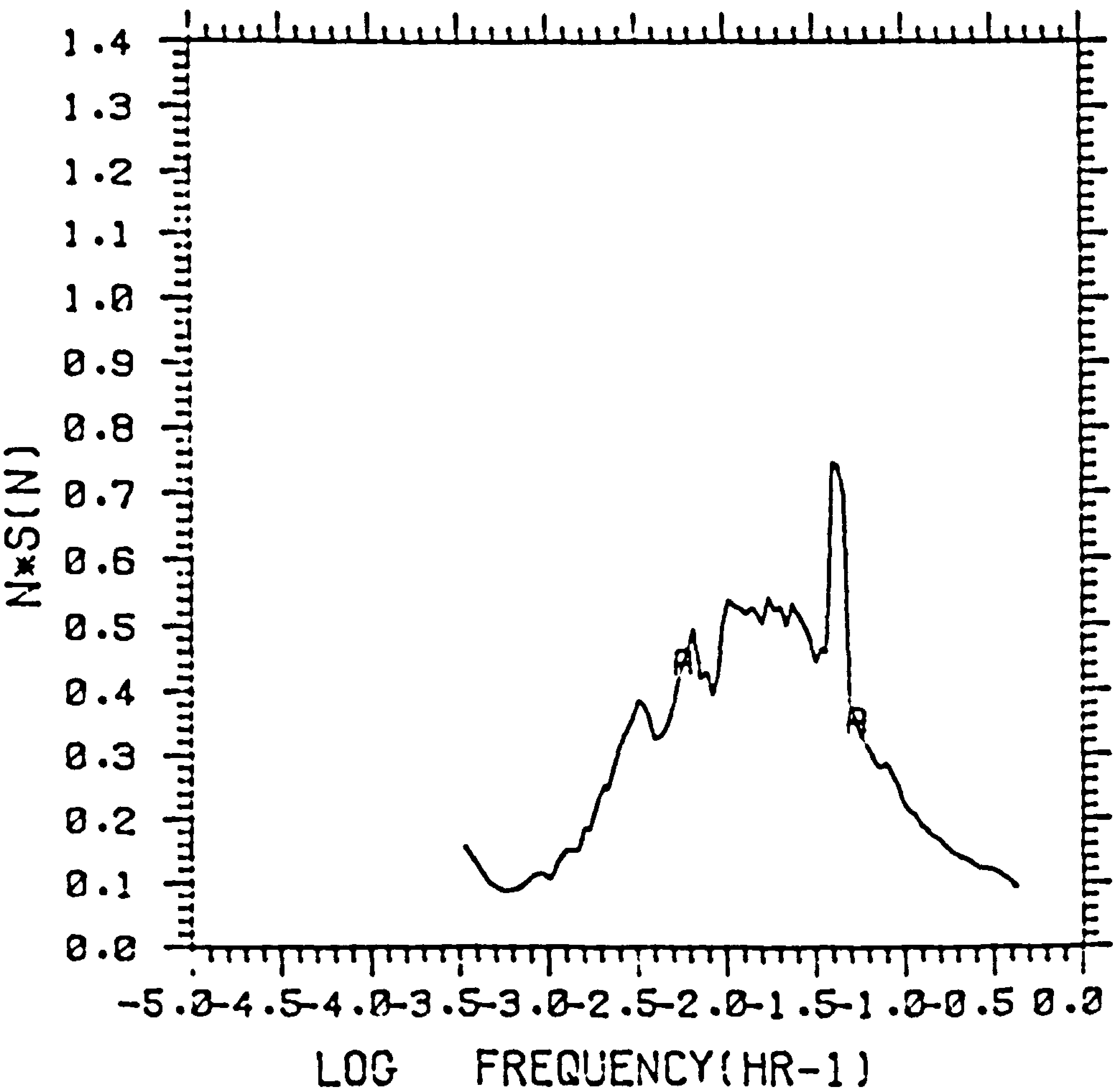


KEY

A ---

RUN 1

EFFECT OF SITE DISPERSAL. SITE WIK MOD2

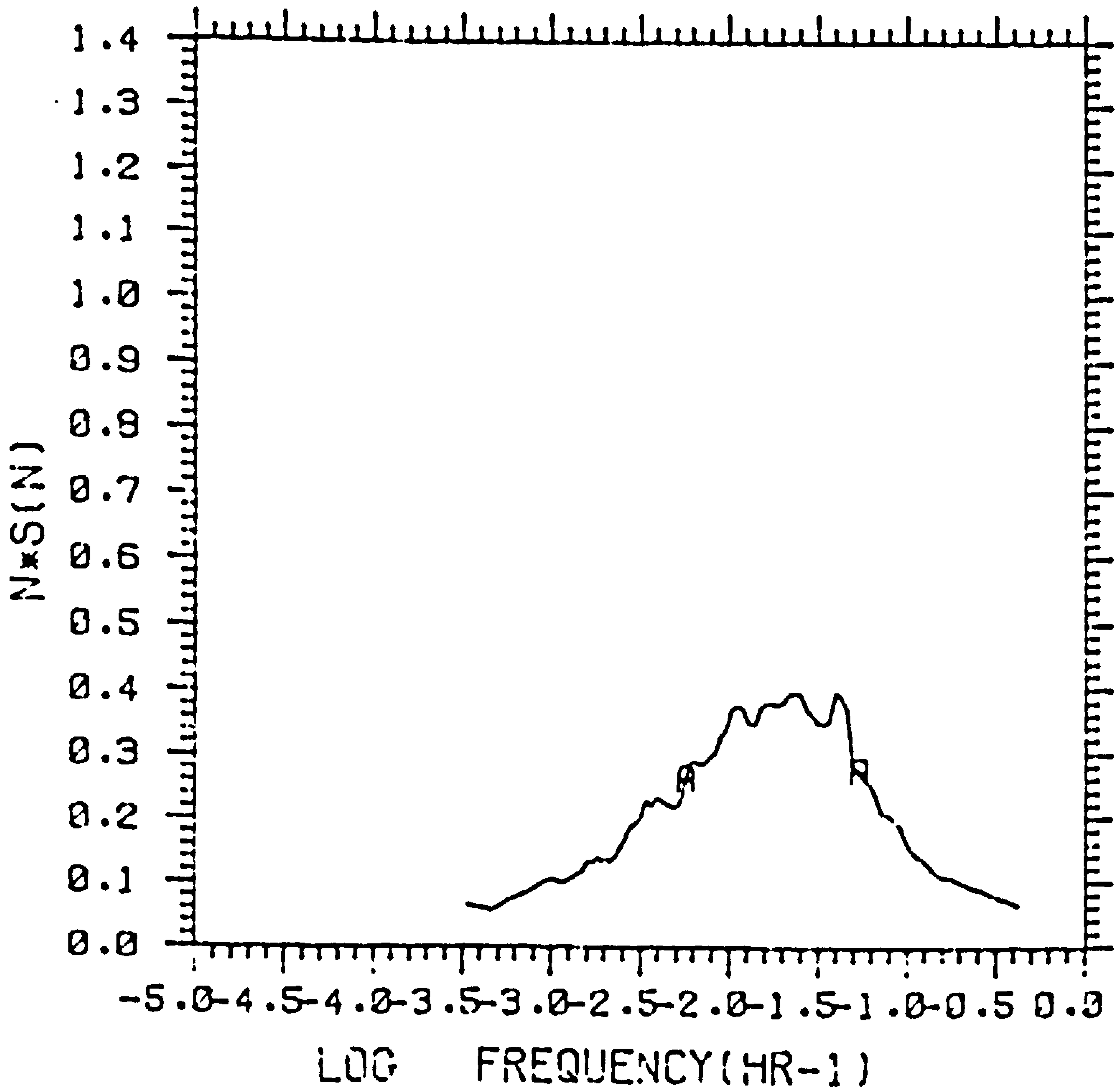


KEY

A ---

RUN 2

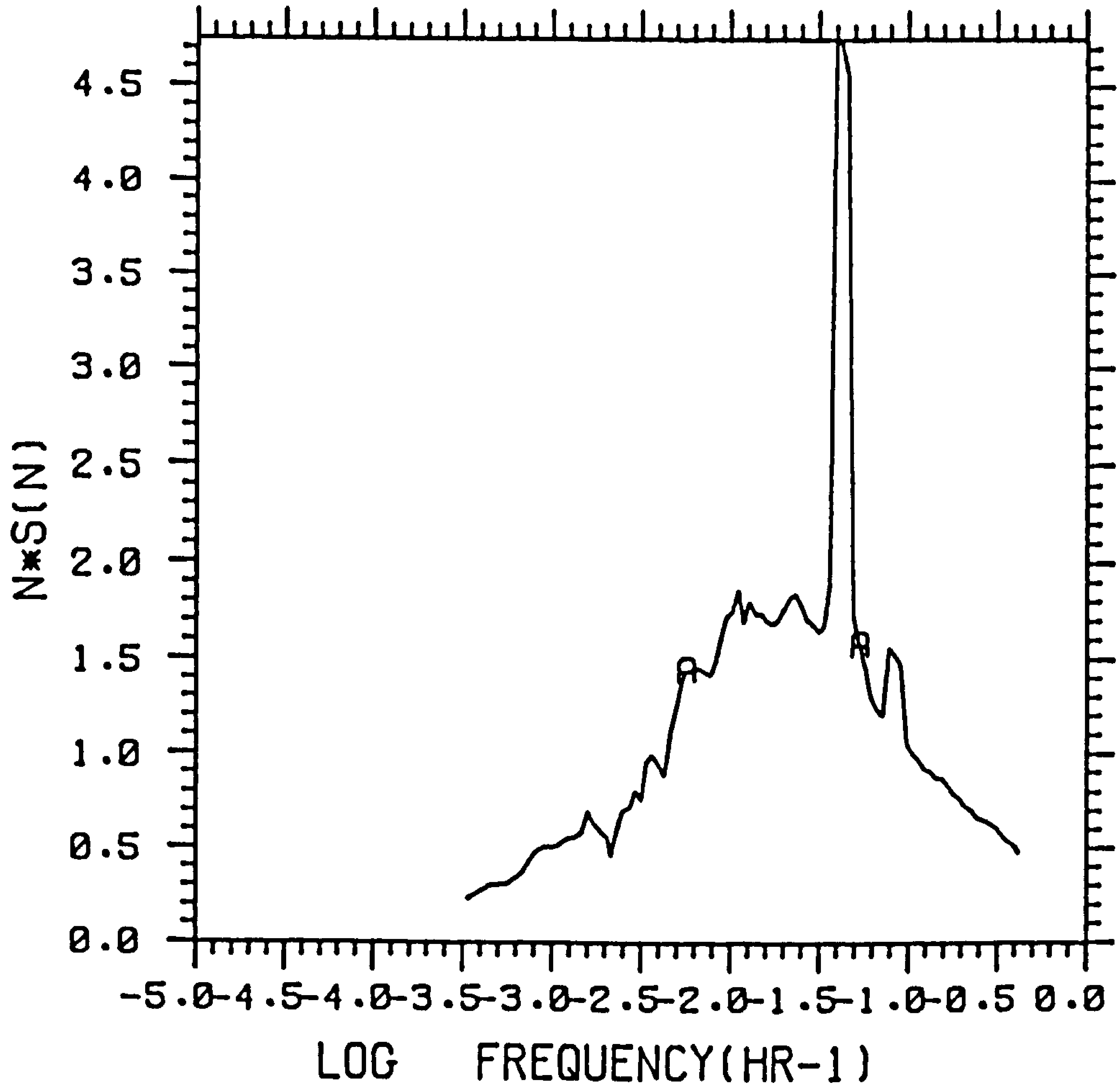
EFFECT OF SITE DISPERSAL. SITE TIR MOD2



KEY

A ----
SIMULATION 3

EFFECT OF SITE DISPERSAL. SITE FAU MOD2

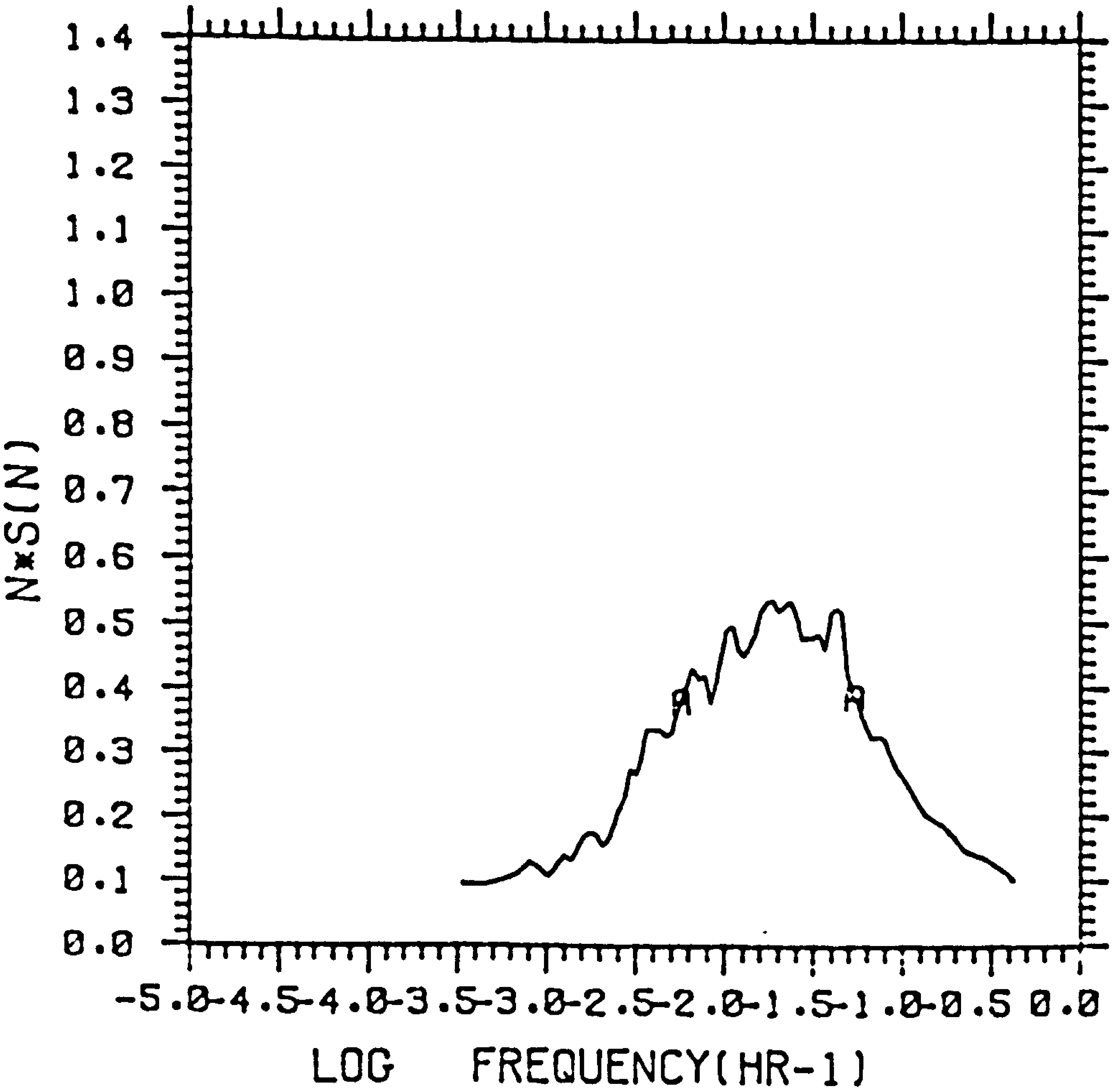


KEY

A ---

RUN 4

EFFECT OF SITE DISPERSAL. SITE FGH MOD2

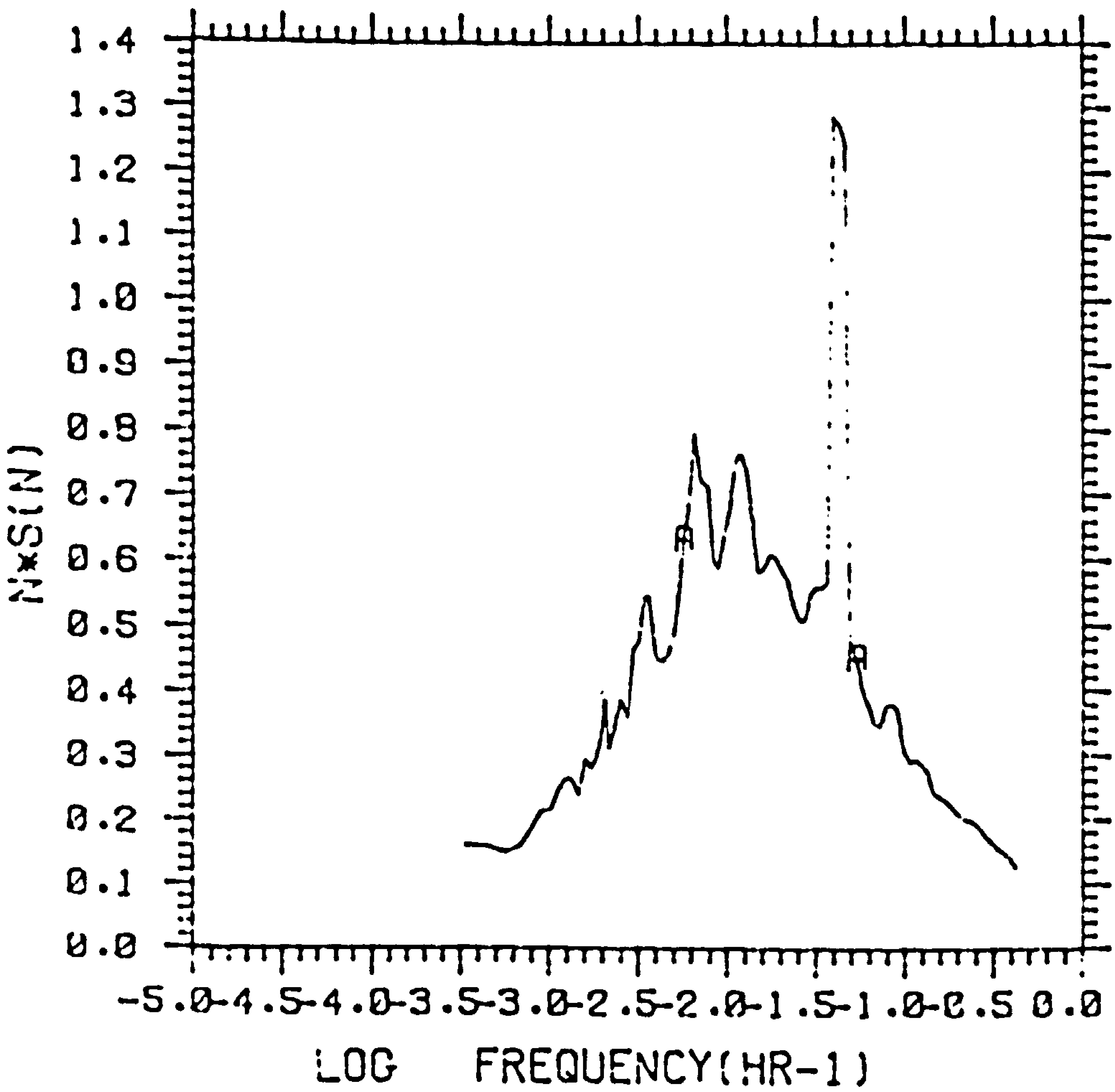


KEY

A ---

RUN 5

EFFECT OF SITE DISPERSAL. SITE LCH MOD2

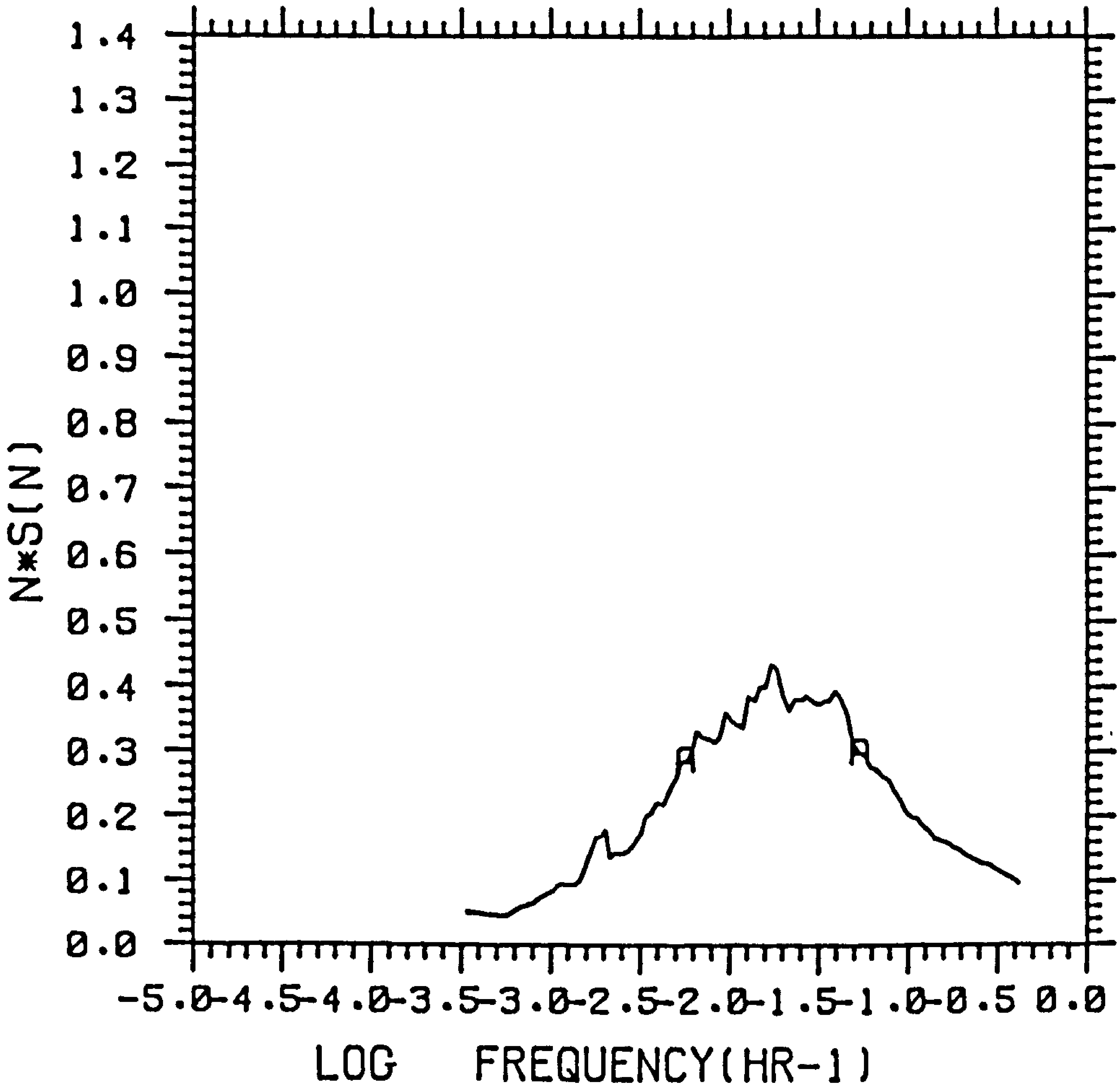


KEY

A ---

RUN 6

EFFECT OF SITE DISPERSAL. SITE BLR MOD2

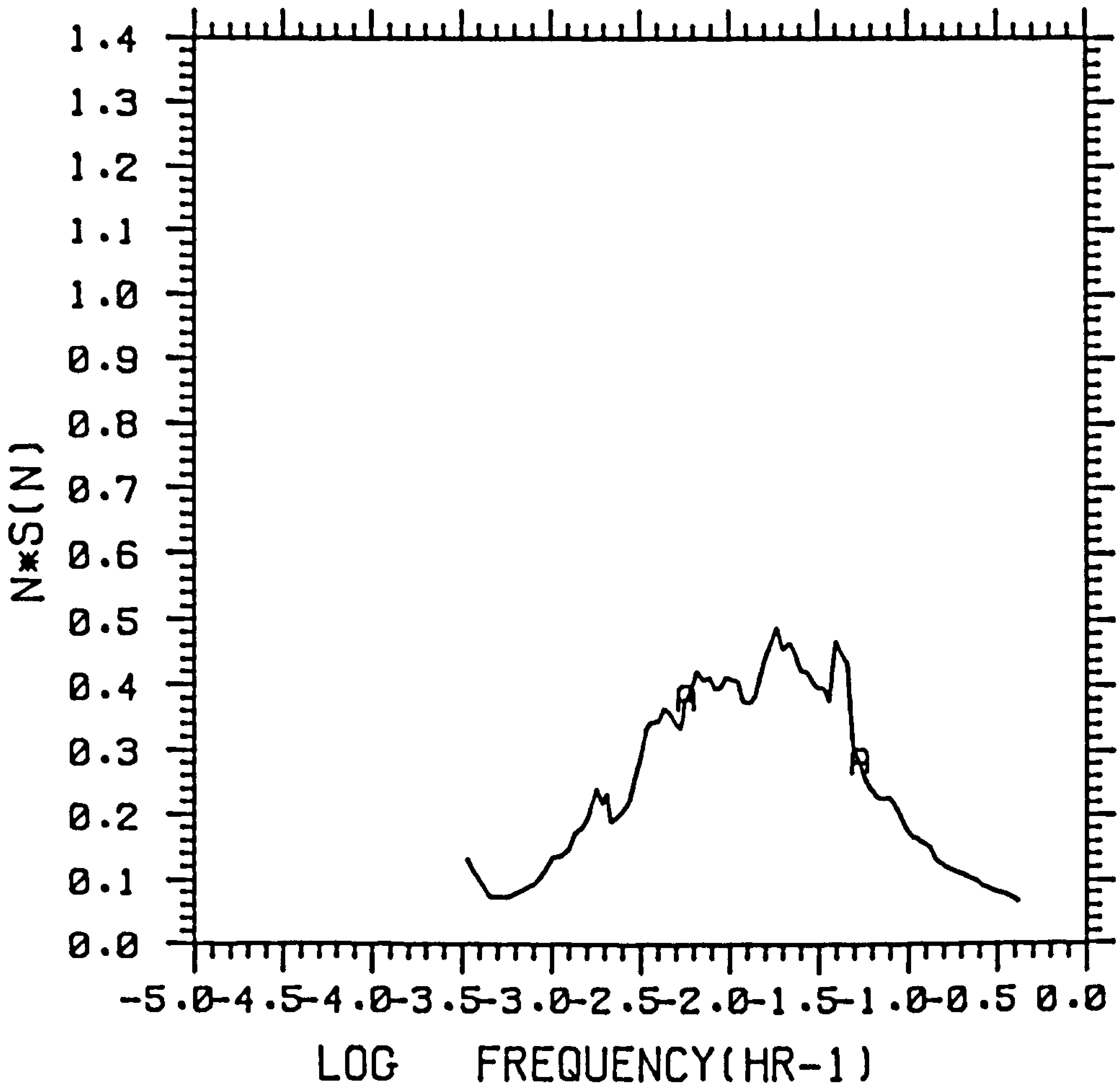


KEY

A ---

RUN 7

EFFECT OF SITE DISPERSAL. SITE BLY MOD2

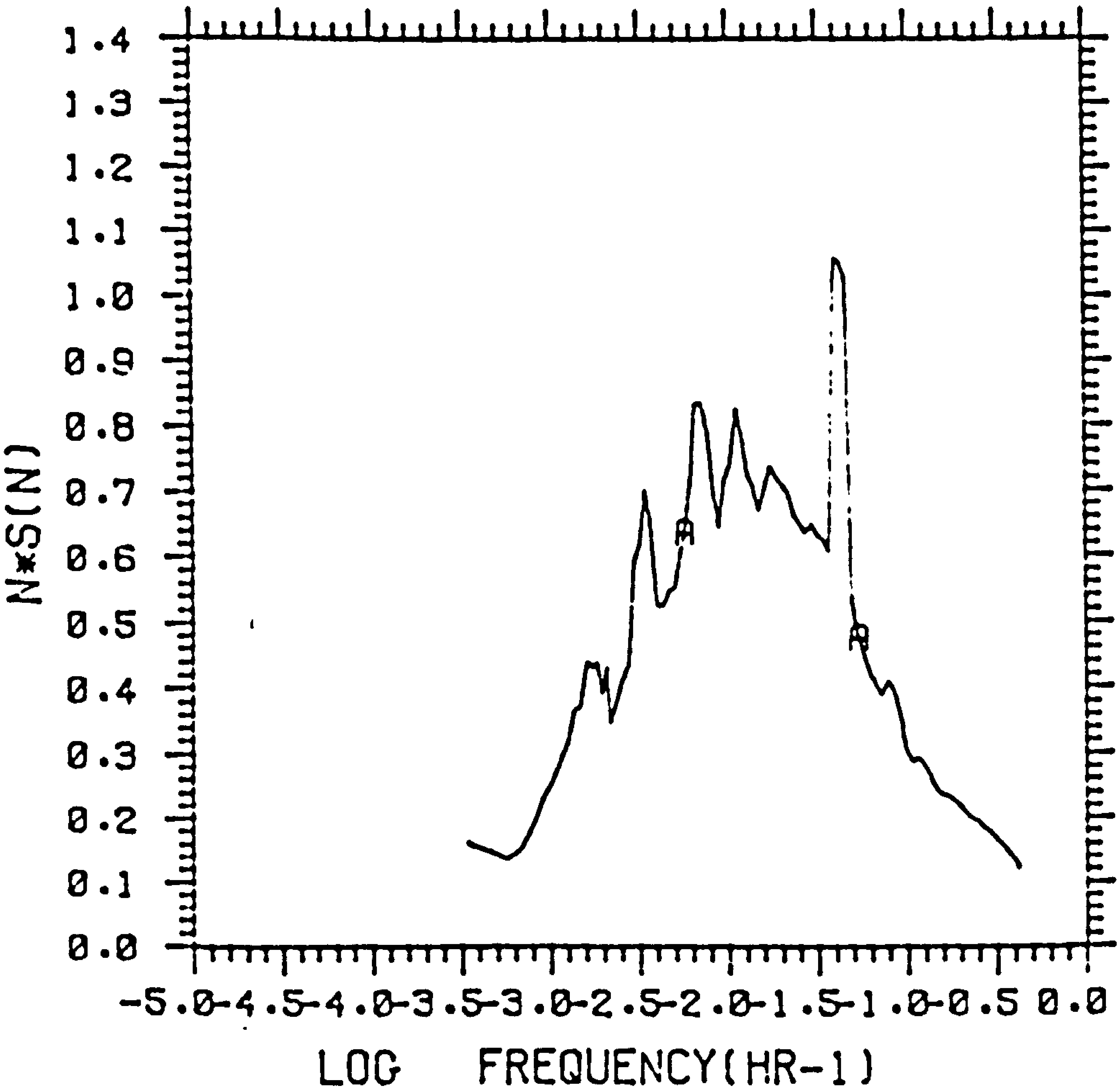


KEY

A ---

RUN 8

EFFECT OF SITE DISPERSAL. SITE PWK MOD2

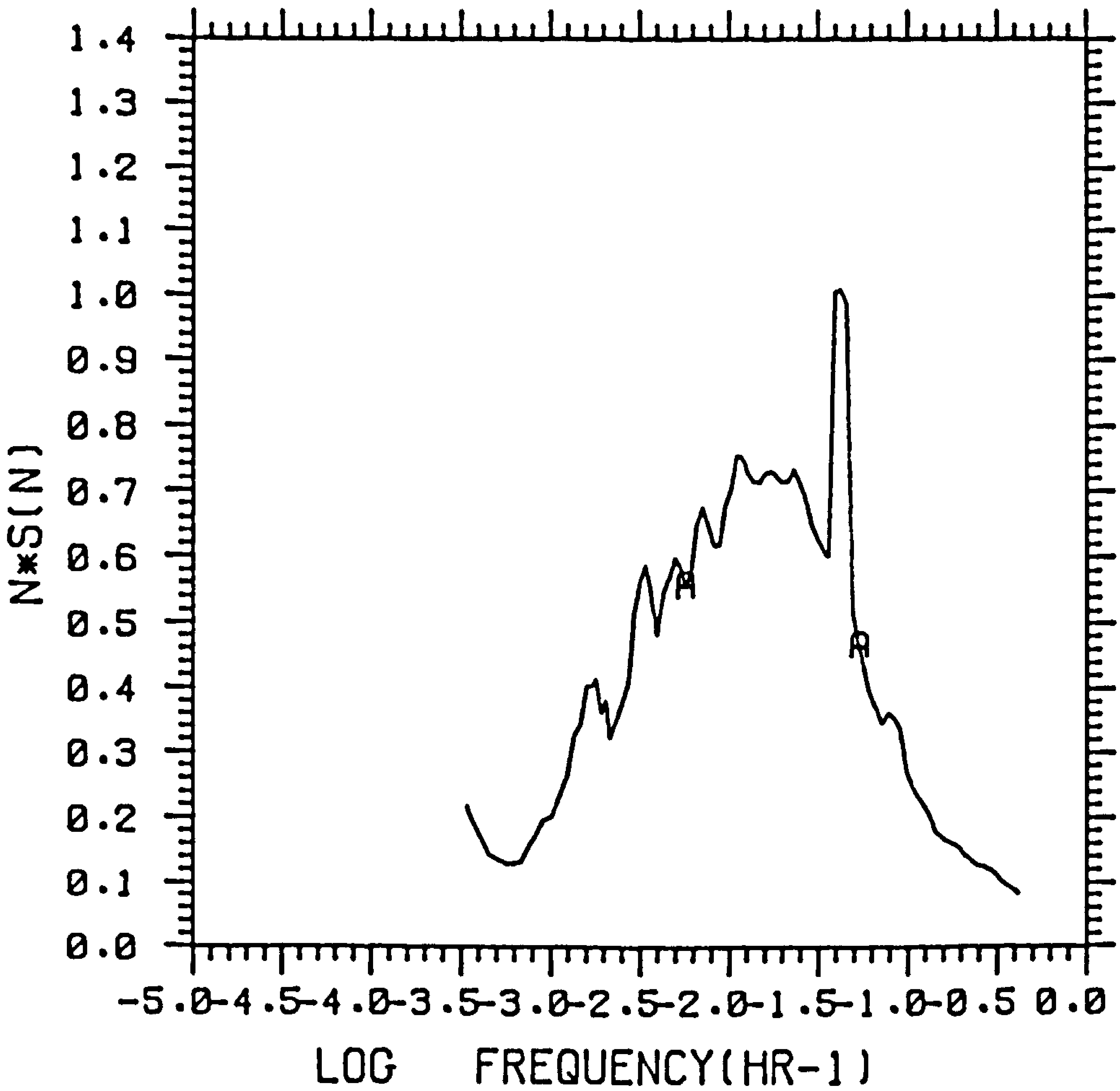


KEY

A ---

RUN 9

EFFECT OF SITE DISPERSAL. SITE WFR MOD2

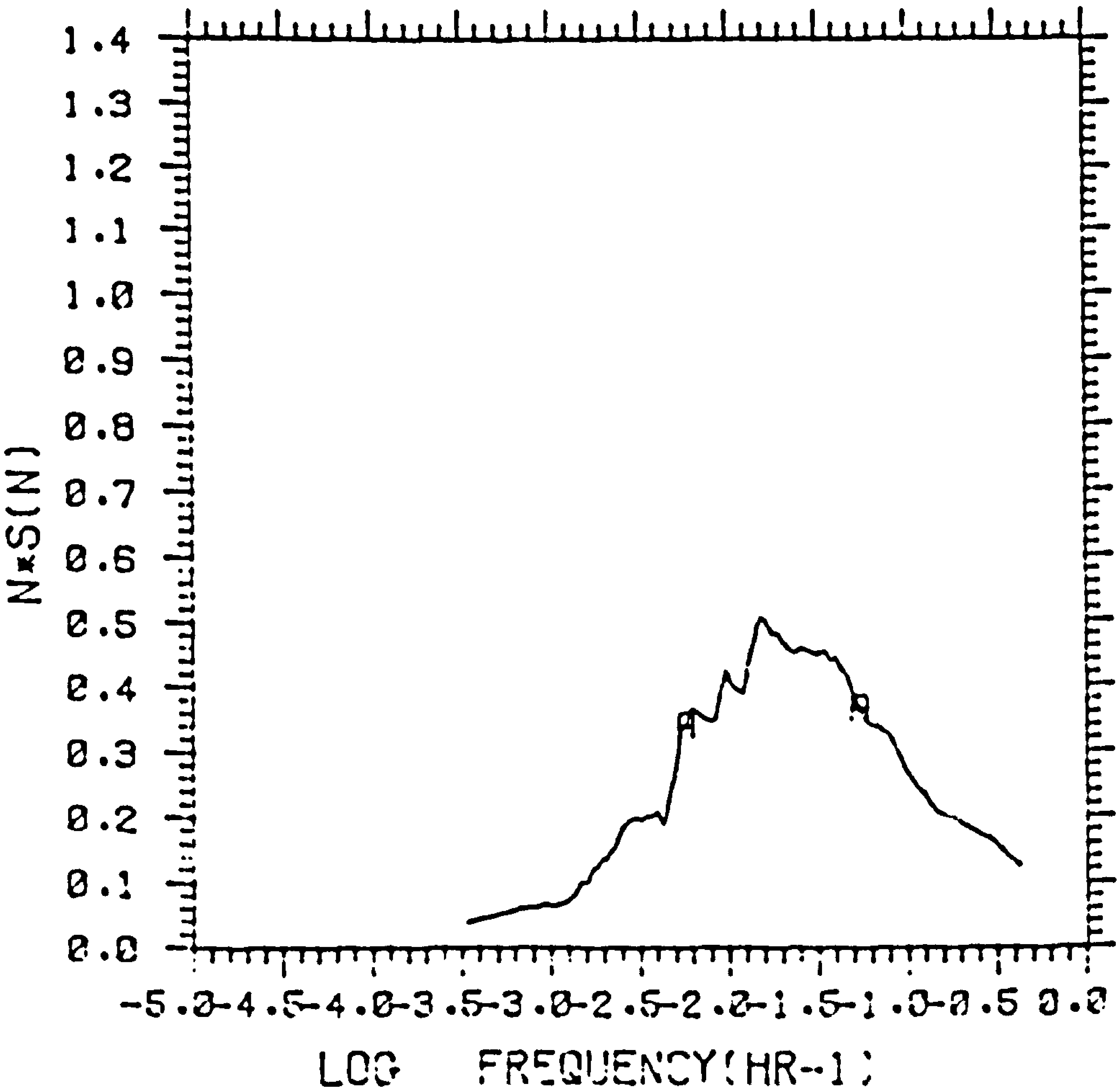


KEY

A ---

RUN 10

EFFECT OF SITE DISPERSAL. SITE GTD MOD2

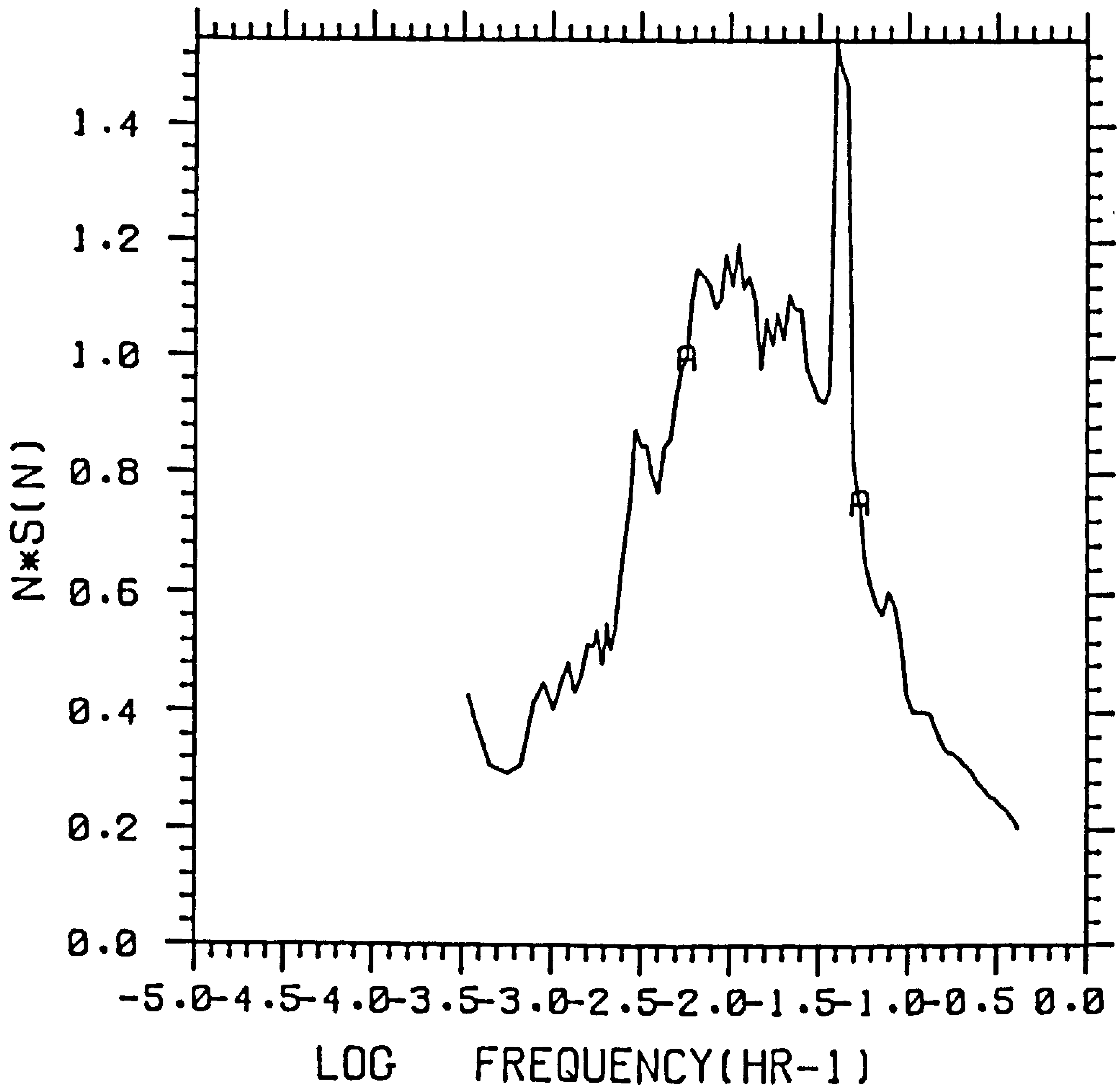


KEY

A ---

RUN 11

EFFECT OF SITE DISPERSAL. SITE DRH MOD2

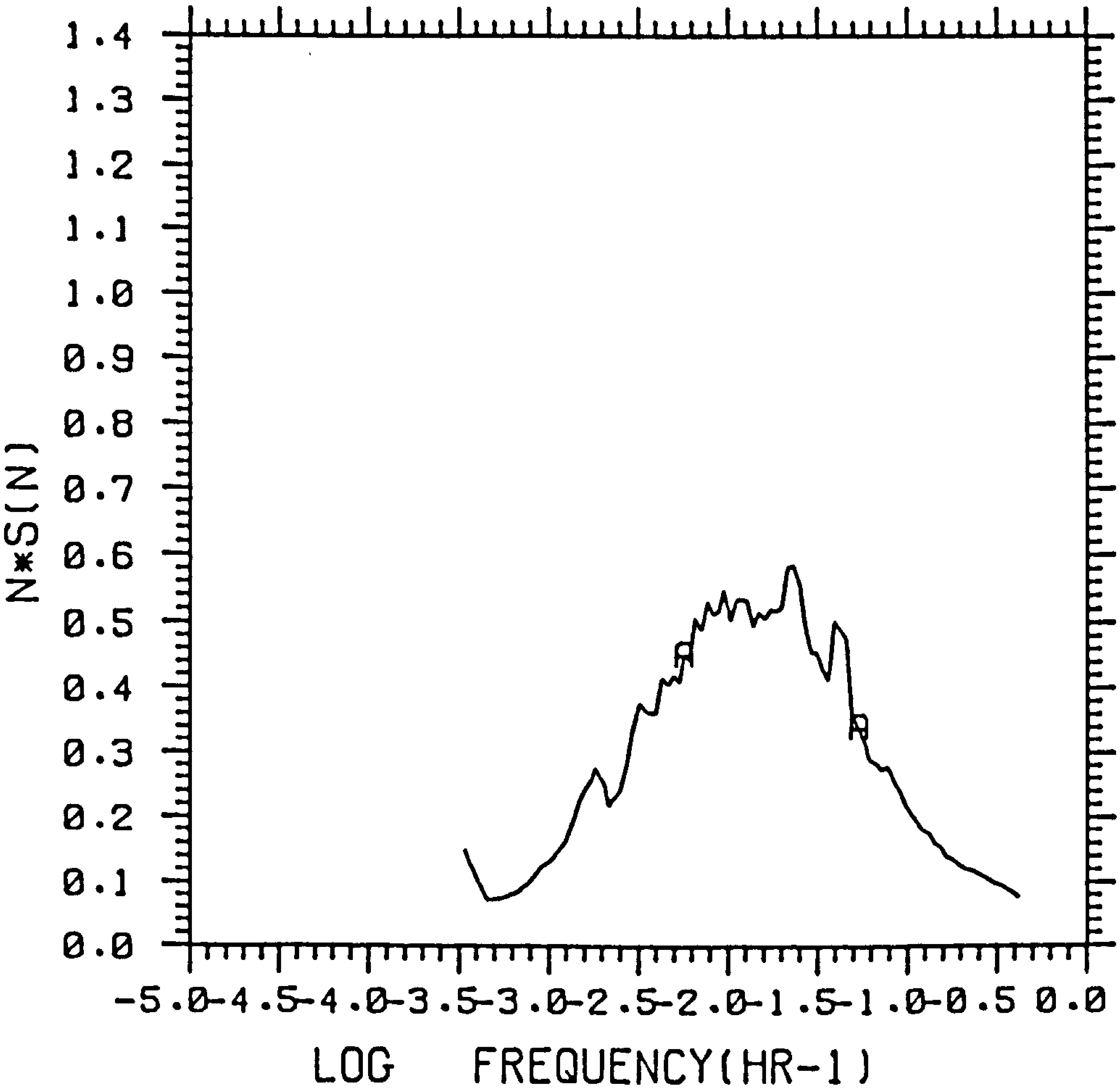


KEY

A ---

RUN 12

EFFECT OF SITE DISPERSAL. SITE RON MOD2

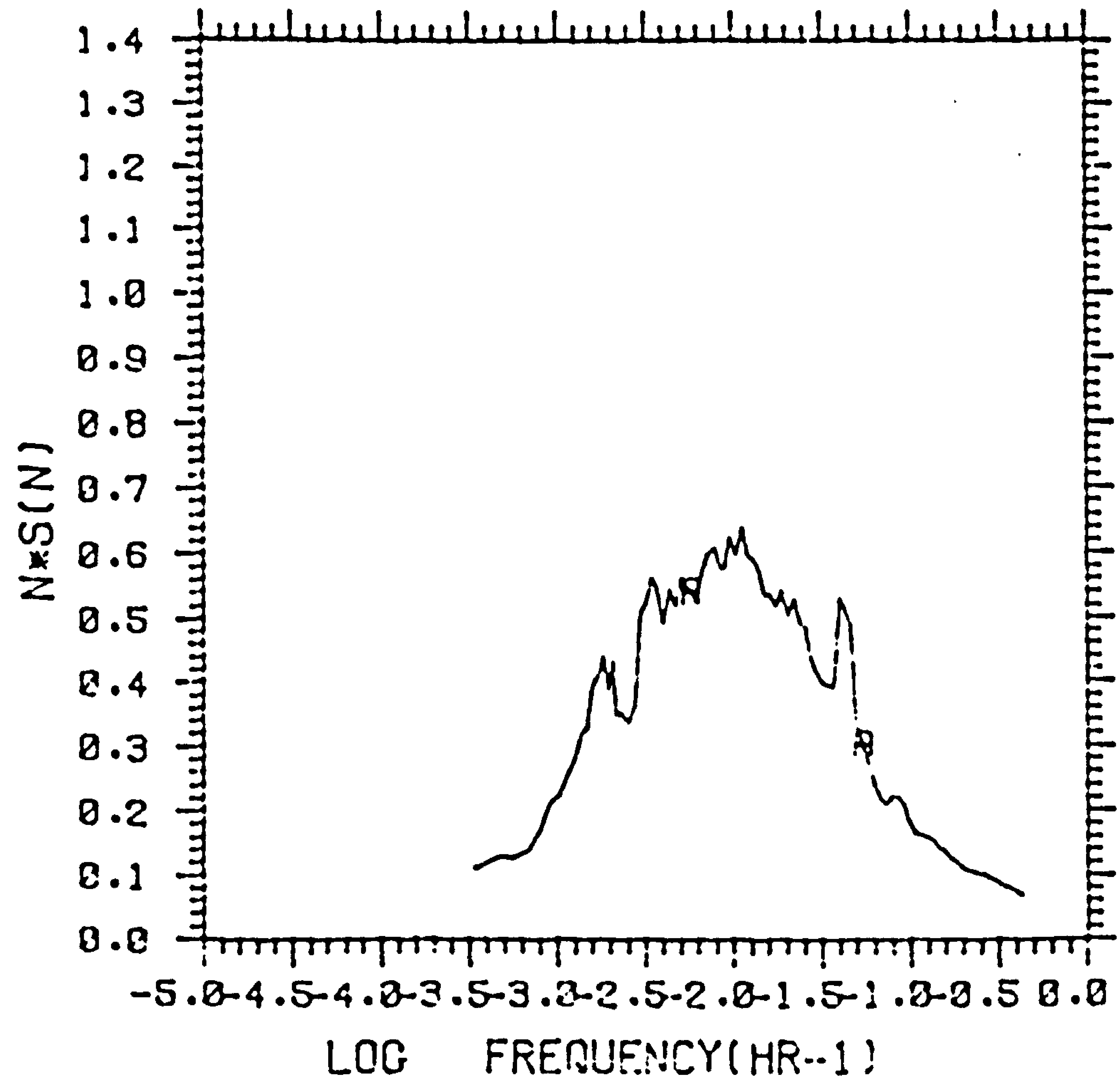


KEY

A ---

RUN 13

EFFECT OF SITE DISPERSAL. SITE SQG MOD2

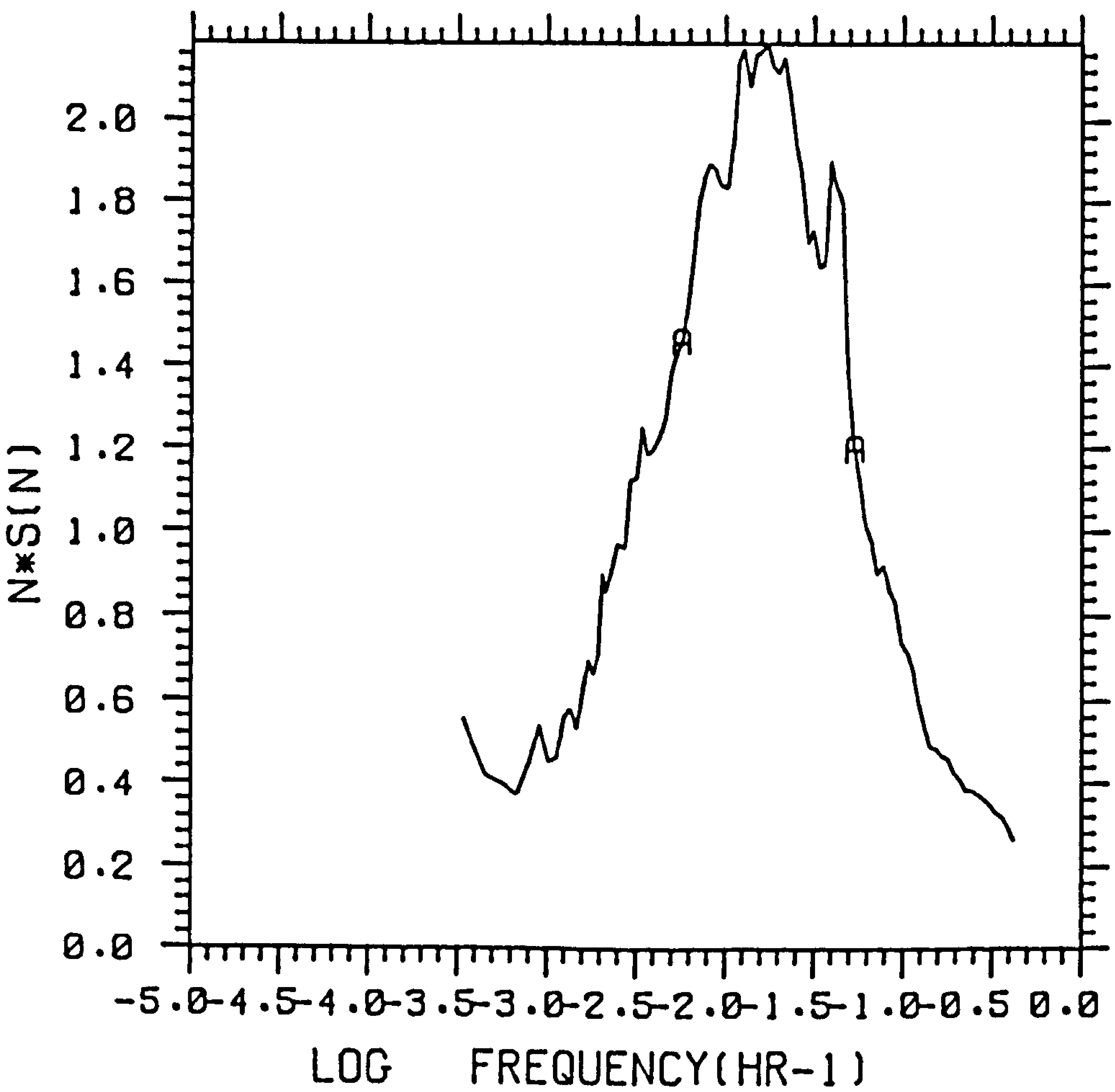


KEY

A ---

RUN 14

EFFECT OF SITE DISPERSAL. SITE SMR MOD2

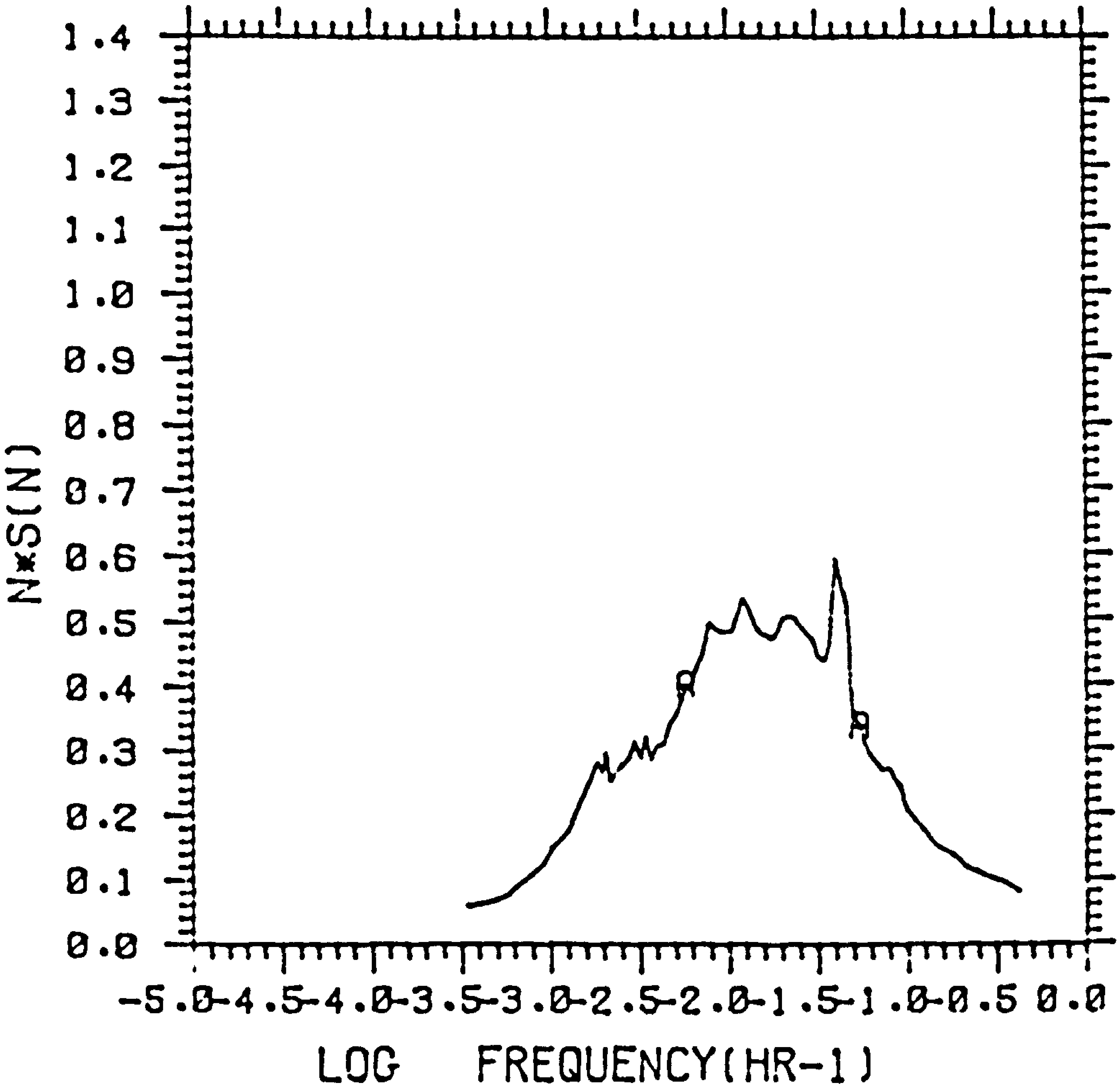


KEY

A ---

RUN 15

EFFECT OF SITE DISPERSAL. SITE VAL MOD2

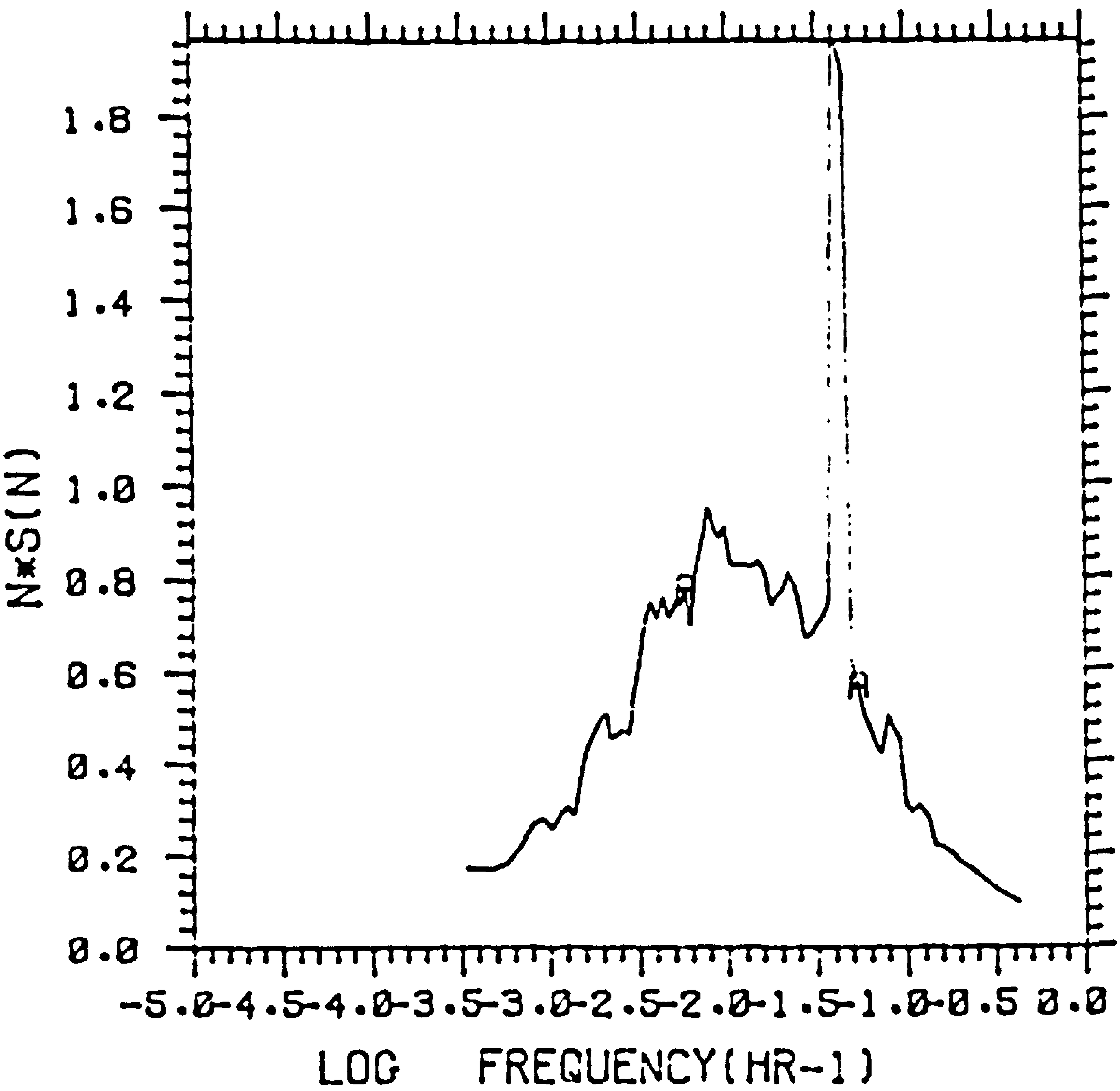


KEY

A ---

RUN 16

EFFECT OF SITE DISPERSAL. SITE CON MOD2

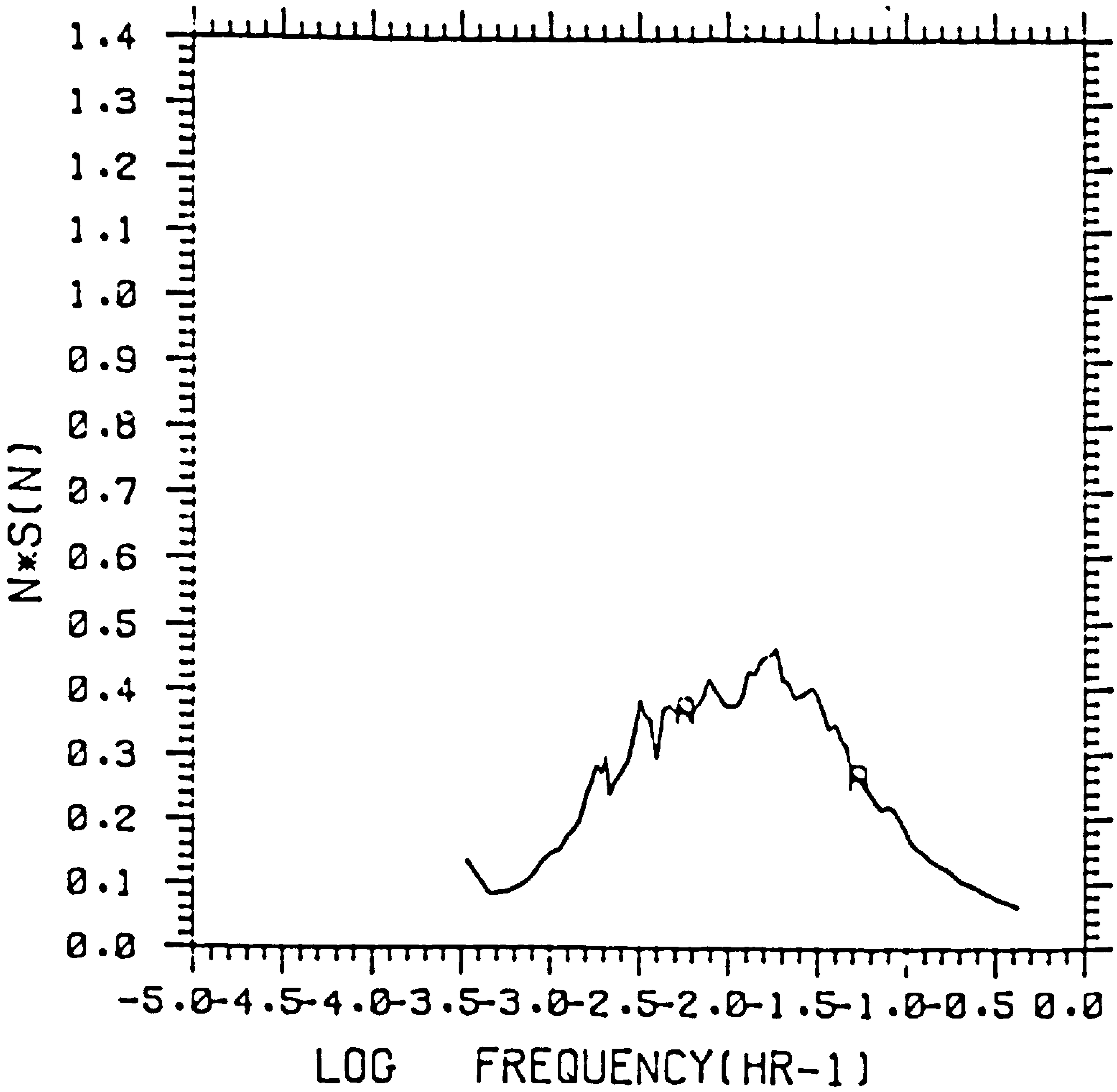


KEY

A ---

RUN 17

EFFECT OF SITE DISPERSAL. SITE ABP MOD2

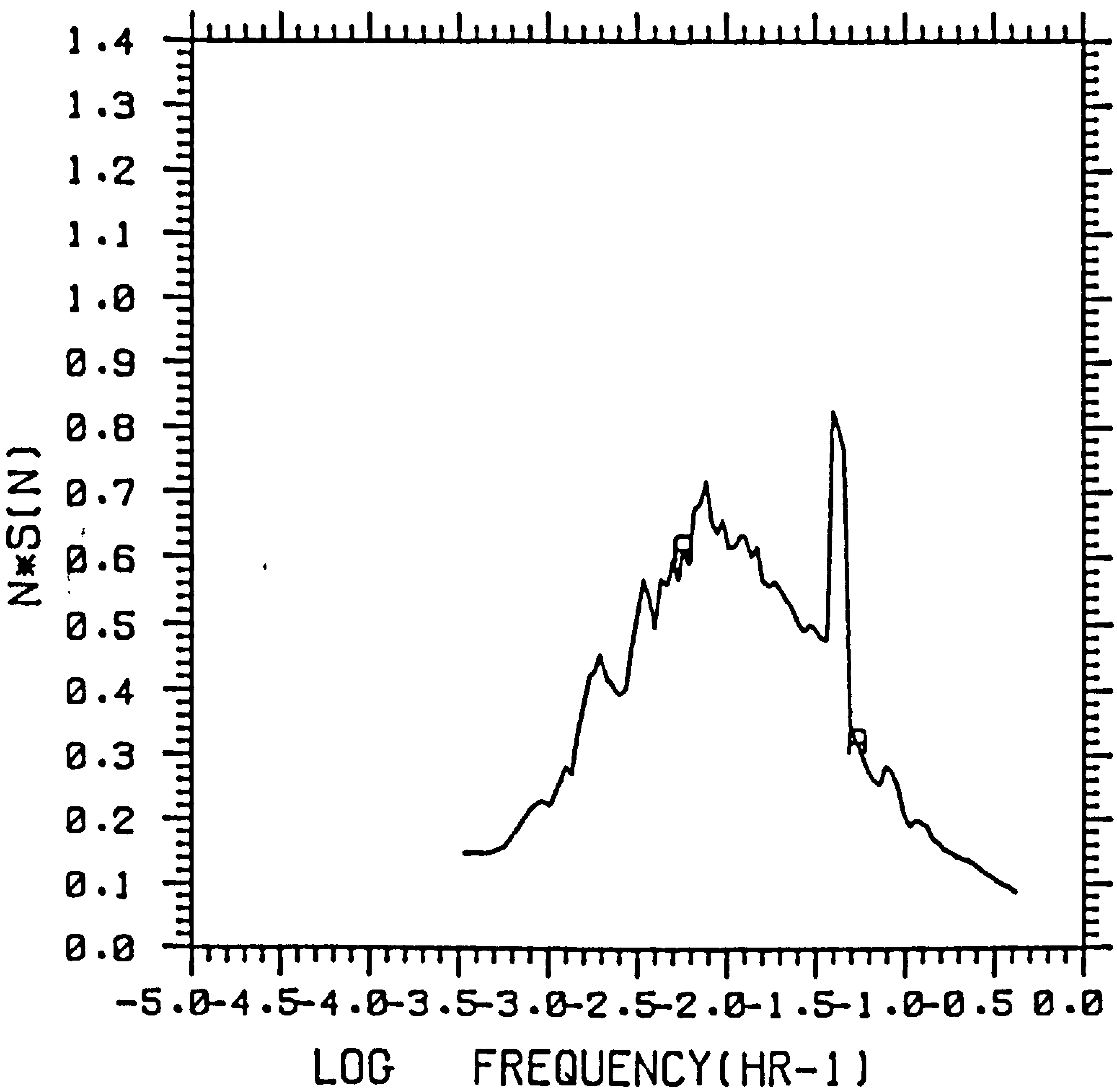


KEY

A ---

RUN 18

EFFECT OF SITE DISPERSAL. SITE CAR MOD2

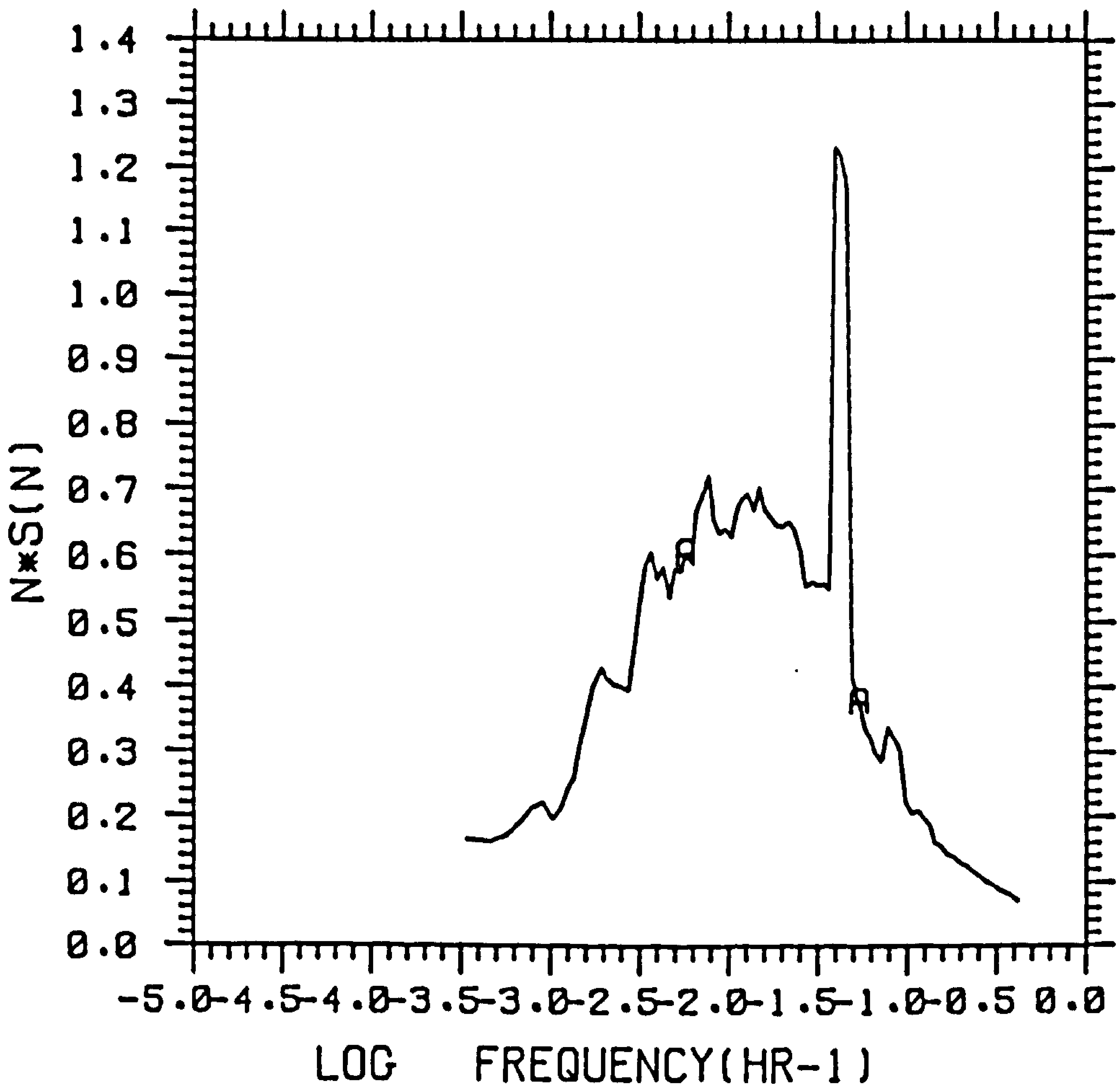


KEY

A ---

RUN 19

EFFECT OF SITE DISPERSAL. SITE WAT MOD2

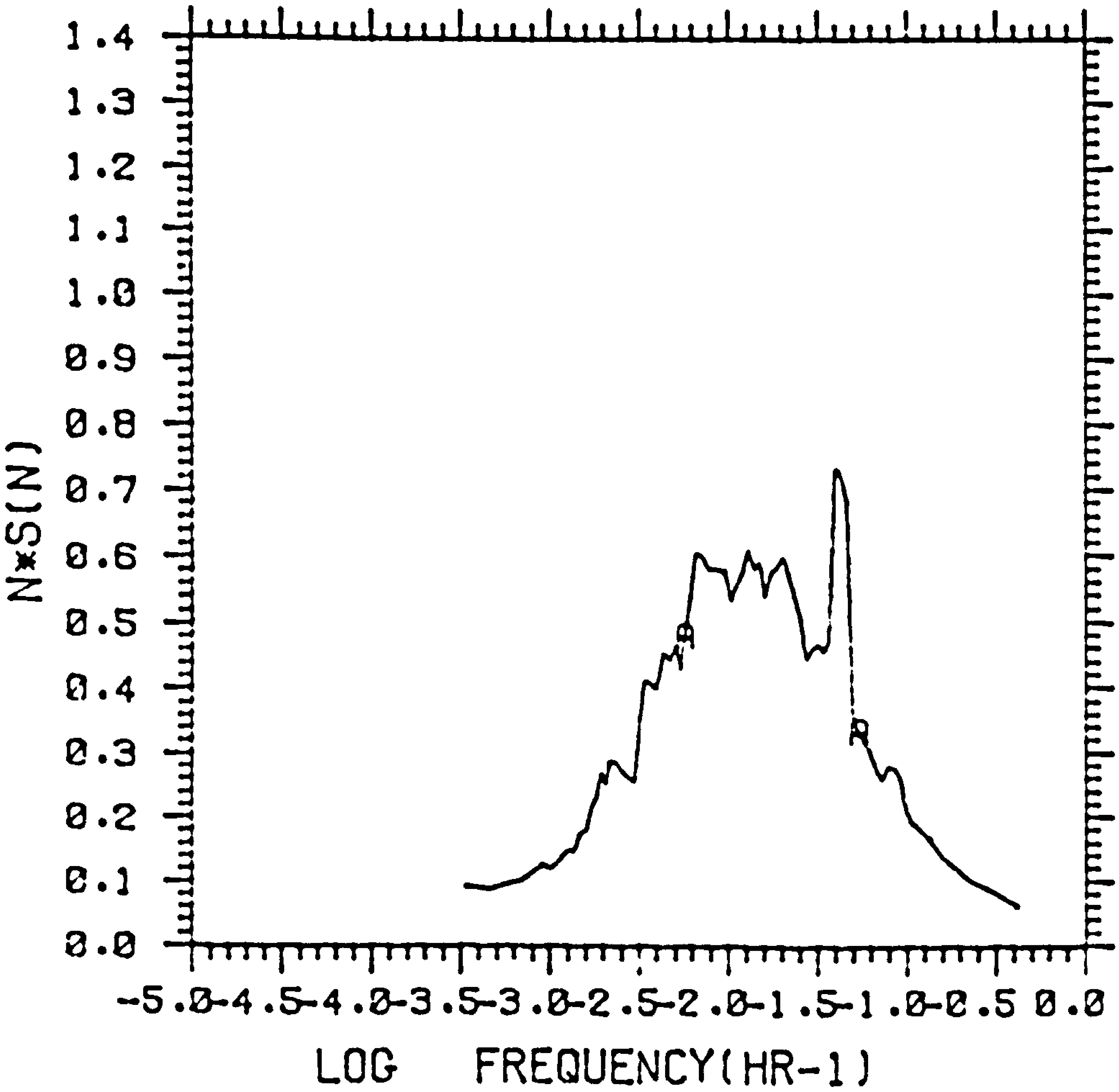


KEY

A ---

RUN 20

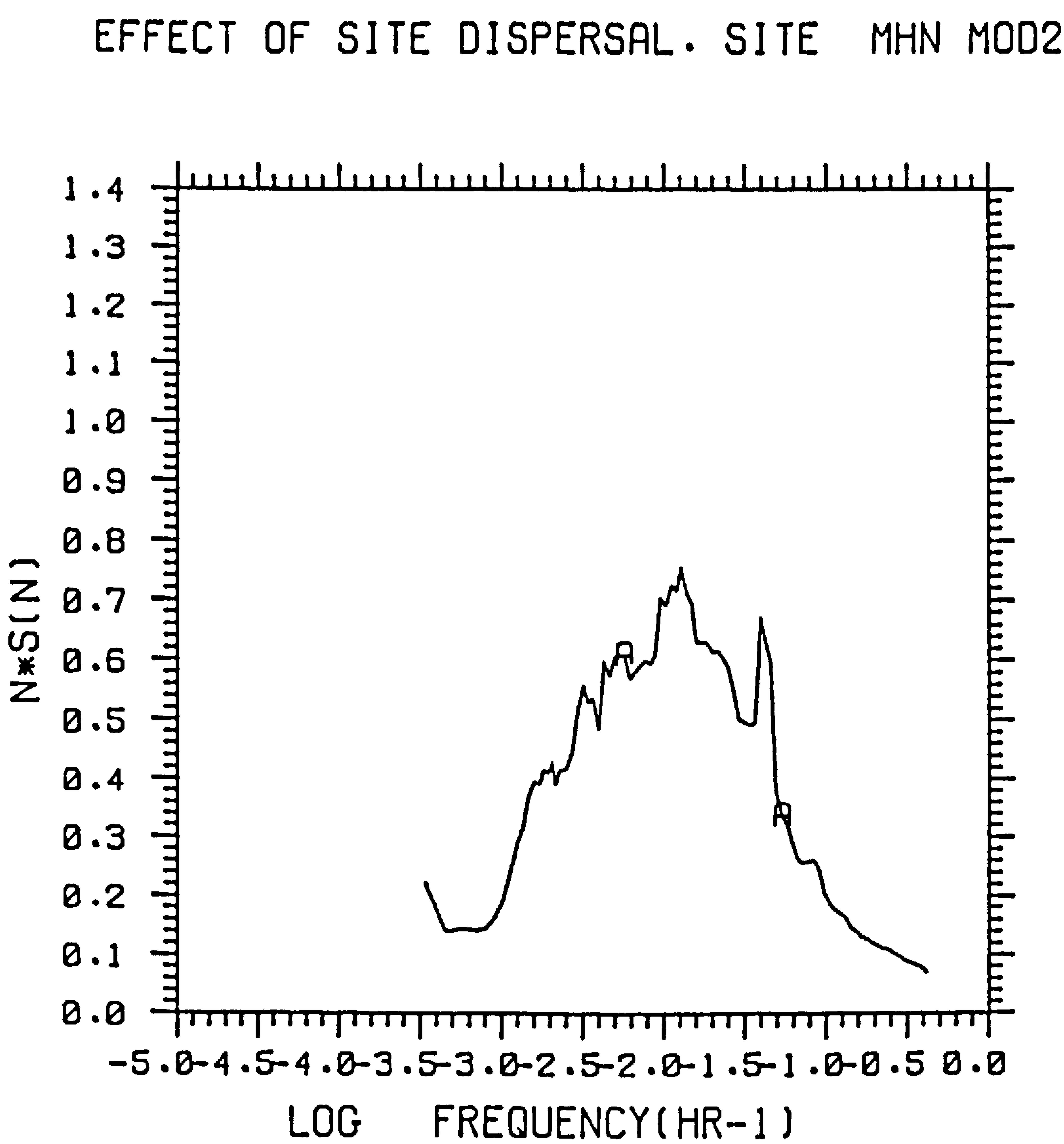
EFFECT OF SITE DISPERSAL. SITE GOR MOD2



KEY

A ---

RUN 21



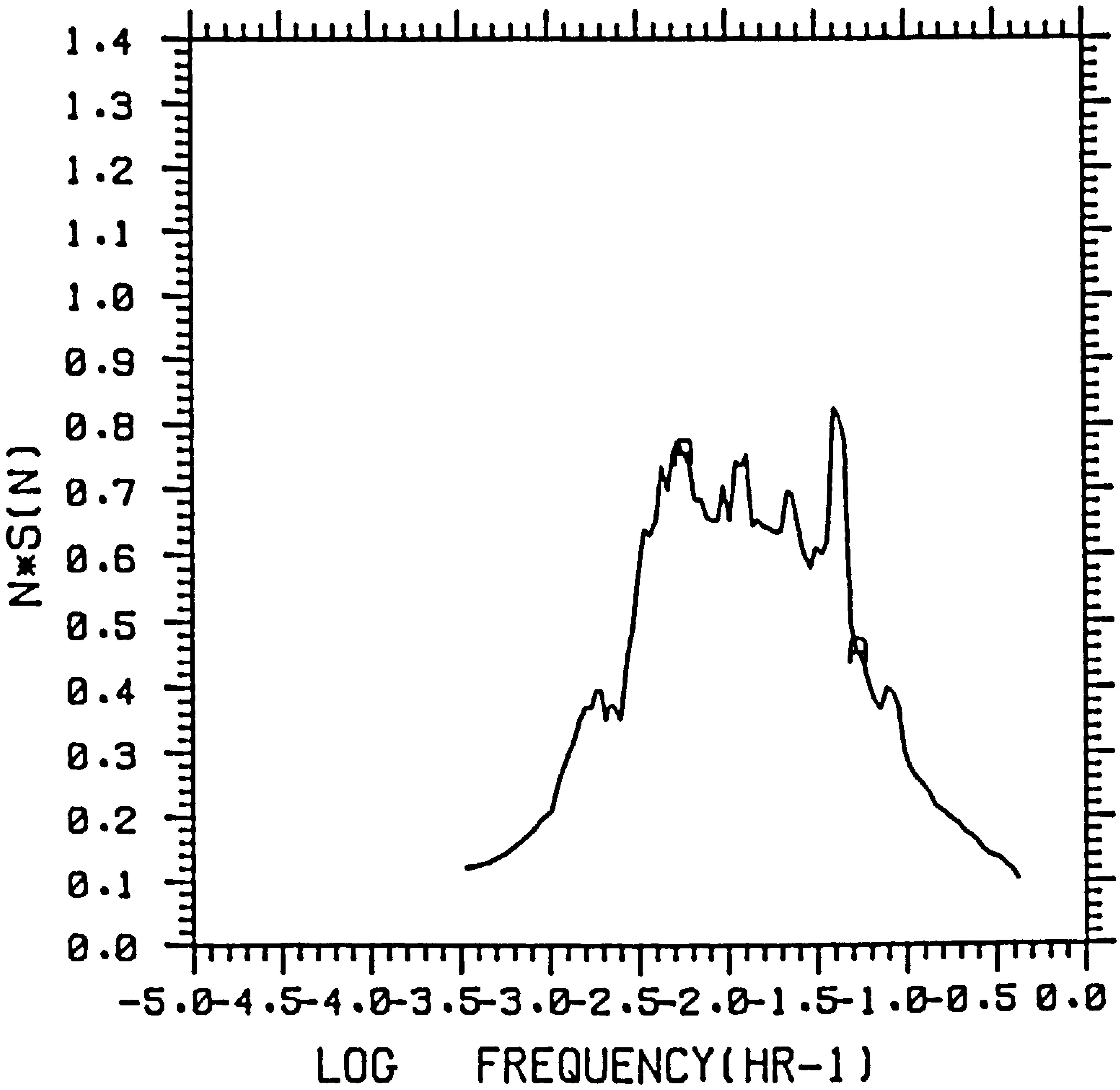
EFFECT OF SITE DISPERSAL. SITE MHN MOD2
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
N*S(N)
A
A
-5.0
-4.5
-4.0
-3.5
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0.0
LOG FREQUENCY(HR-1)

KEY

A ---

RUN 22

EFFECT OF SITE DISPERSAL. SITE PRT MOD2

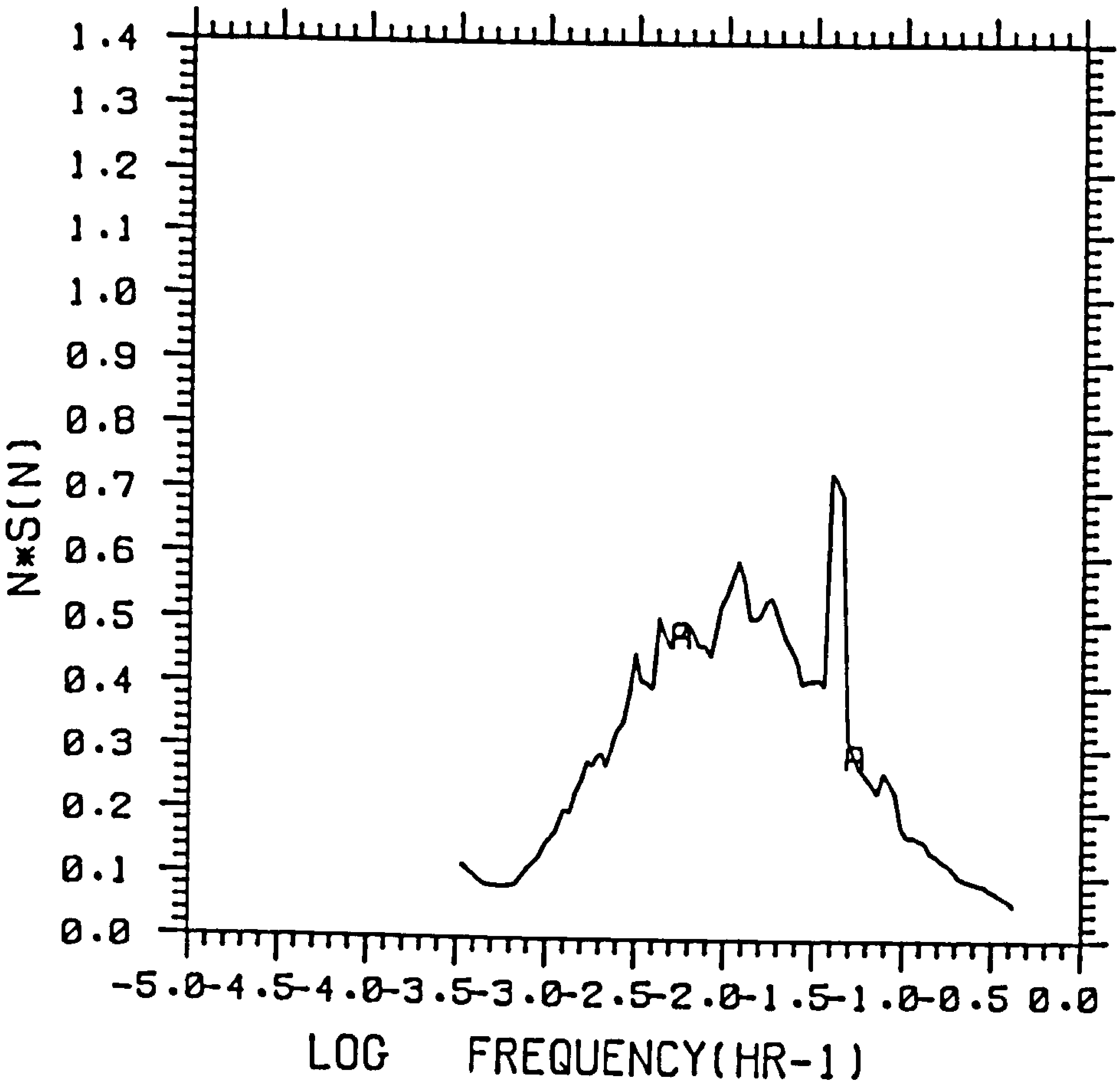


KEY

A ---

RUN 23

EFFECT OF SITE DISPERSAL. SITE STM MOD2

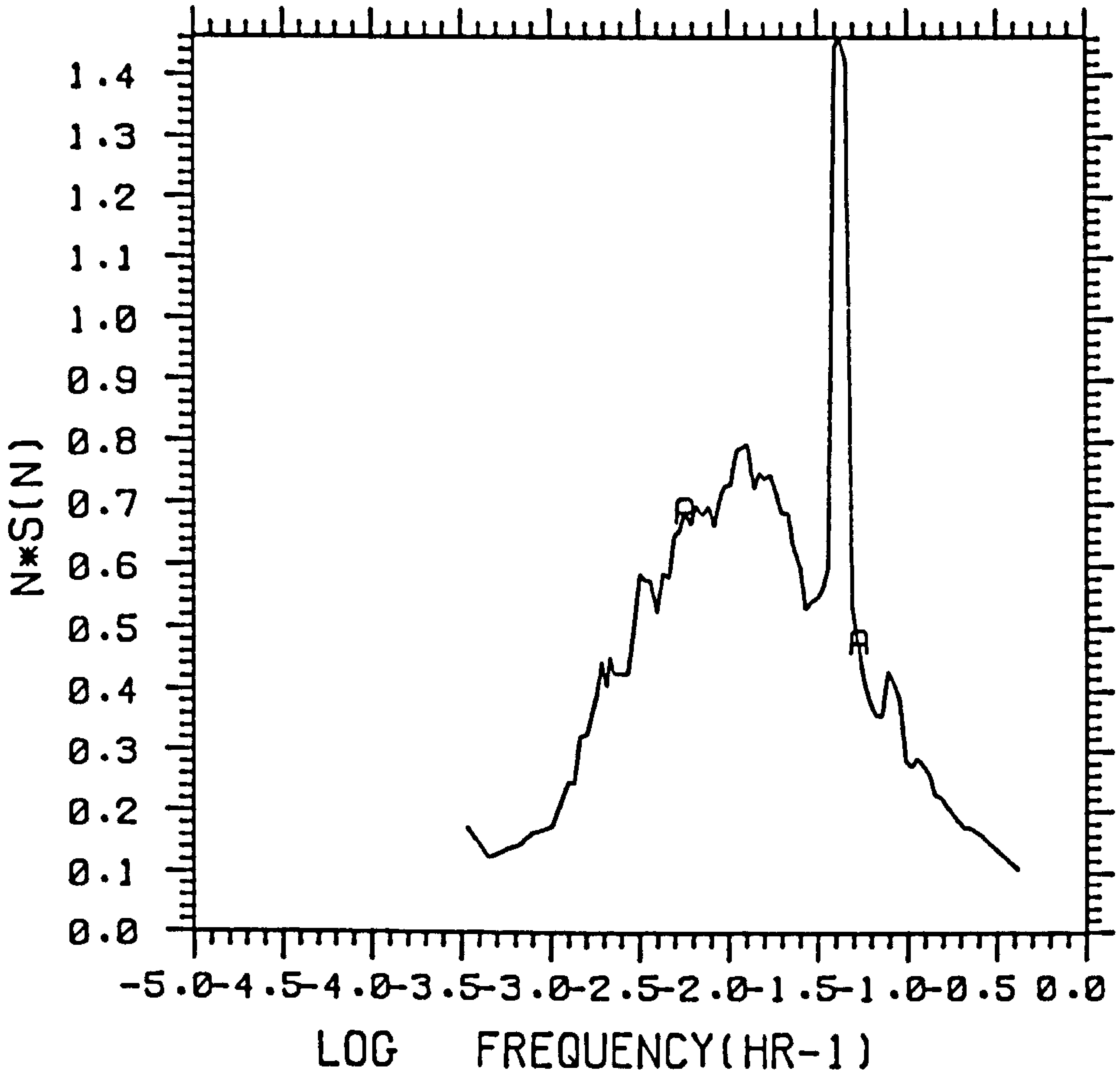


KEY

A ---

RUN 24

EFFECT OF SITE DISPERSAL. SITE BDN MOD2

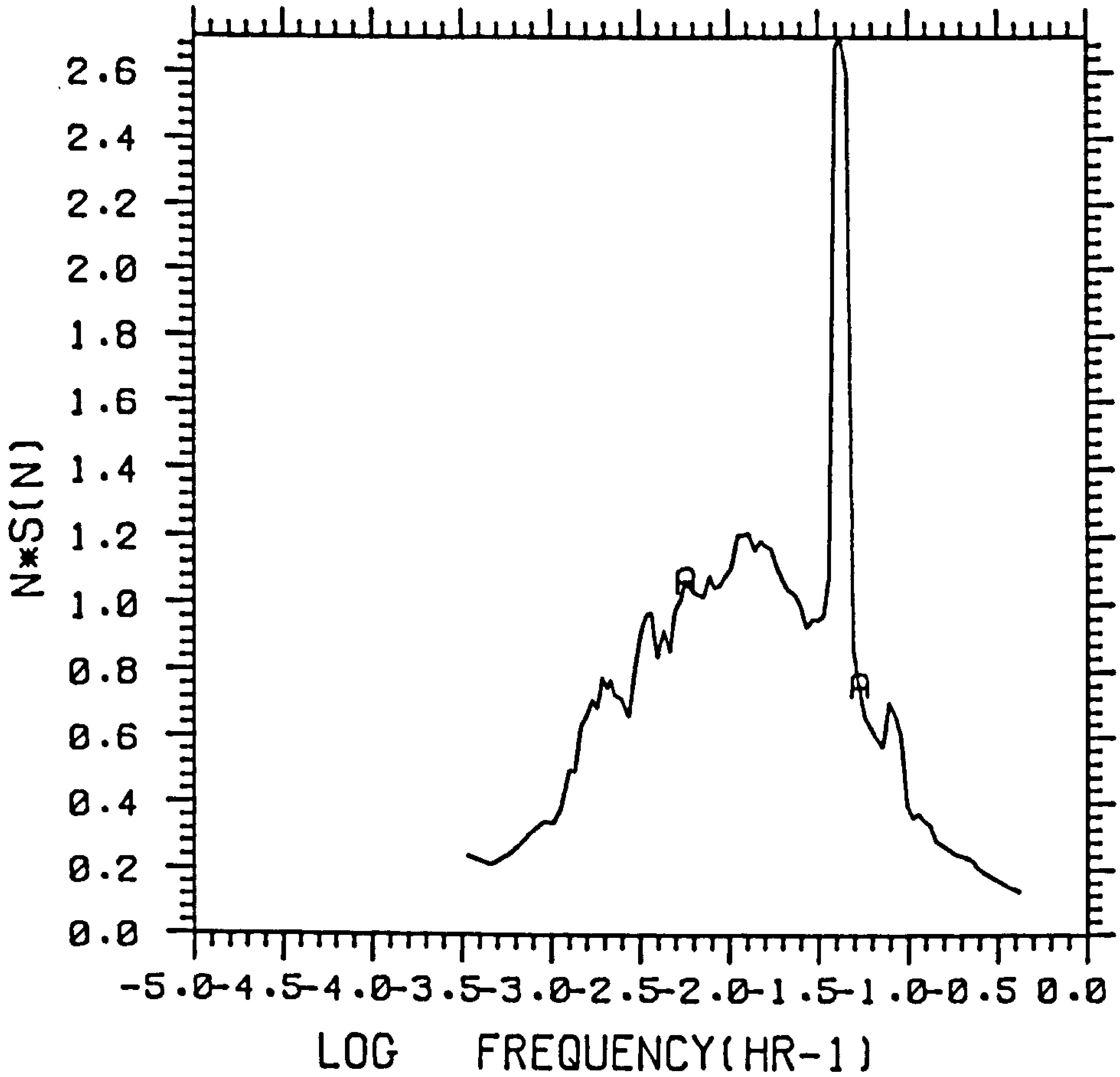


KEY

A ---

RUN 25

EFFECT OF SITE DISPERSAL. SITE GWK MOD2

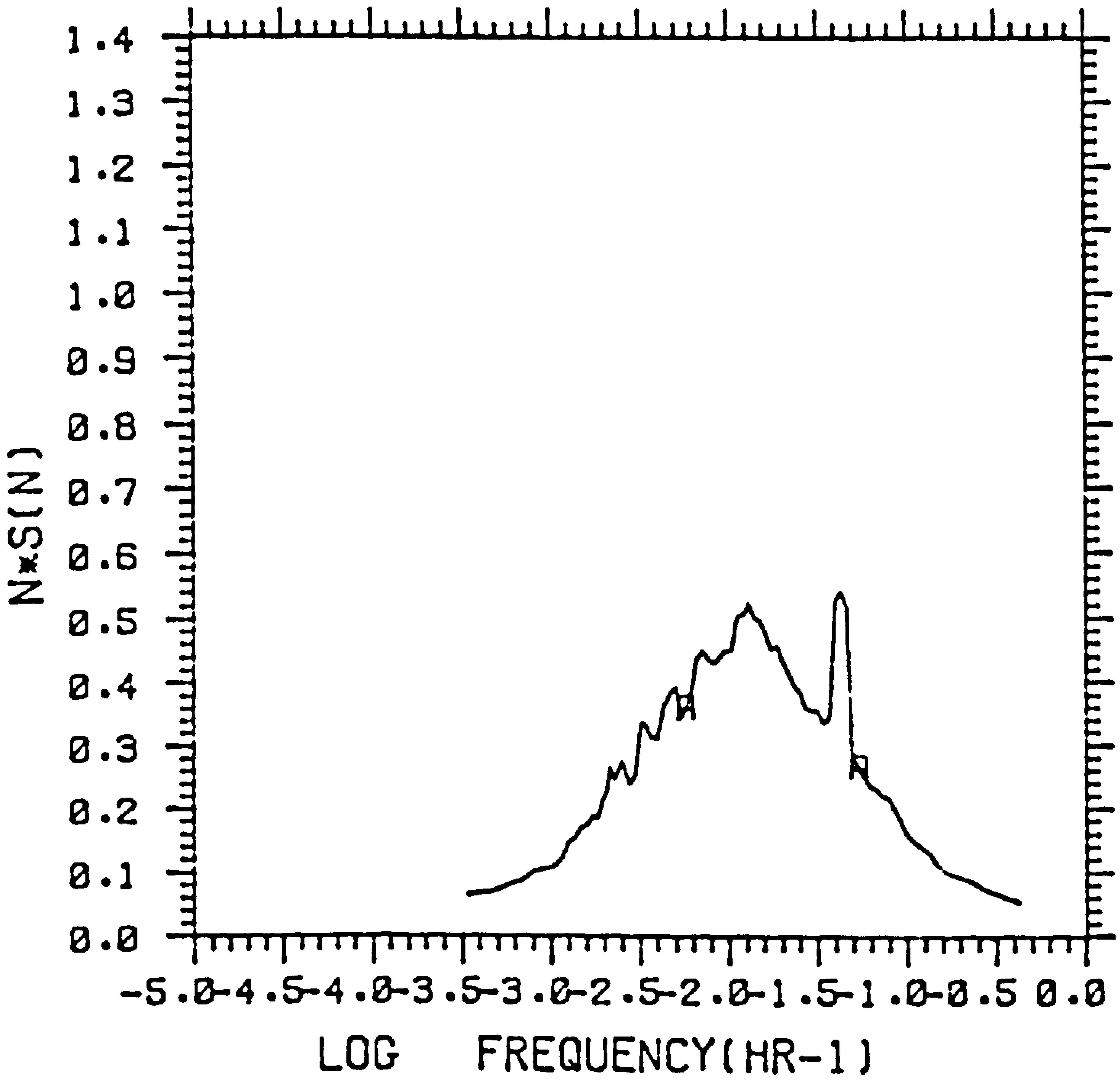


KEY

A ---

RUN 26

EFFECT OF SITE DISPERSAL. SITE DNG MOD2

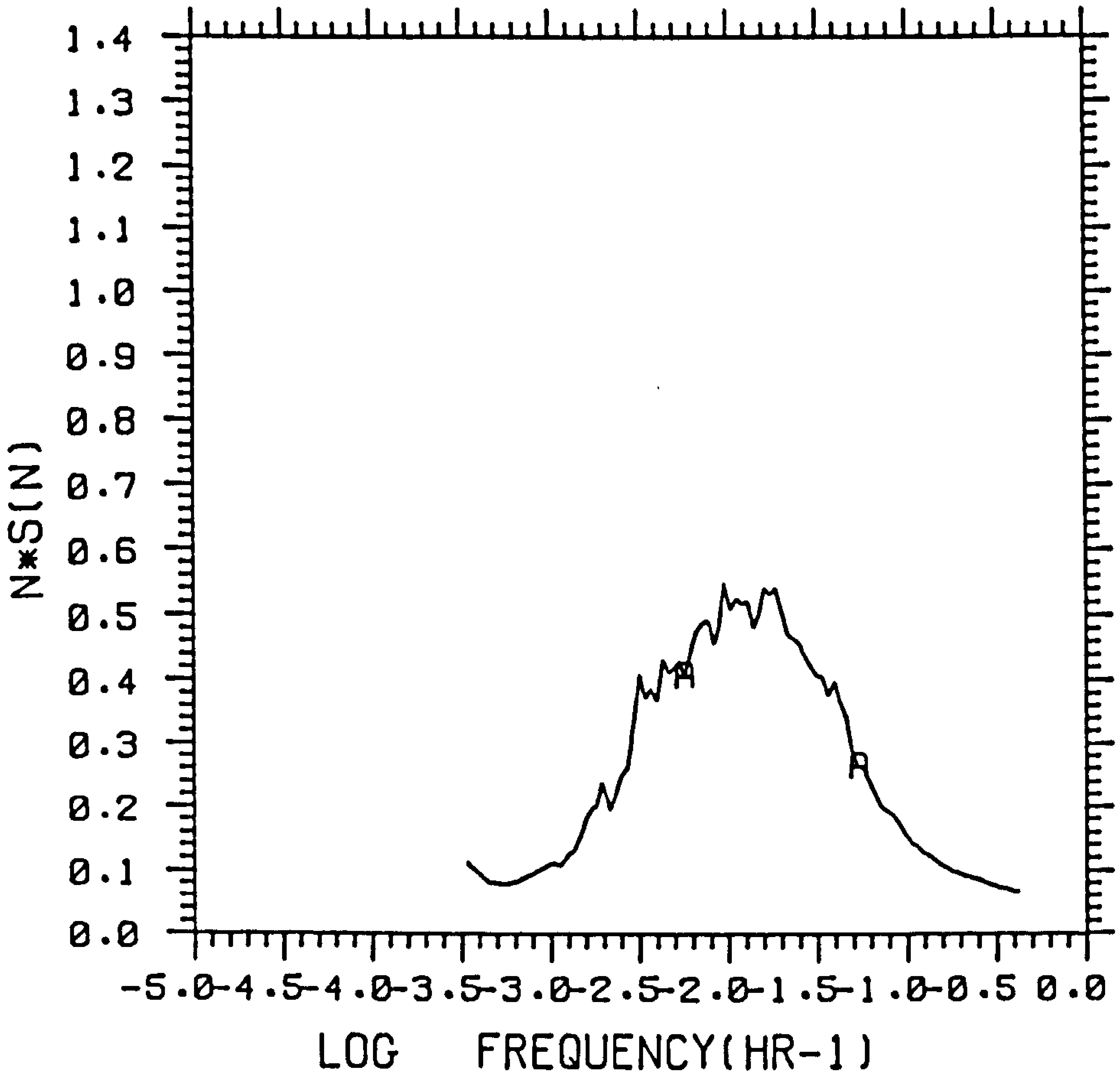


KEY

A ---

RUN 27

EFFECT OF SITE DISPERSAL. SITE SCI MOD2

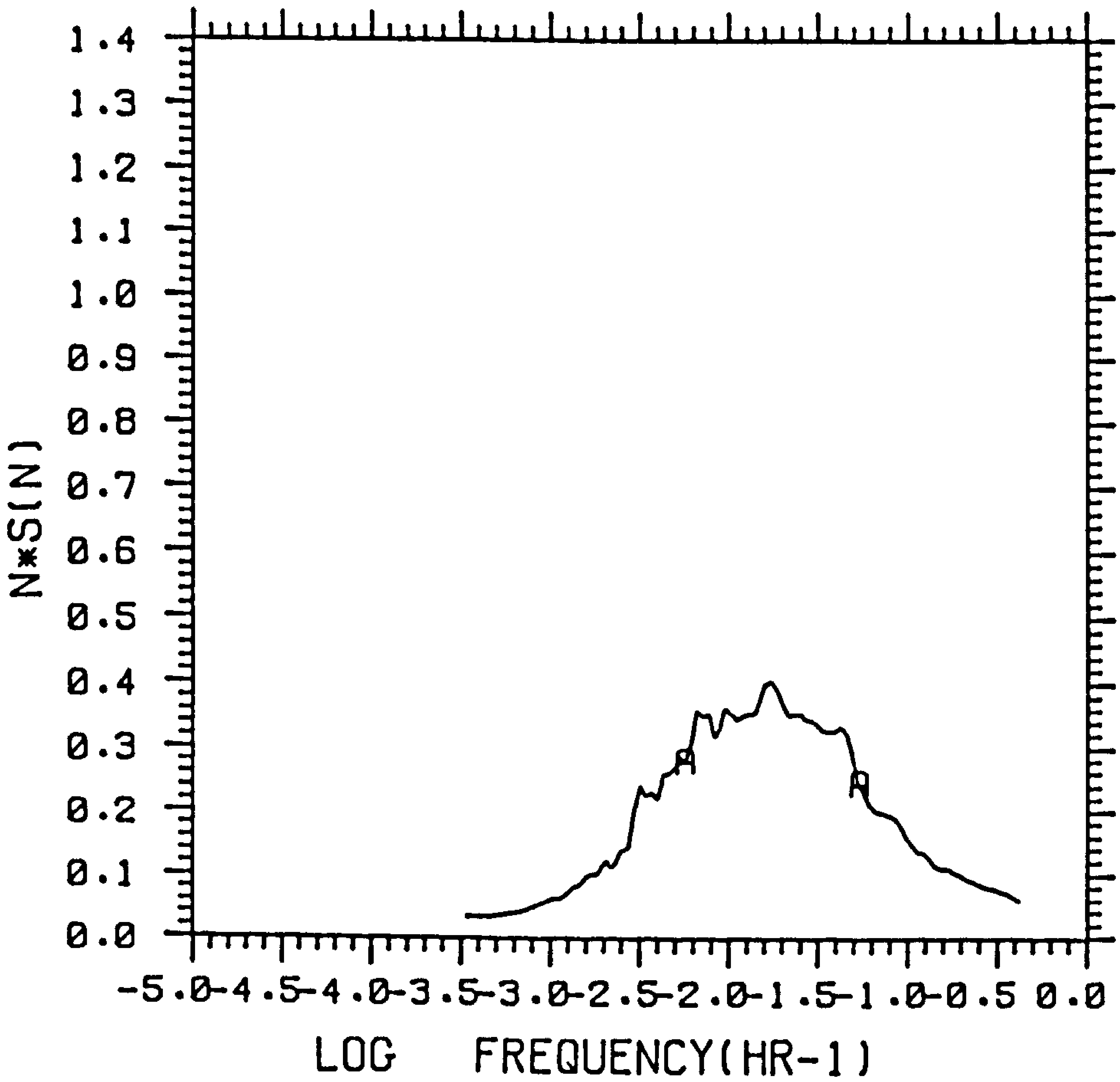


KEY

A ---

RUN 28

EFFECT OF SITE DISPERSAL. SITE LZA MOD2

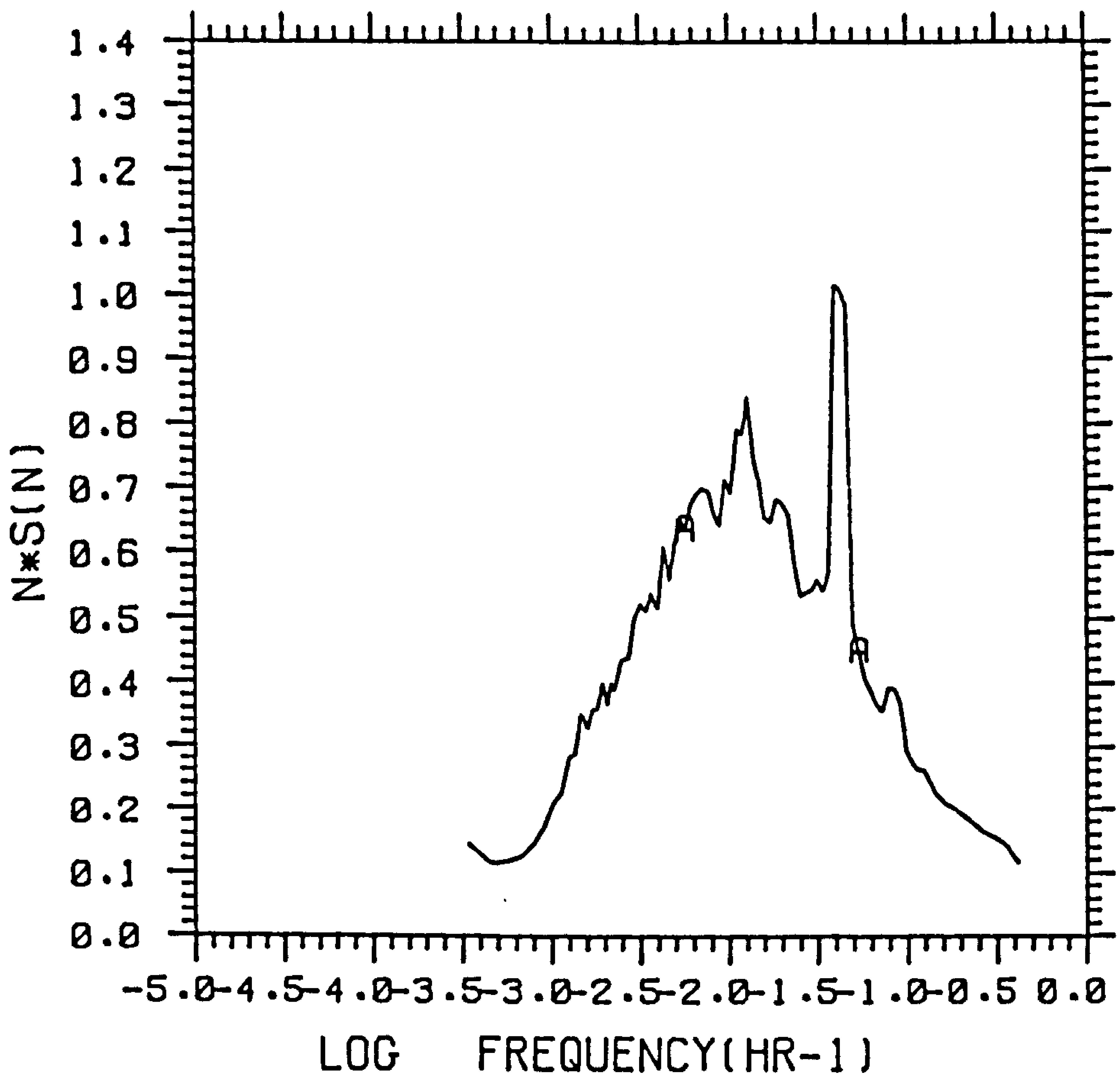


KEY

A ---

RUN 29

EFFECT OF SITE DISPERSAL. SITE MNT MOD2



KEY

A ---

RUN 30

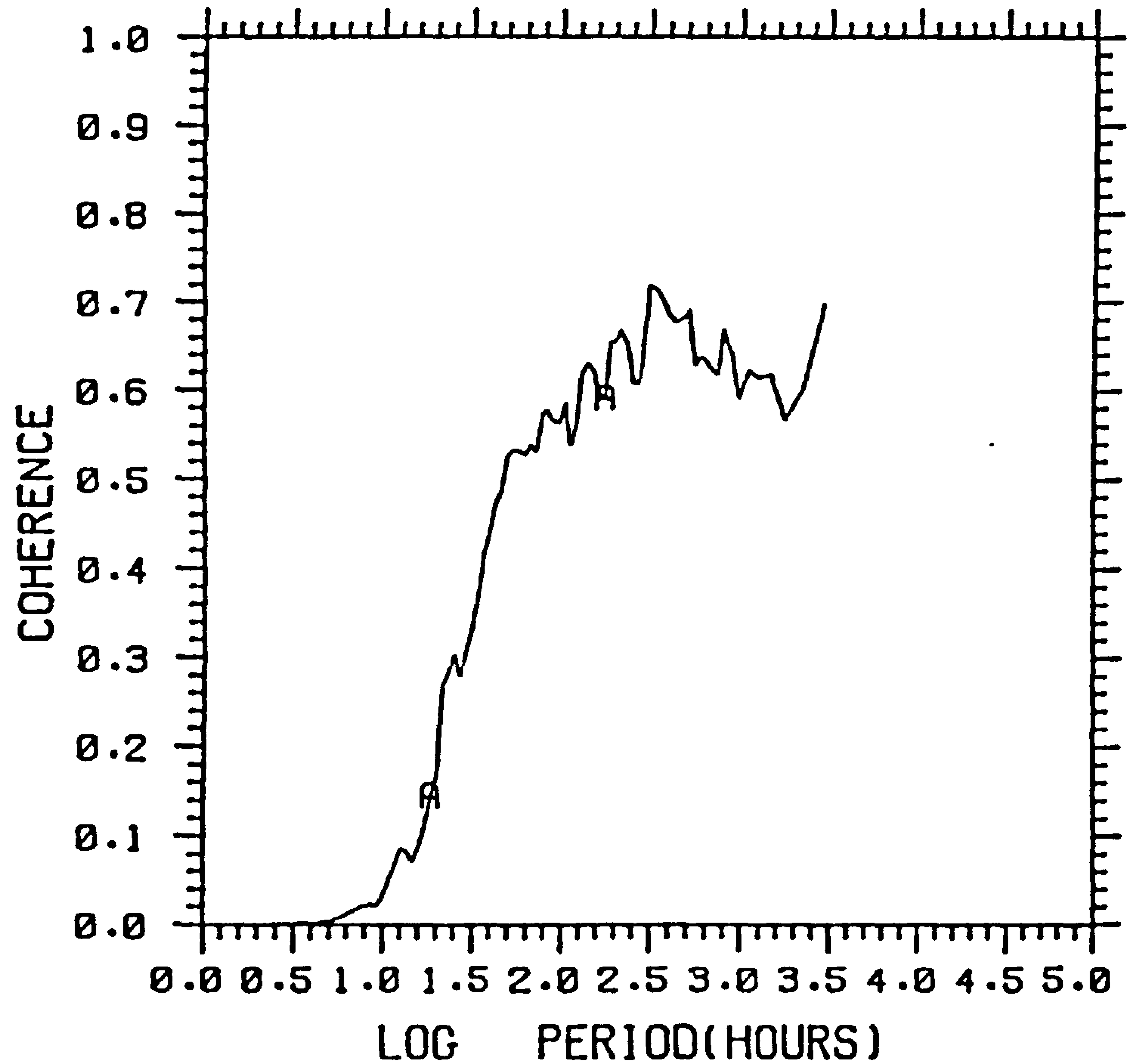
Site pair analysis.

Coherence function and cross correlation plots are presented for the following pairs of wind power sites:

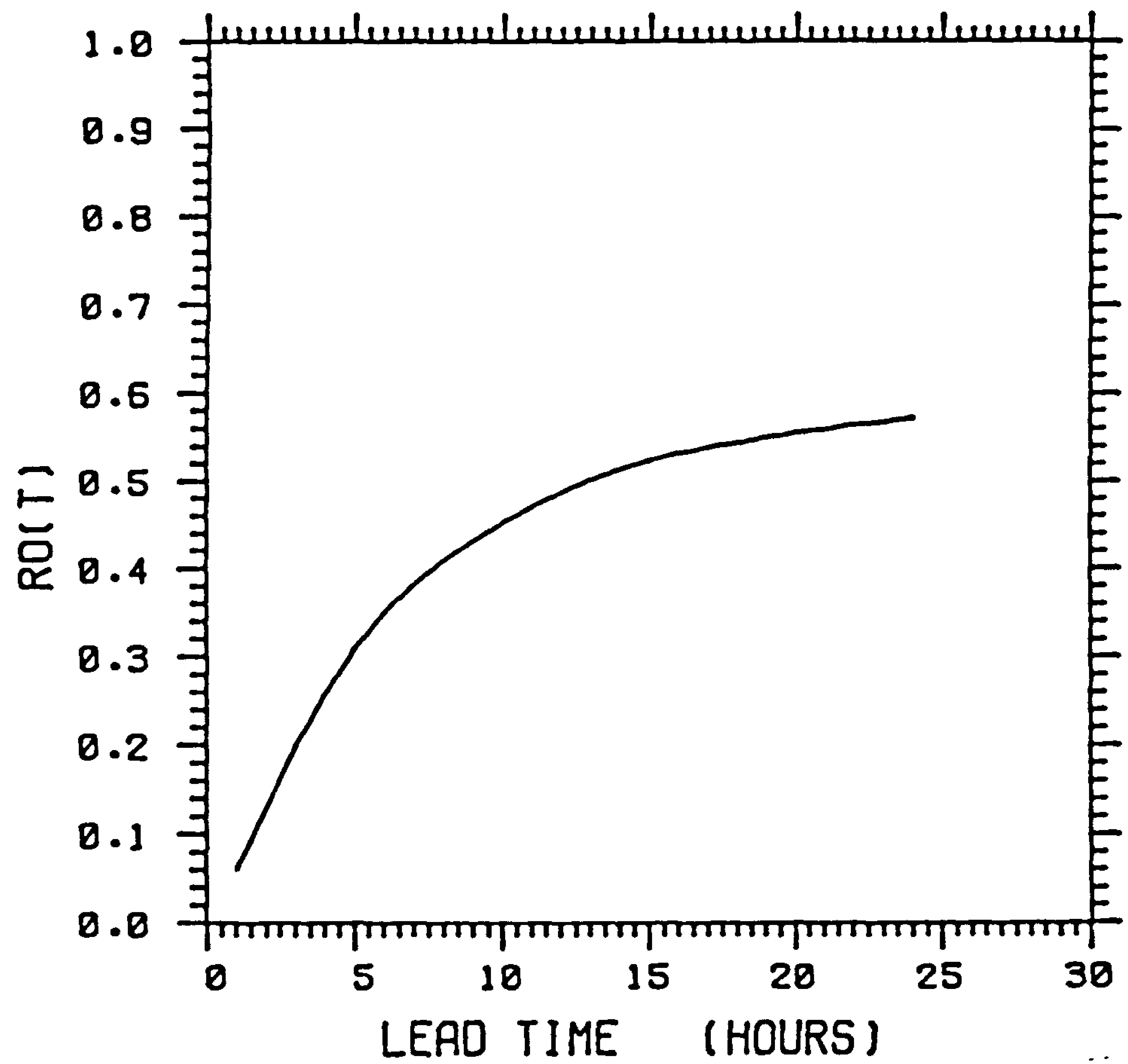
Common site St. Mawgan		Separation (km)
1.	Scilly	107
2.	Valley	314
3.	Dungeness	417
4.	Wattisham	451
5.	Gorleston	517
6.	Lizard	55
7.	Mount Batten	66
8.	Porth Talbot	151
9.	Milford Haven	134
10.	Aberporth	190
11.	Boscombe Down	238
12.	Coningsby	441
13.	Wick	893

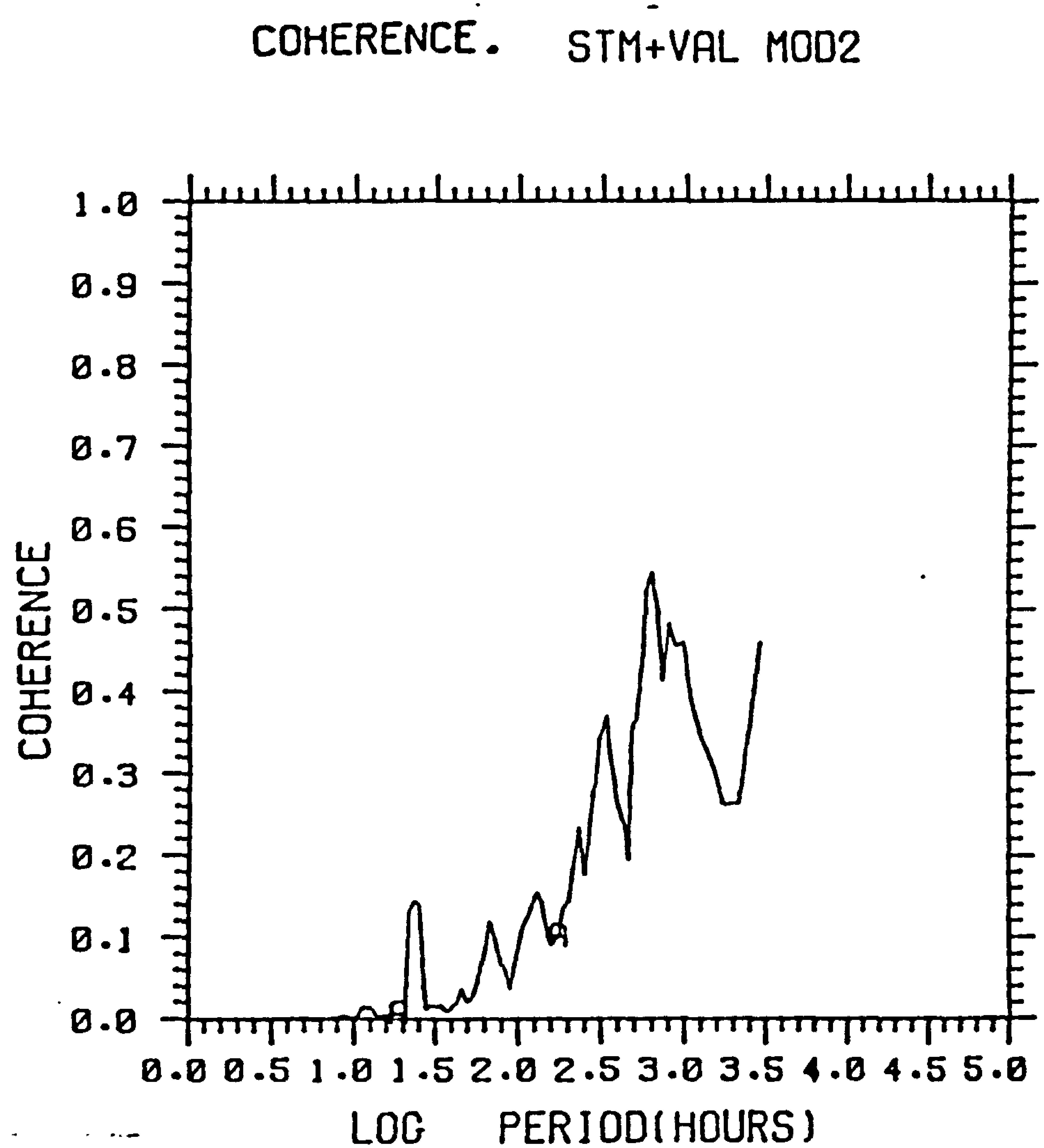
Common site Gorleston		Separation (km)
21.	Wattisham	69
22.	Cardington	152
23.	Gatwick	203
24.	Dungeness	190
25.	Coningsby	138
26.	Silpho Moor	245
27.	Durham	338
28.	Leuchars	510
29.	Fraserburgh	610
30.	Prestwick	517
31.	Wick	713
32.	Stornoway	800
33.	West Freugh	503

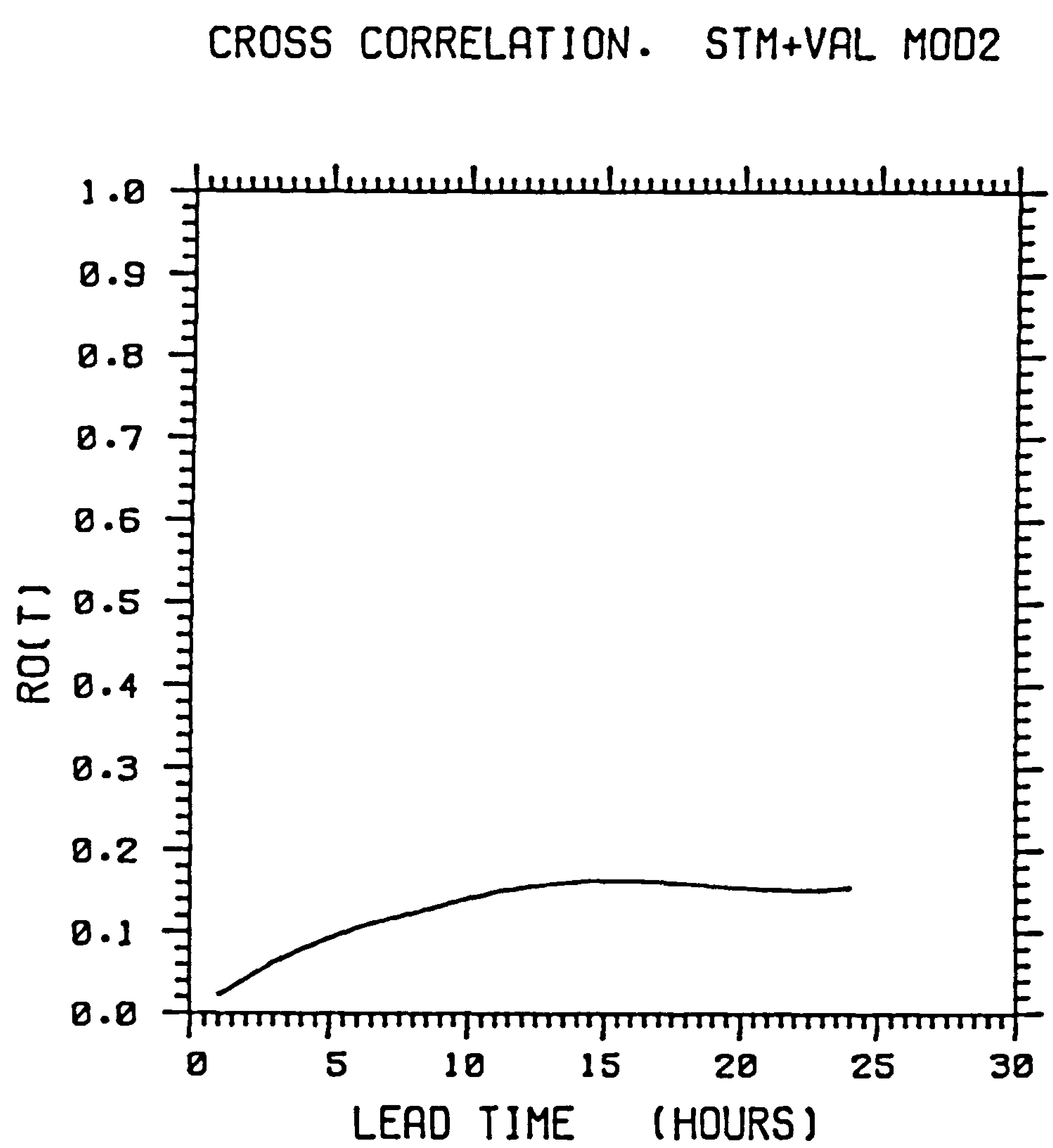
COHERENCE. SITES STM+SCI MOD2



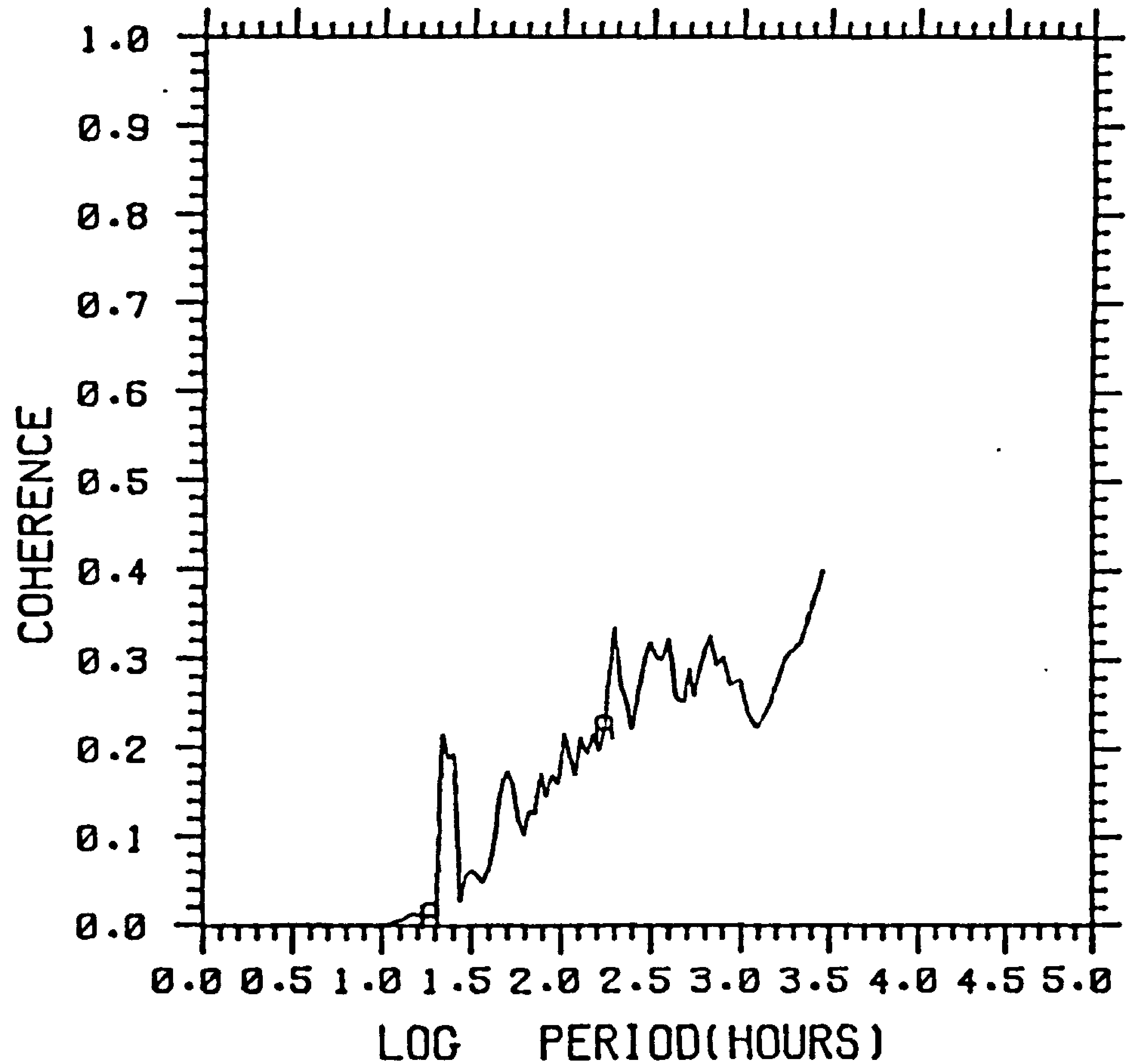
CROSS CORRELATION. STM+SCI MOD2



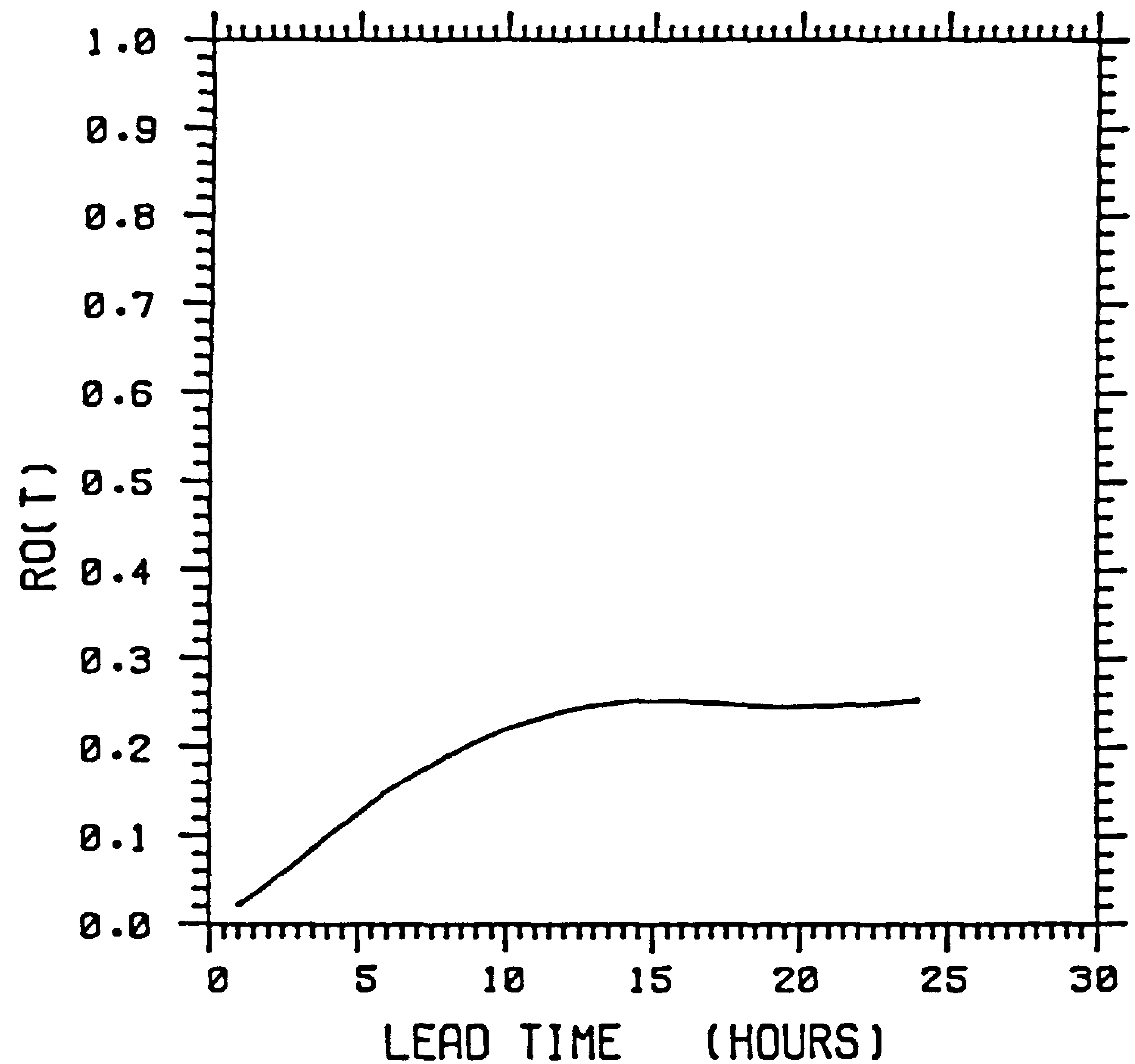

COHERENCE. STM+VAL MOD2
COHERENCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
LOG PERIOD(HOURS)


CROSS CORRELATION. STM+VAL MOD2
RO(T)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 5 10 15 20 25 30
LEAD TIME (HOURS)

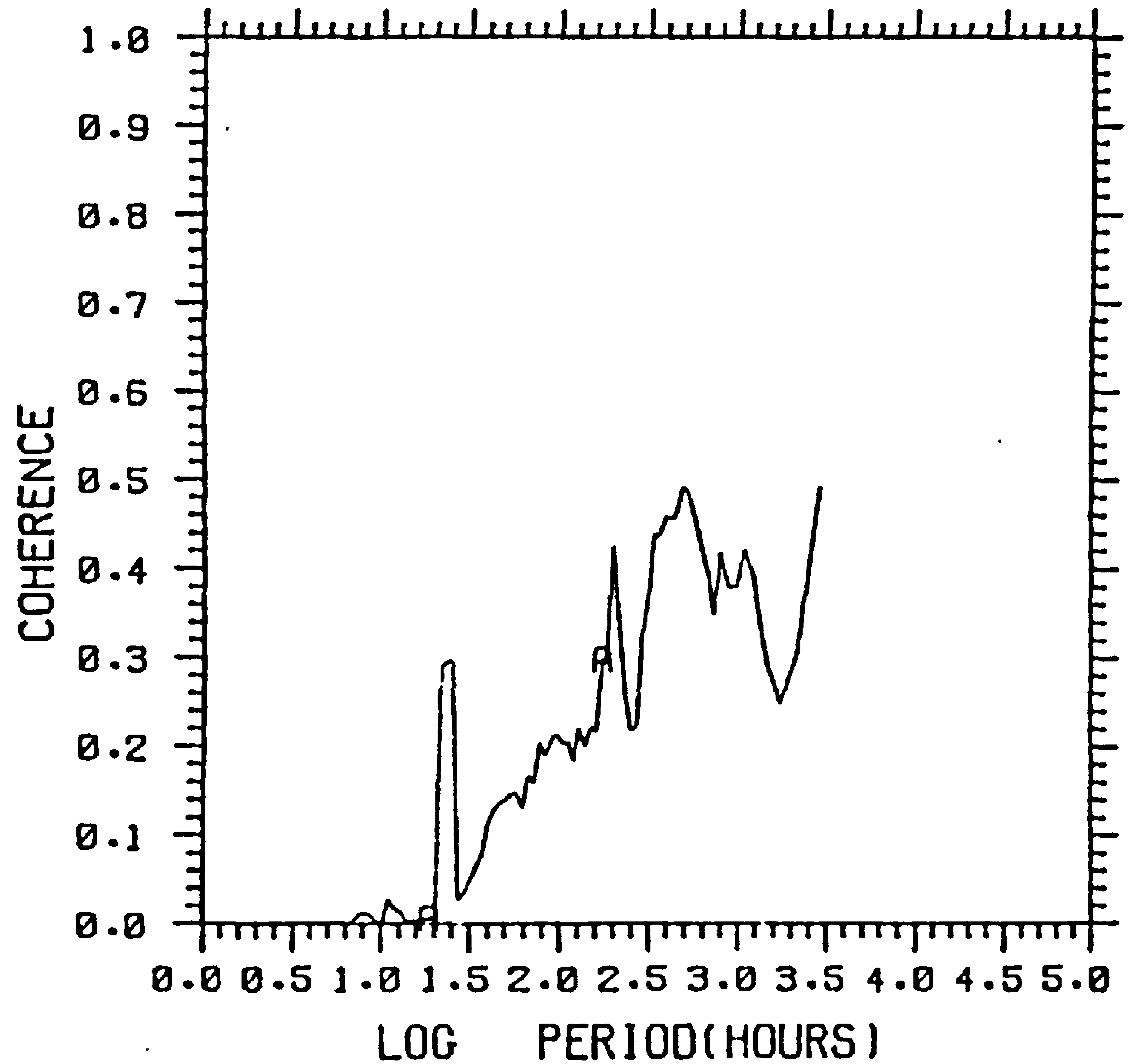
COHERENCE. STM+DNG MOD2



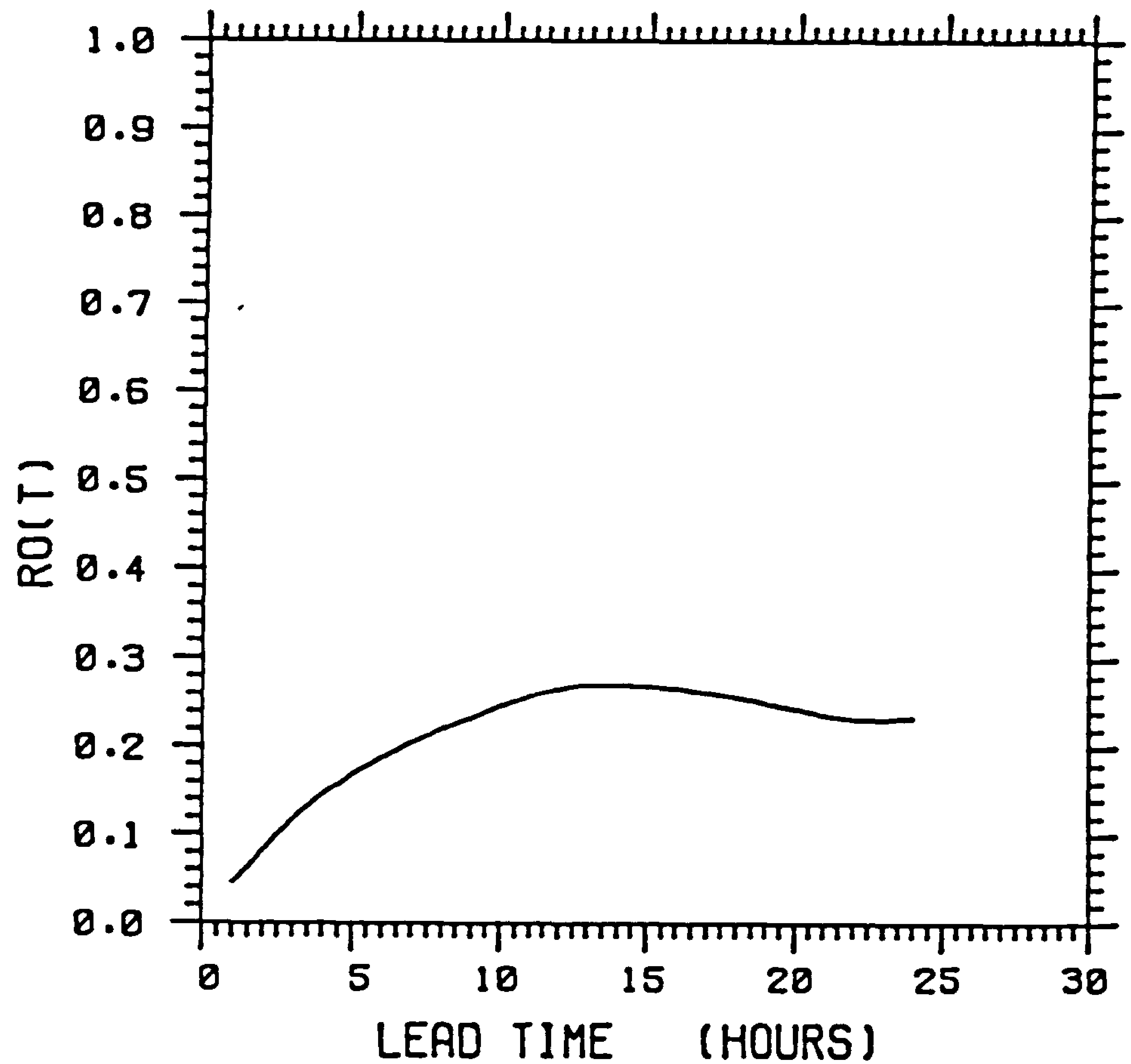
CROSS CORRELATION. STM+DNG MOD2

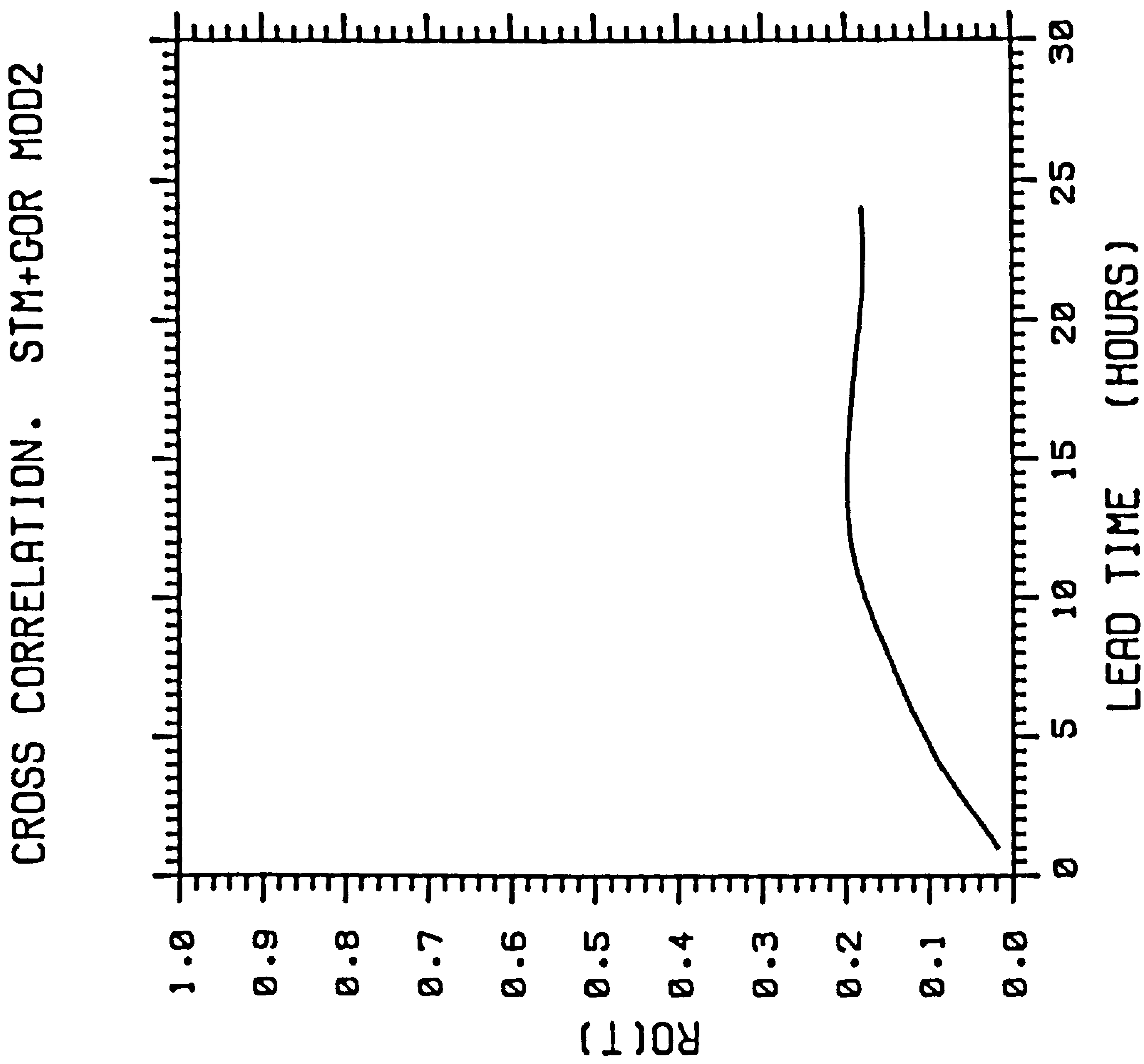


COHERENCE. STM+WAT MOD2

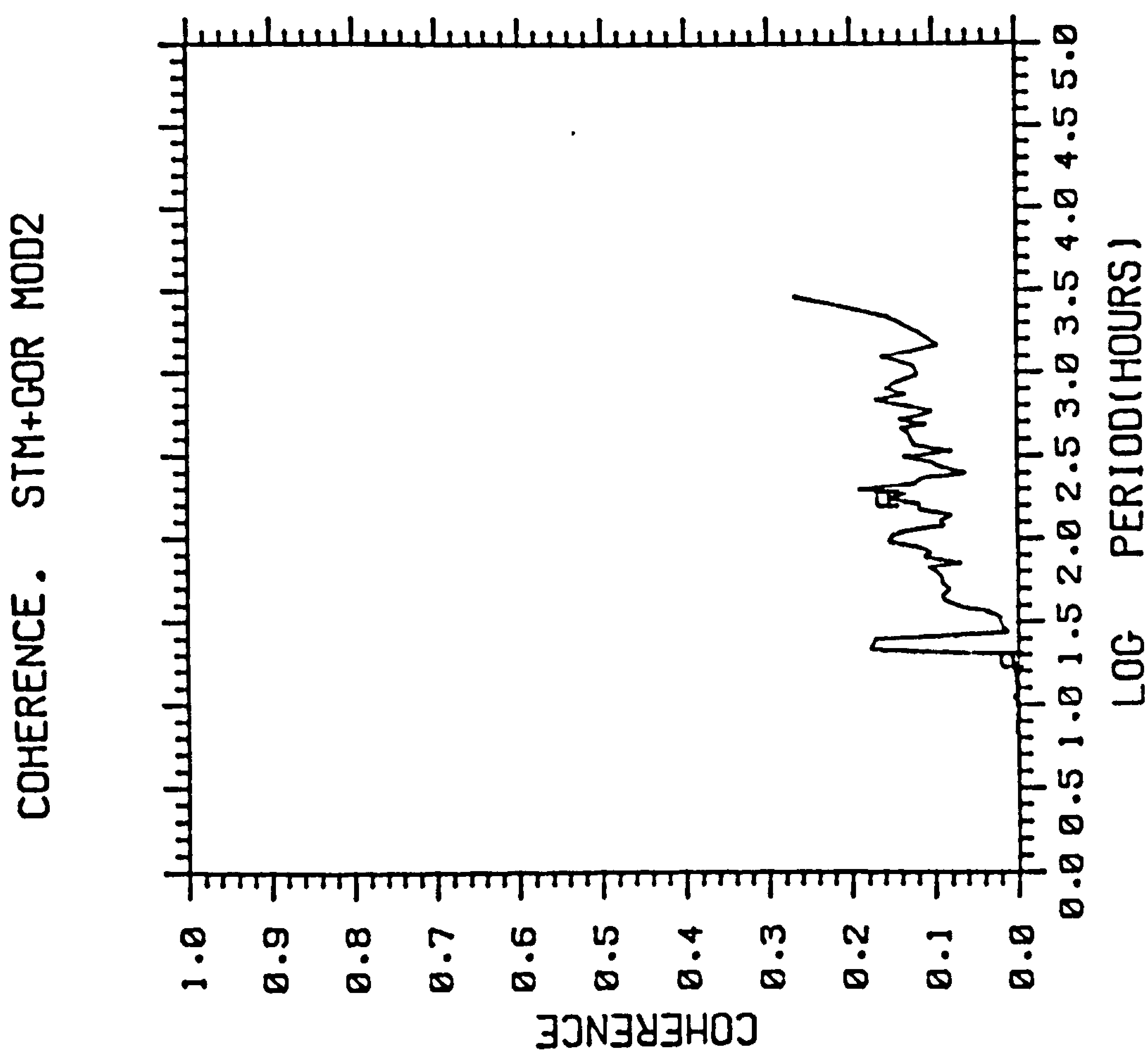


CROSS CORRELATION. STM+WAT MOD2

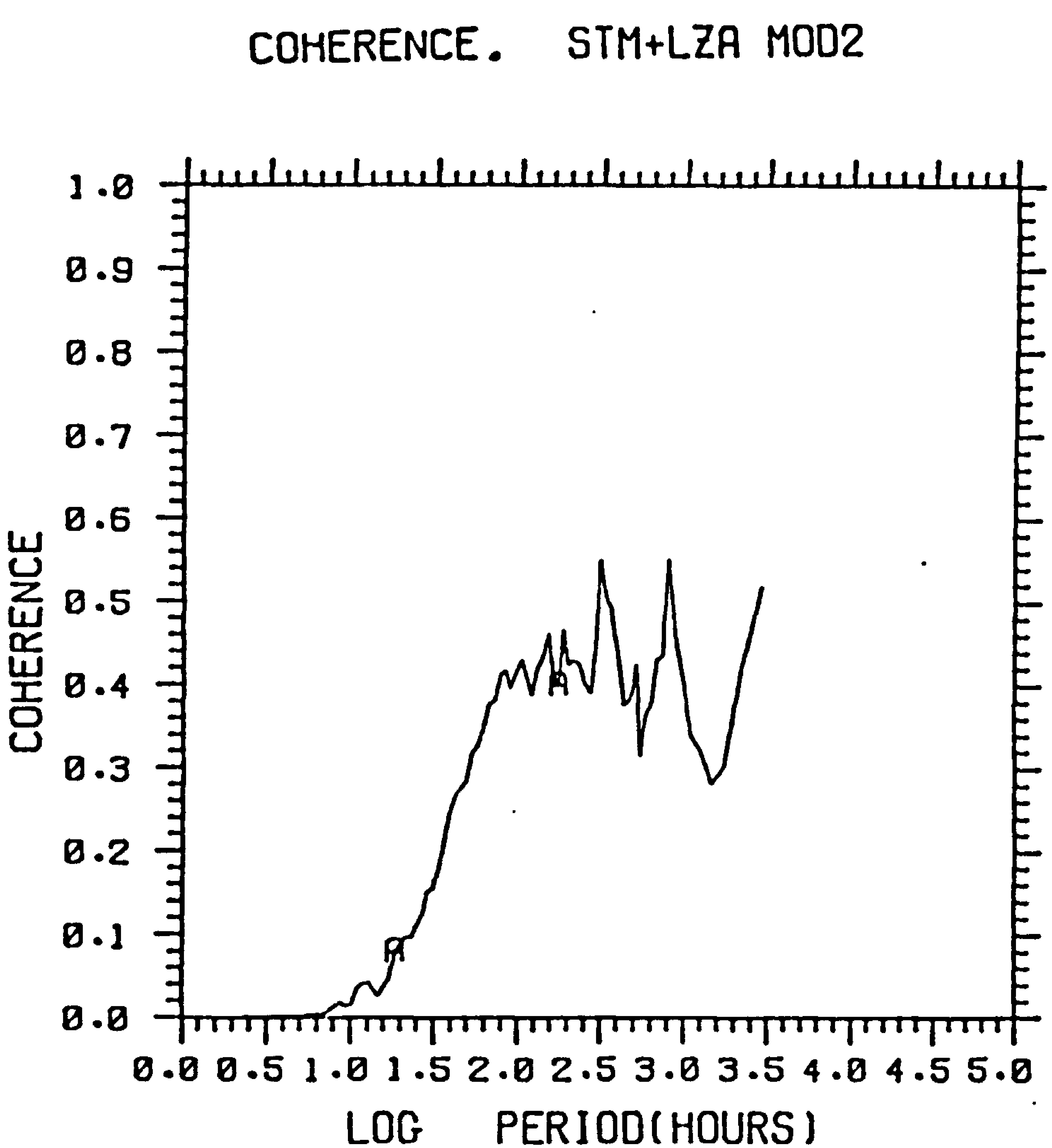




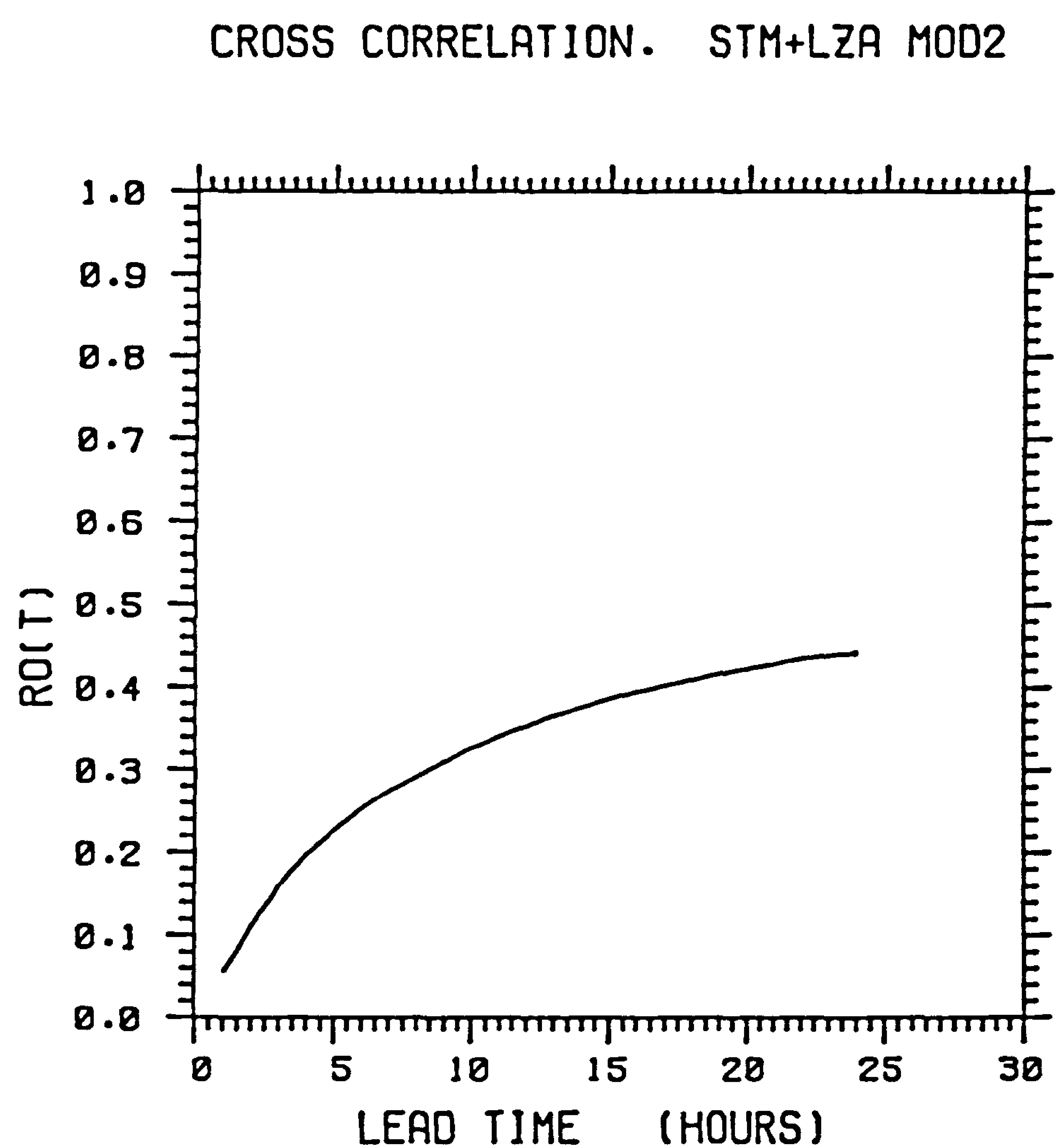
CROSS CORRELATION. STM+GOR MOD2
RO(T)
LEAD TIME (HOURS)



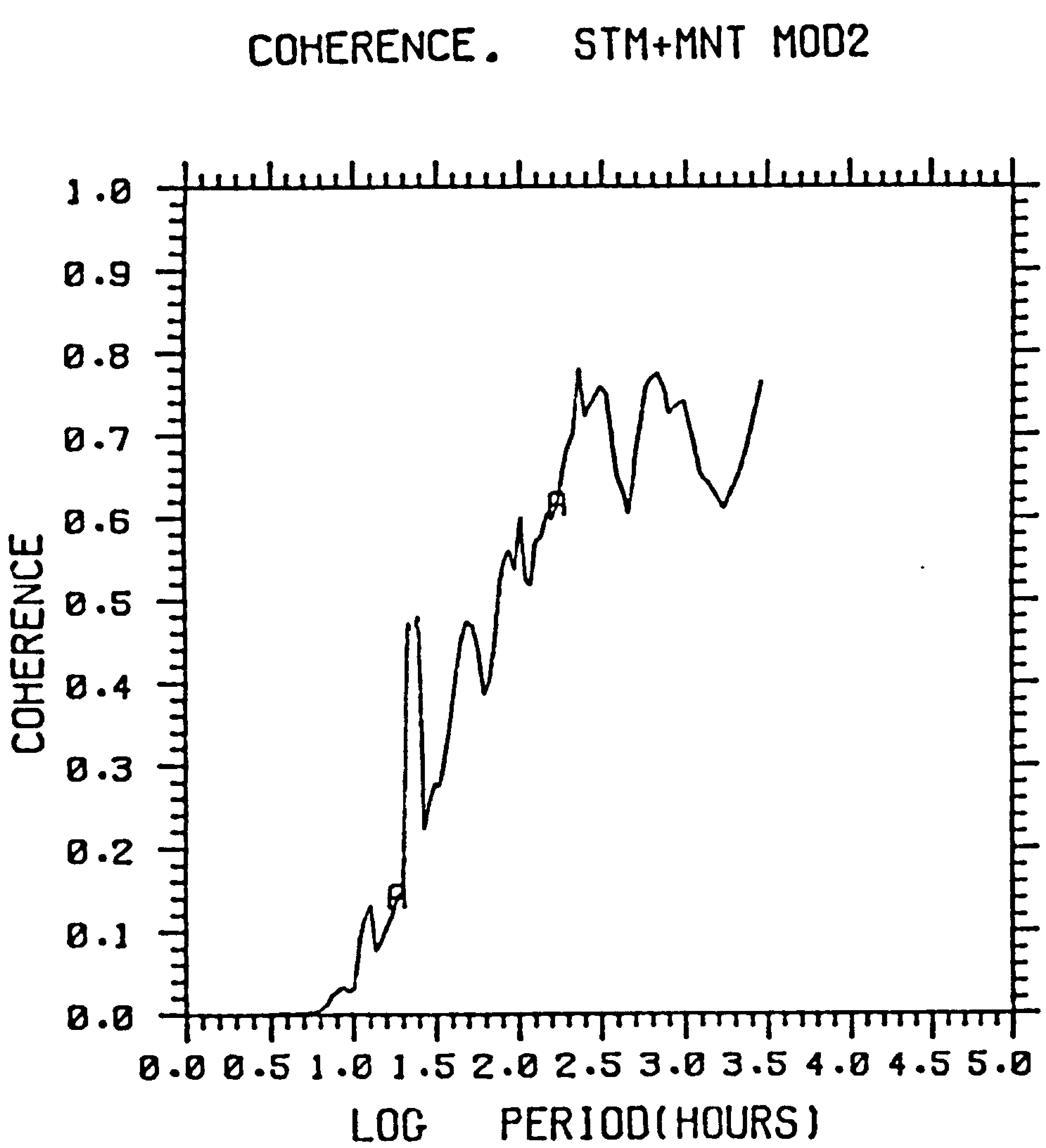
COHERENCE. STM+GOR MOD2
COHERENCE
LOG PERIOD(HOURS)

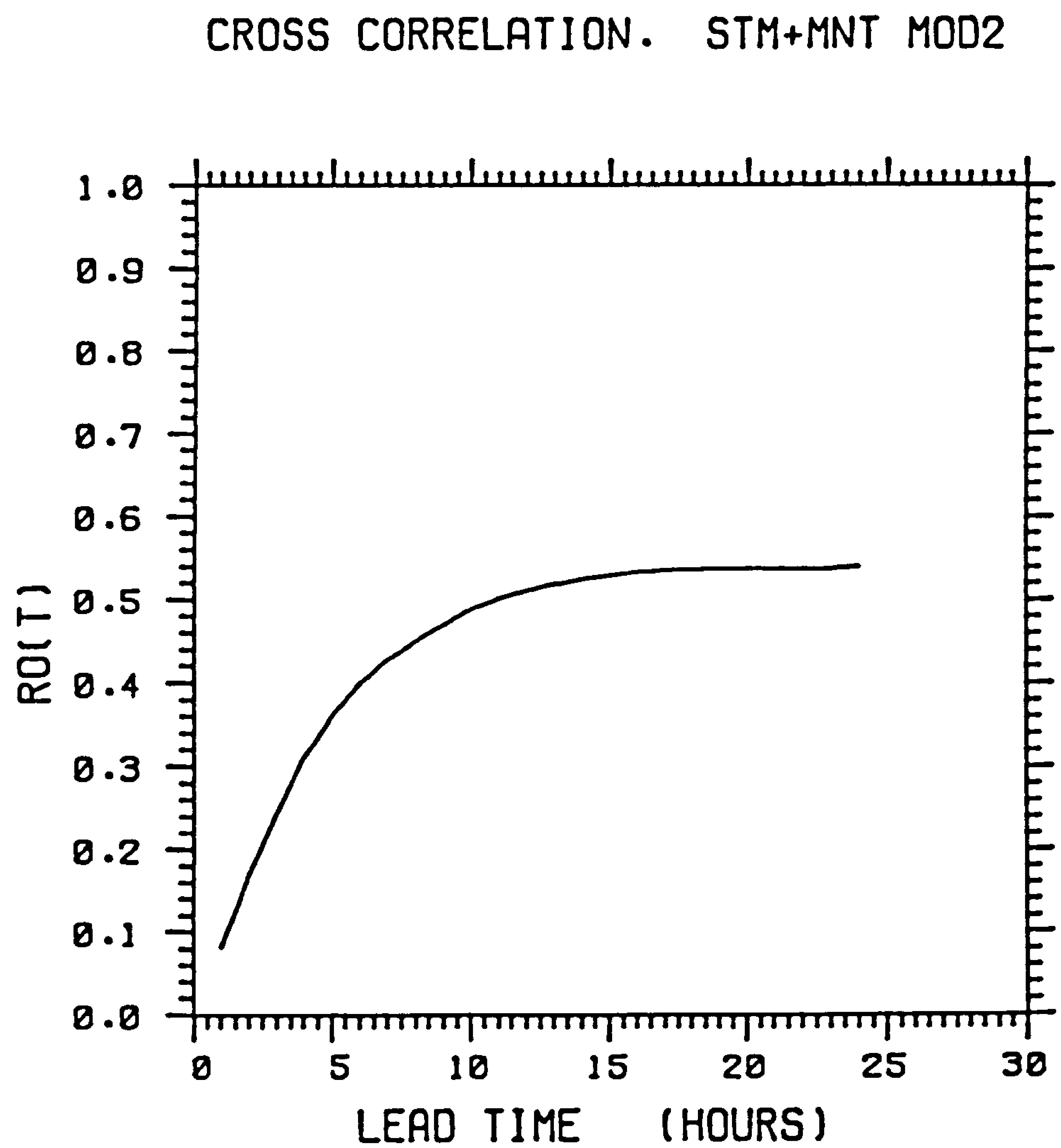


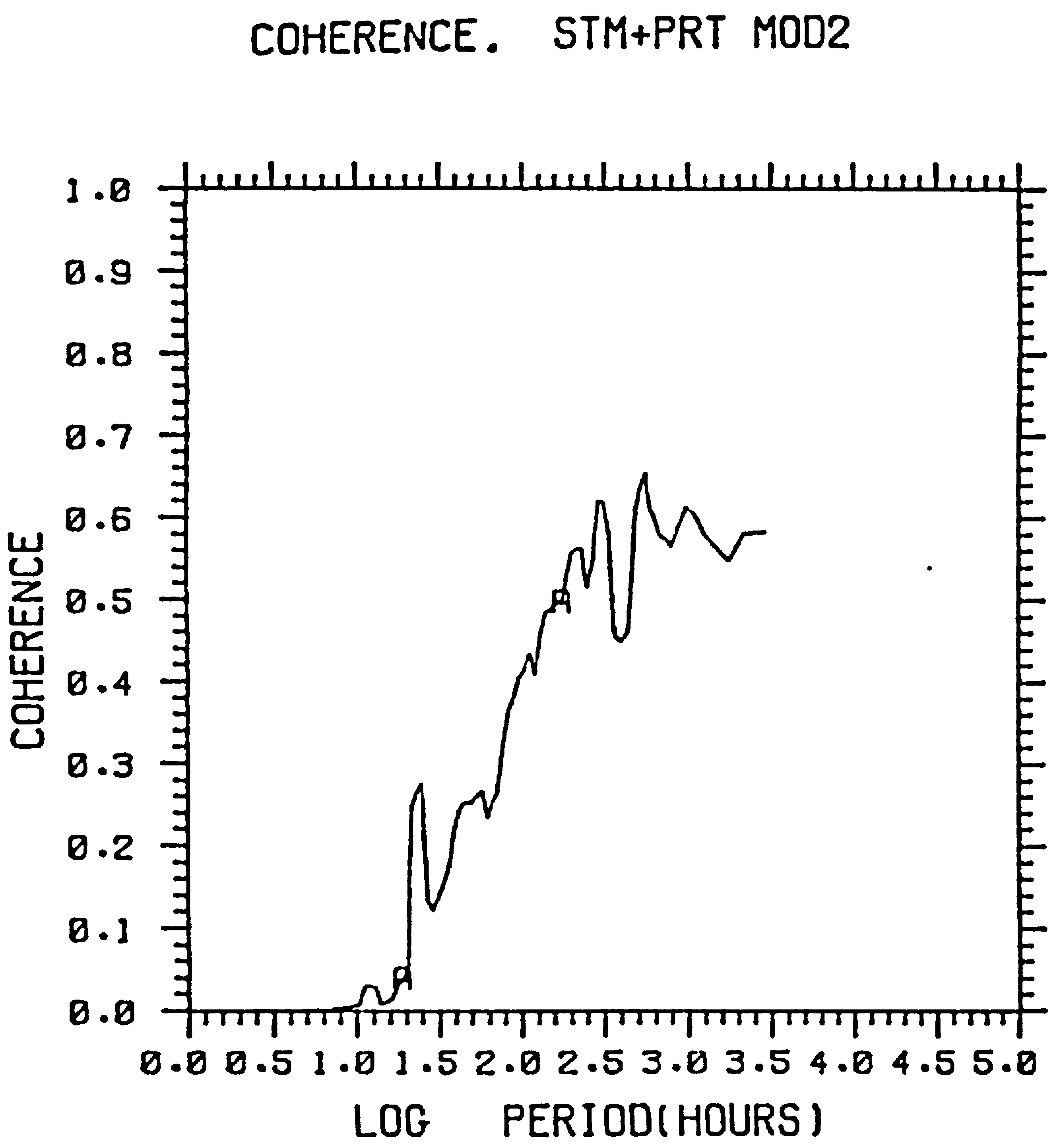
COHERENCE. STM+LZA MOD2
COHERENCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
LOG PERIOD(HOURS)

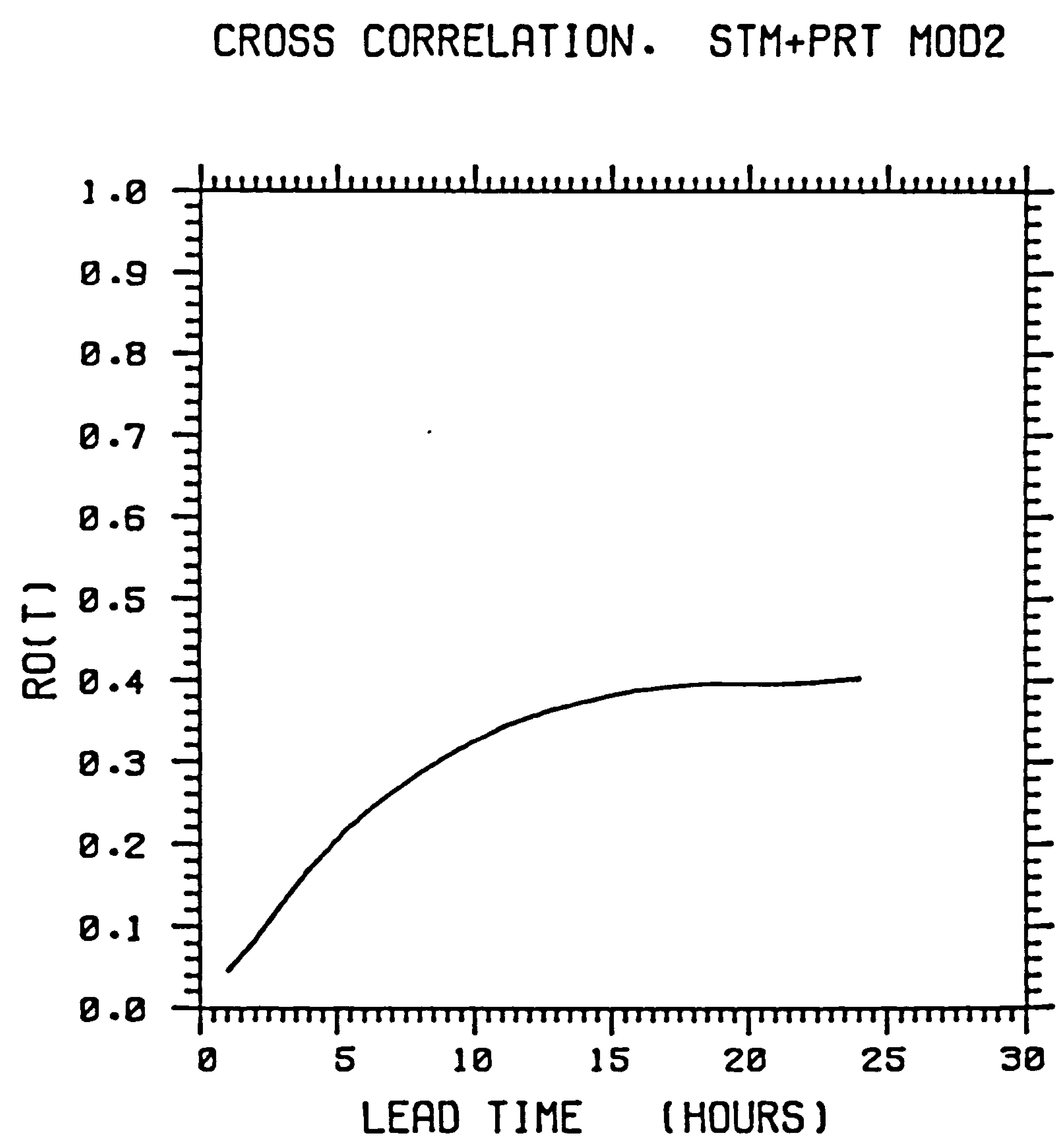


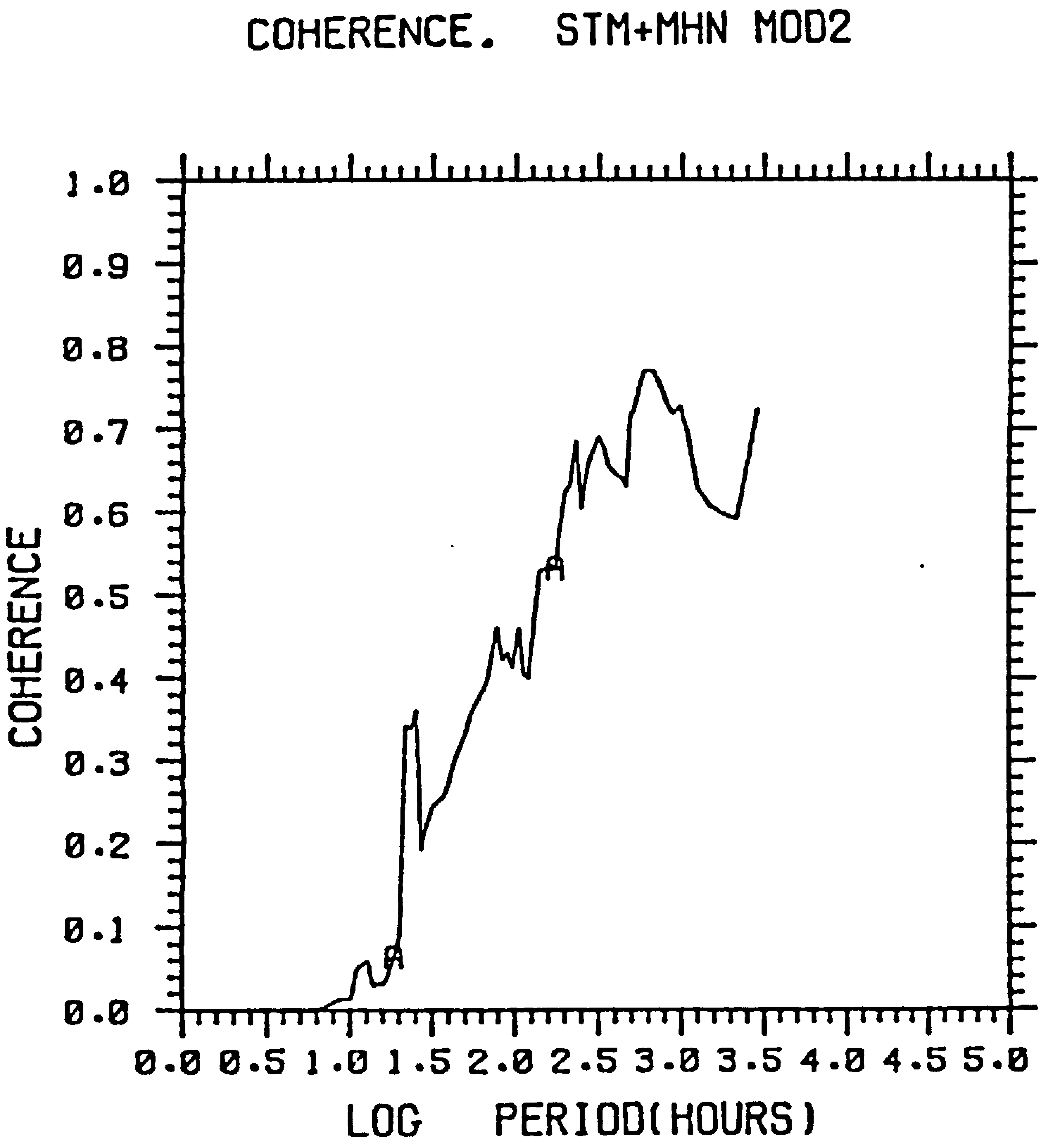
CROSS CORRELATION. STM+LZA MOD2
RO(T)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 5 10 15 20 25 30
LEAD TIME (HOURS)

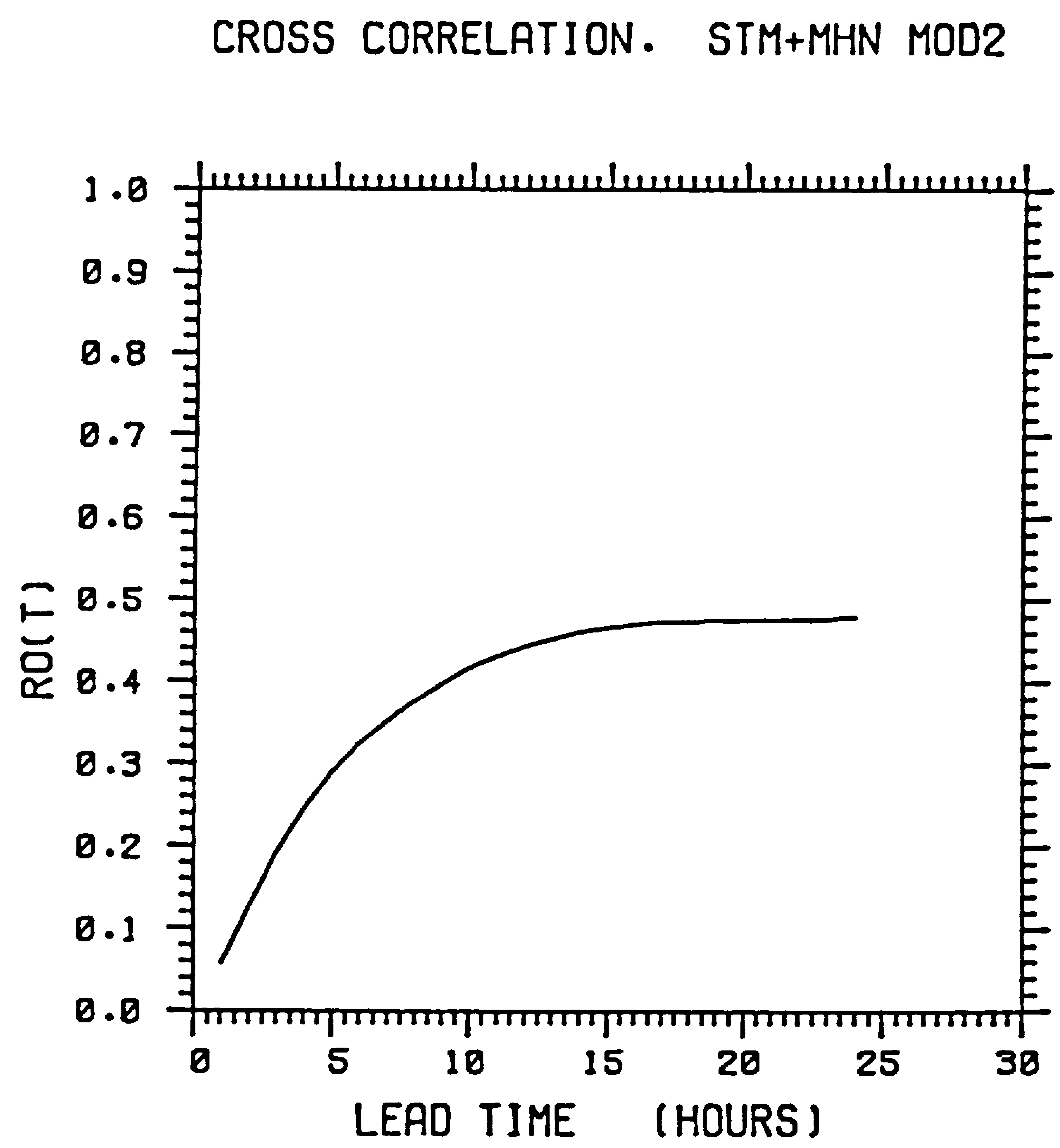

COHERENCE. STM+MNT MOD2
COHERENCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
LOG PERIOD(HOURS)


CROSS CORRELATION. STM+MNT MOD2
RO(T)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 5 10 15 20 25 30
LEAD TIME (HOURS)

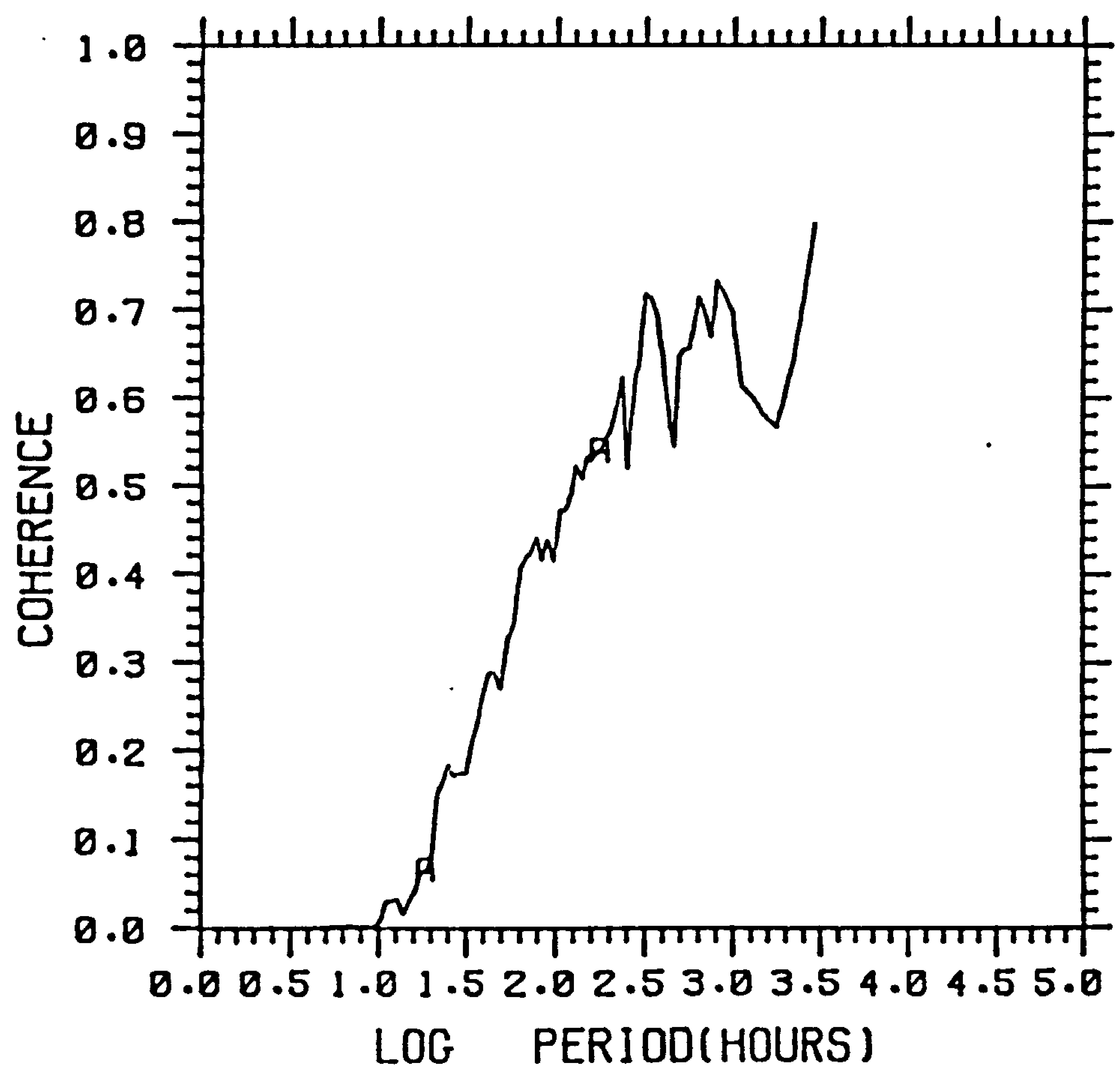

COHERENCE. STM+PRT MOD2
COHERENCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
LOG PERIOD(HOURS)


CROSS CORRELATION. STM+PRT MOD2
RO(T)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 5 10 15 20 25 30
LEAD TIME (HOURS)

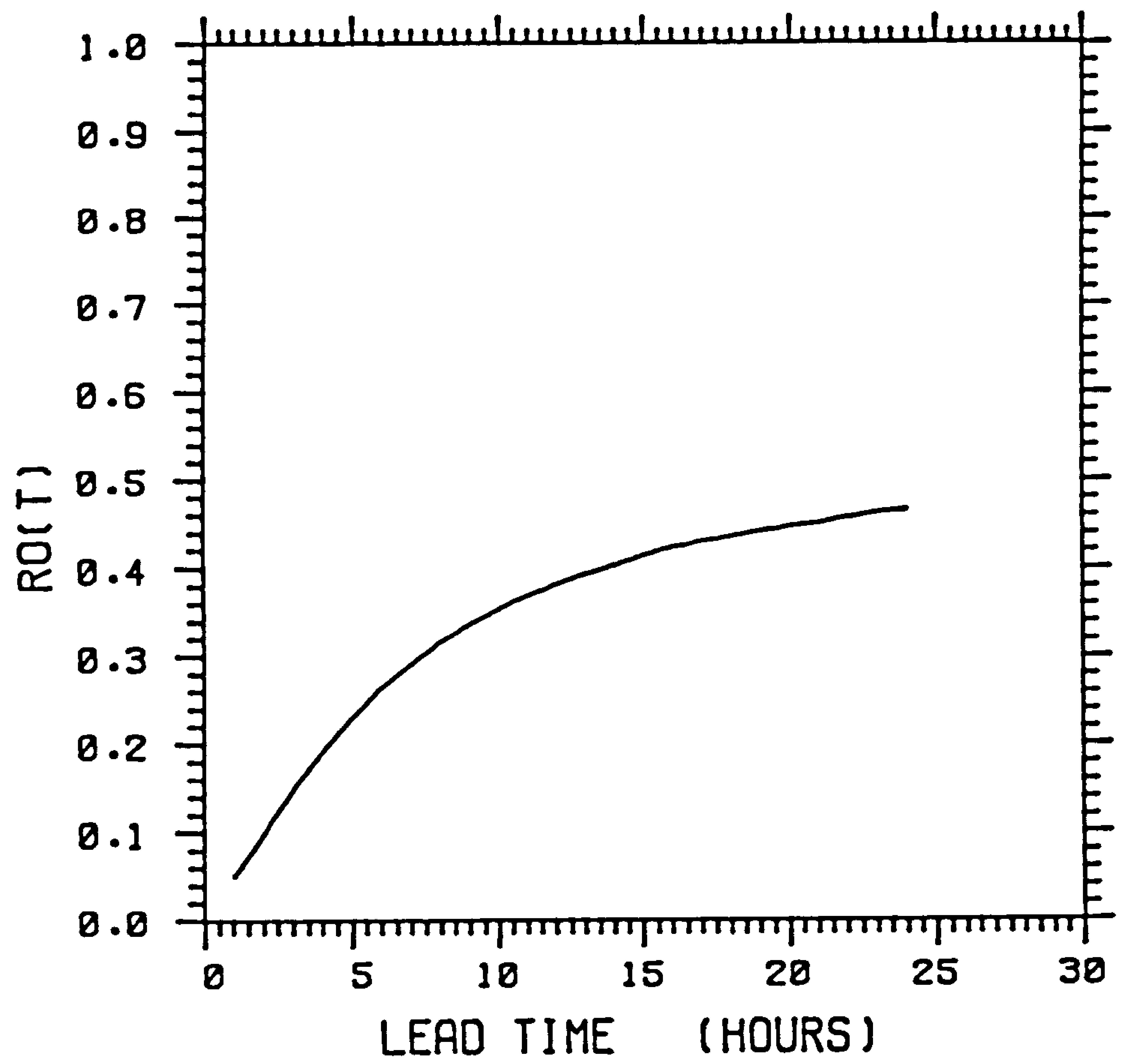

COHERENCE. STM+MHN MOD2
COHERENCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
LOG PERIOD(HOURS)

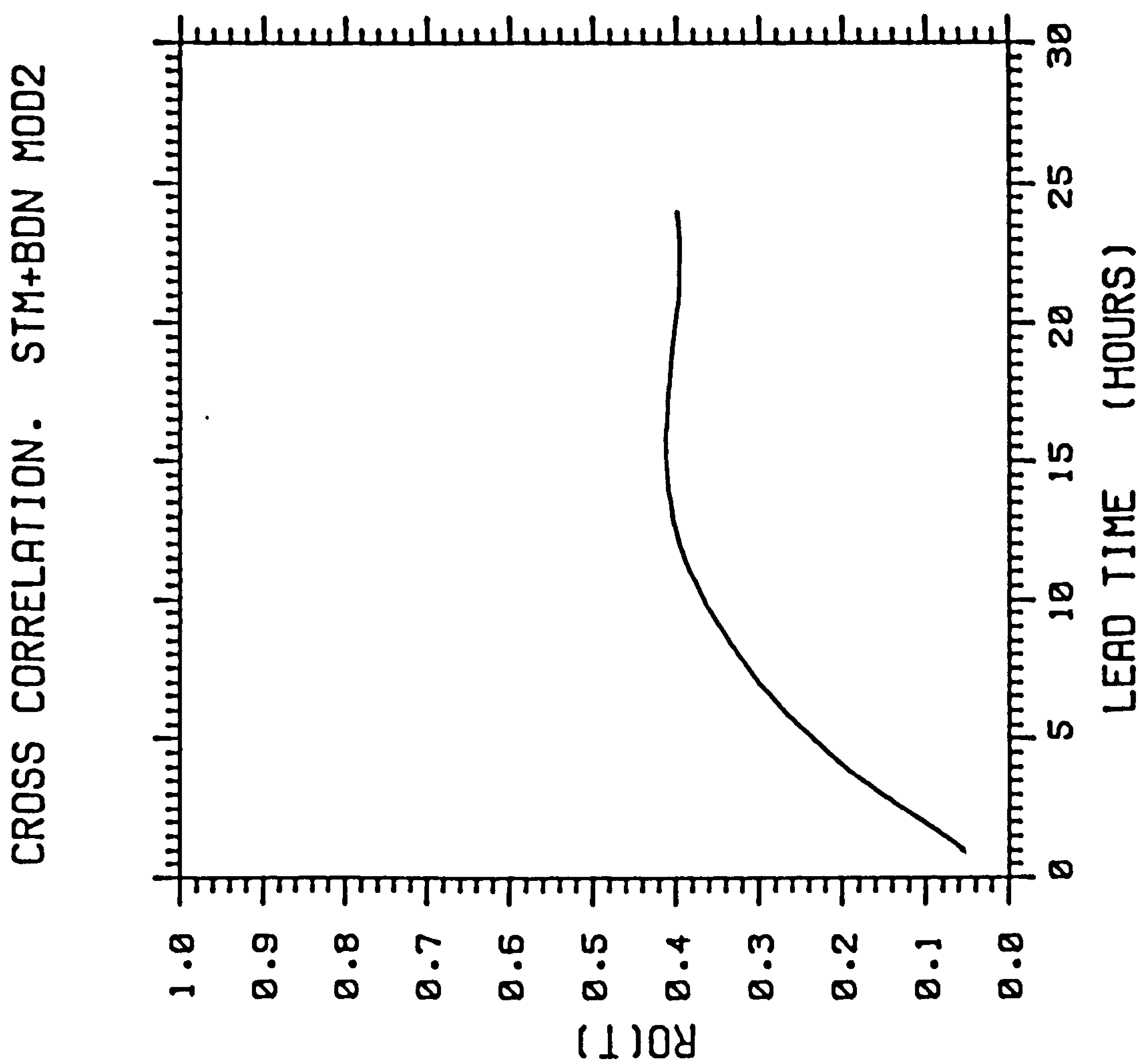

CROSS CORRELATION. STM+MHN MOD2
RO(T)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 5 10 15 20 25 30
LEAD TIME (HOURS)

COHERENCE. STM+ABP MOD2

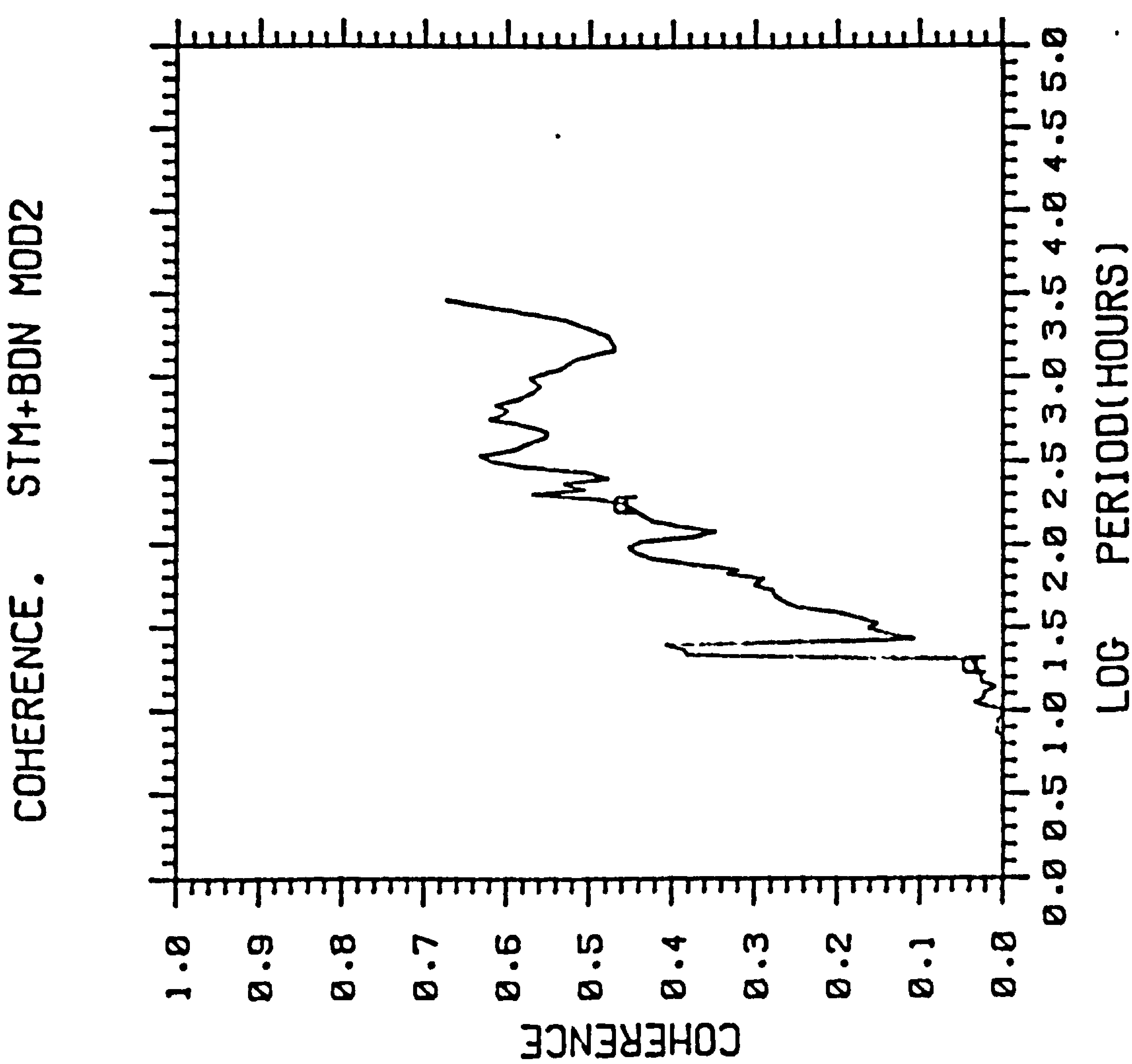


CROSS CORRELATION. STM+ABP MOD2



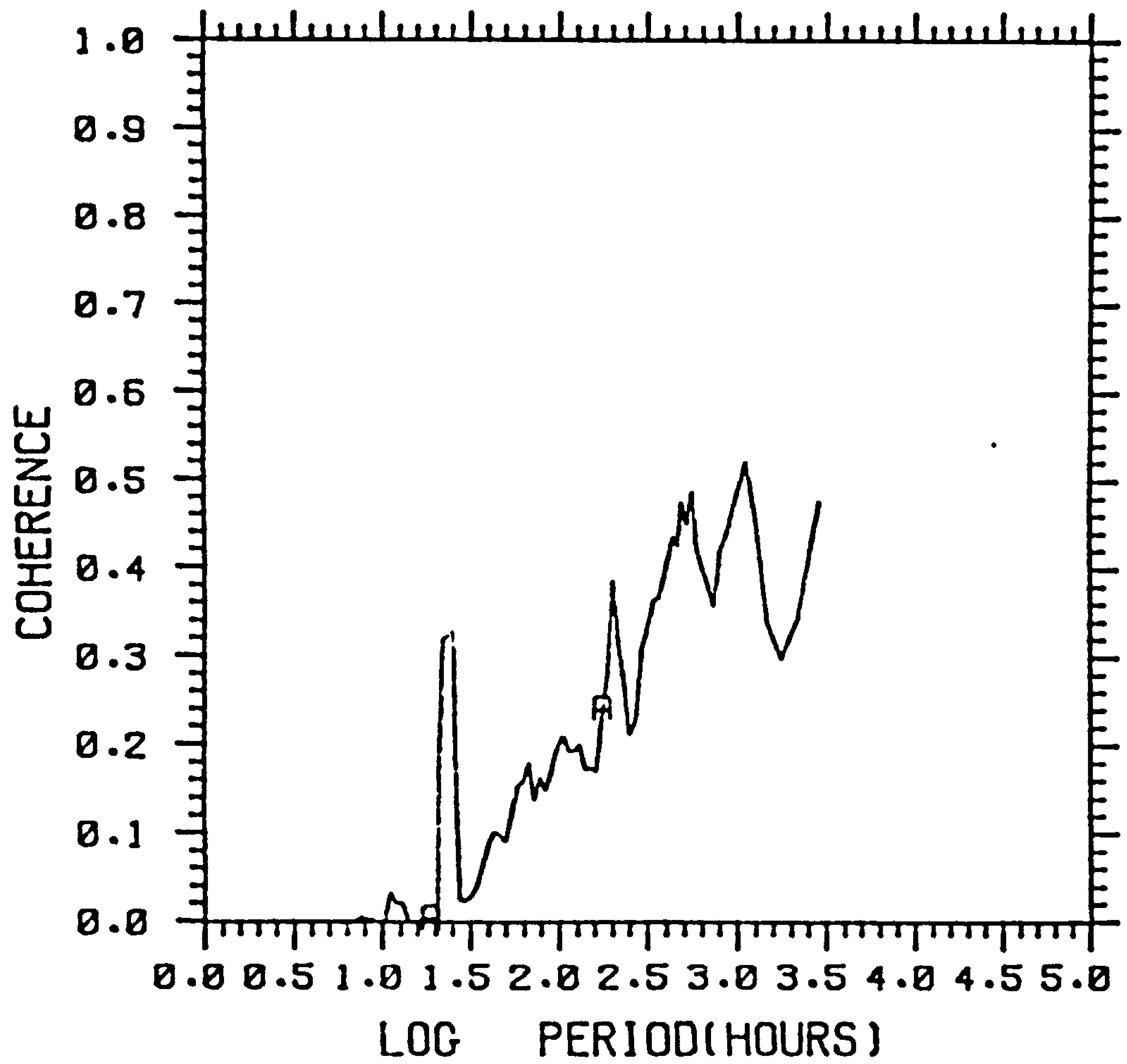


CROSS CORRELATION. STM+BDN MOD2
RO(T)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20
25
30
LEAD TIME (HOURS)

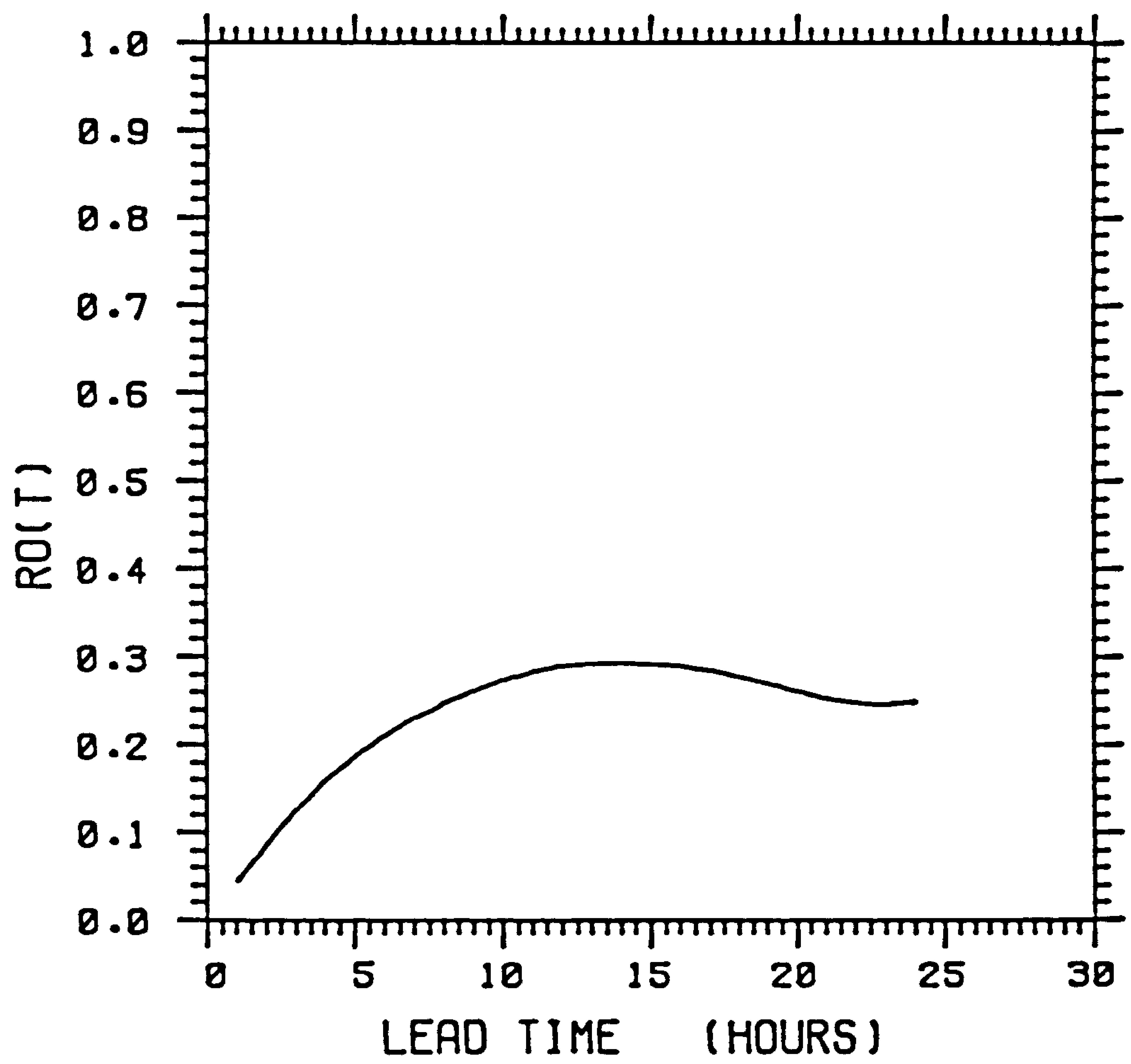


COHERENCE. STM+BDN MOD2
COHERENCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
LOG PERIOD(HOURS)

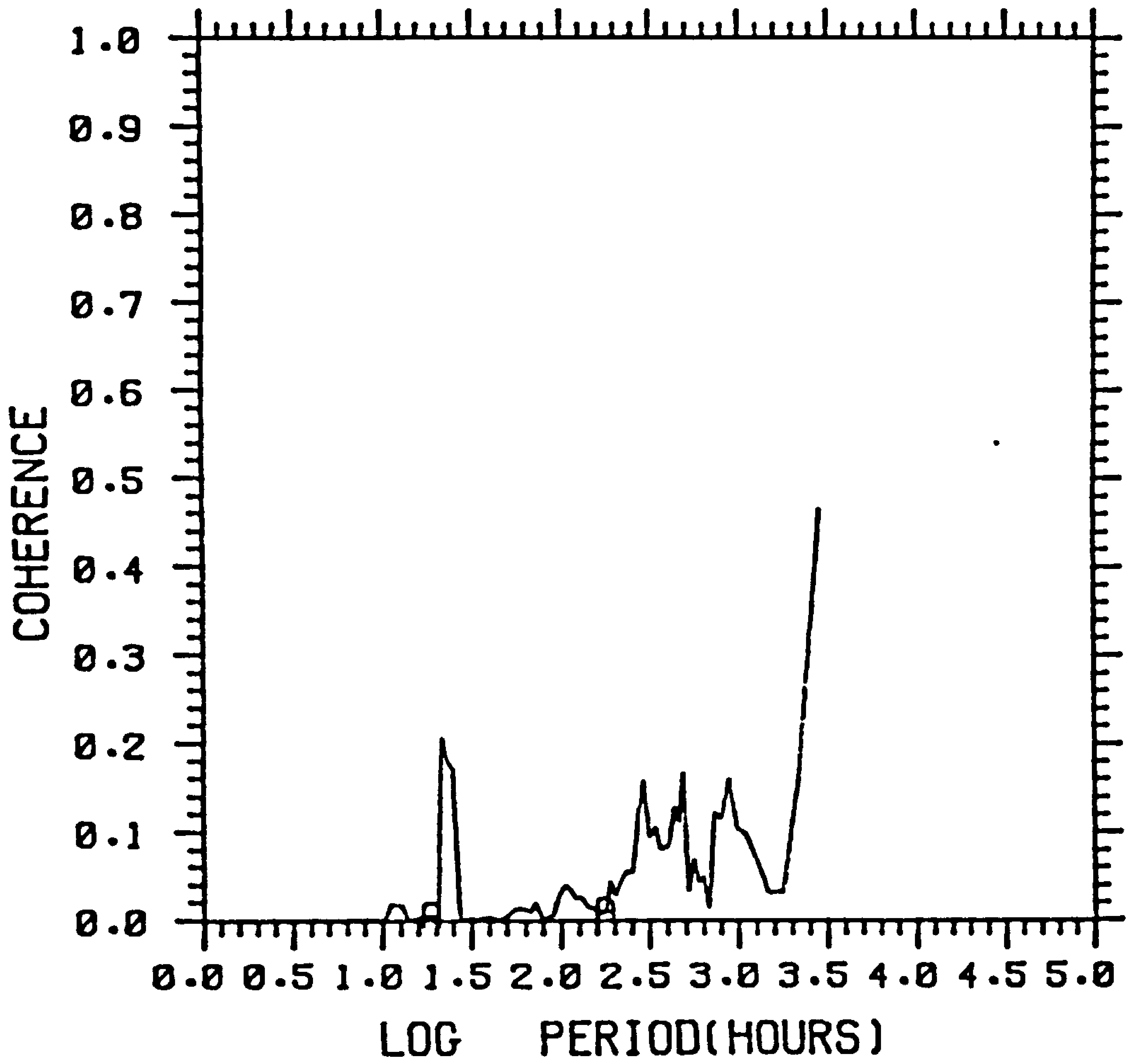
COHERENCE.   STM+CON MOD2



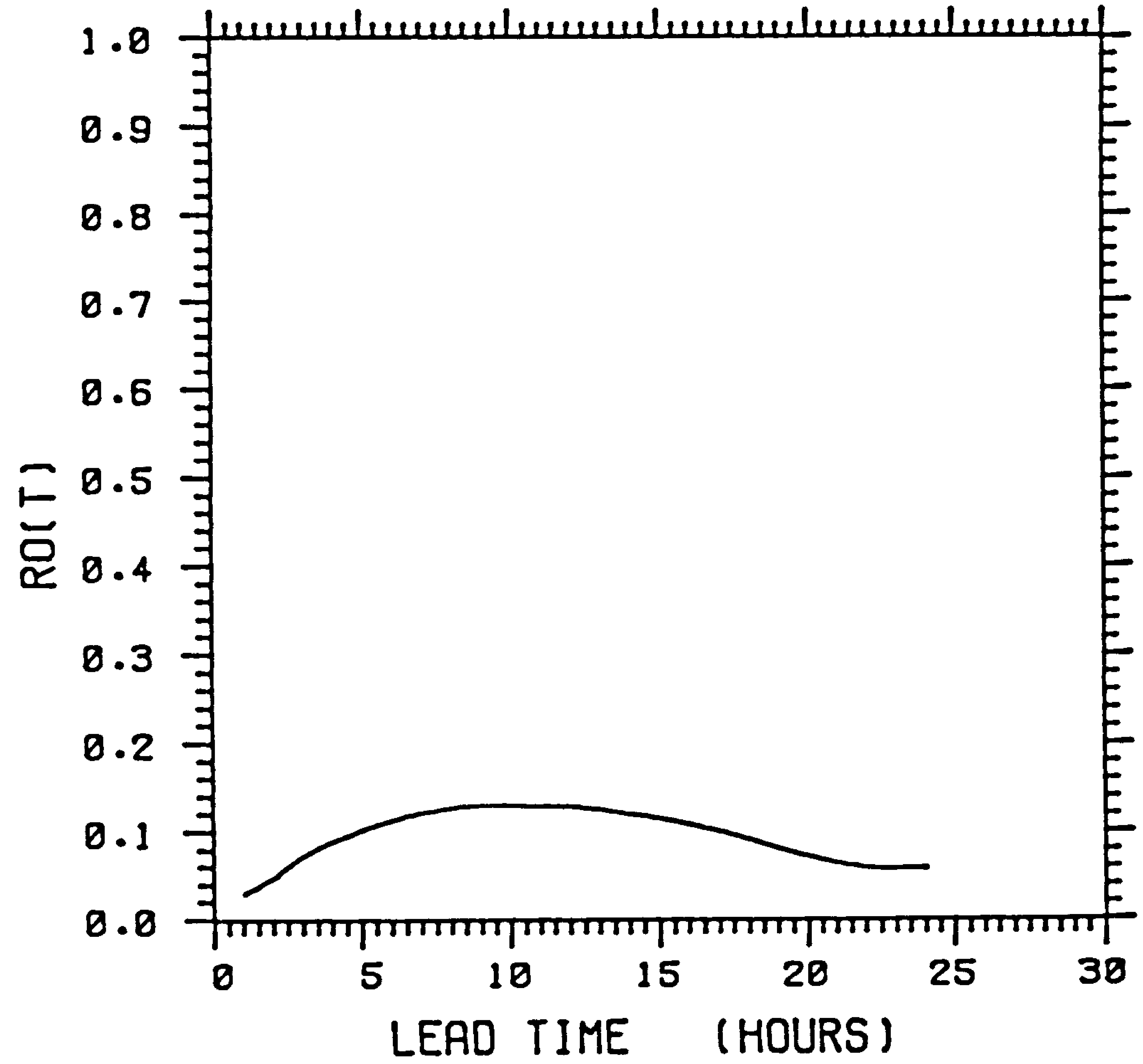
CROSS CORRELATION.   STM+CON MOD2

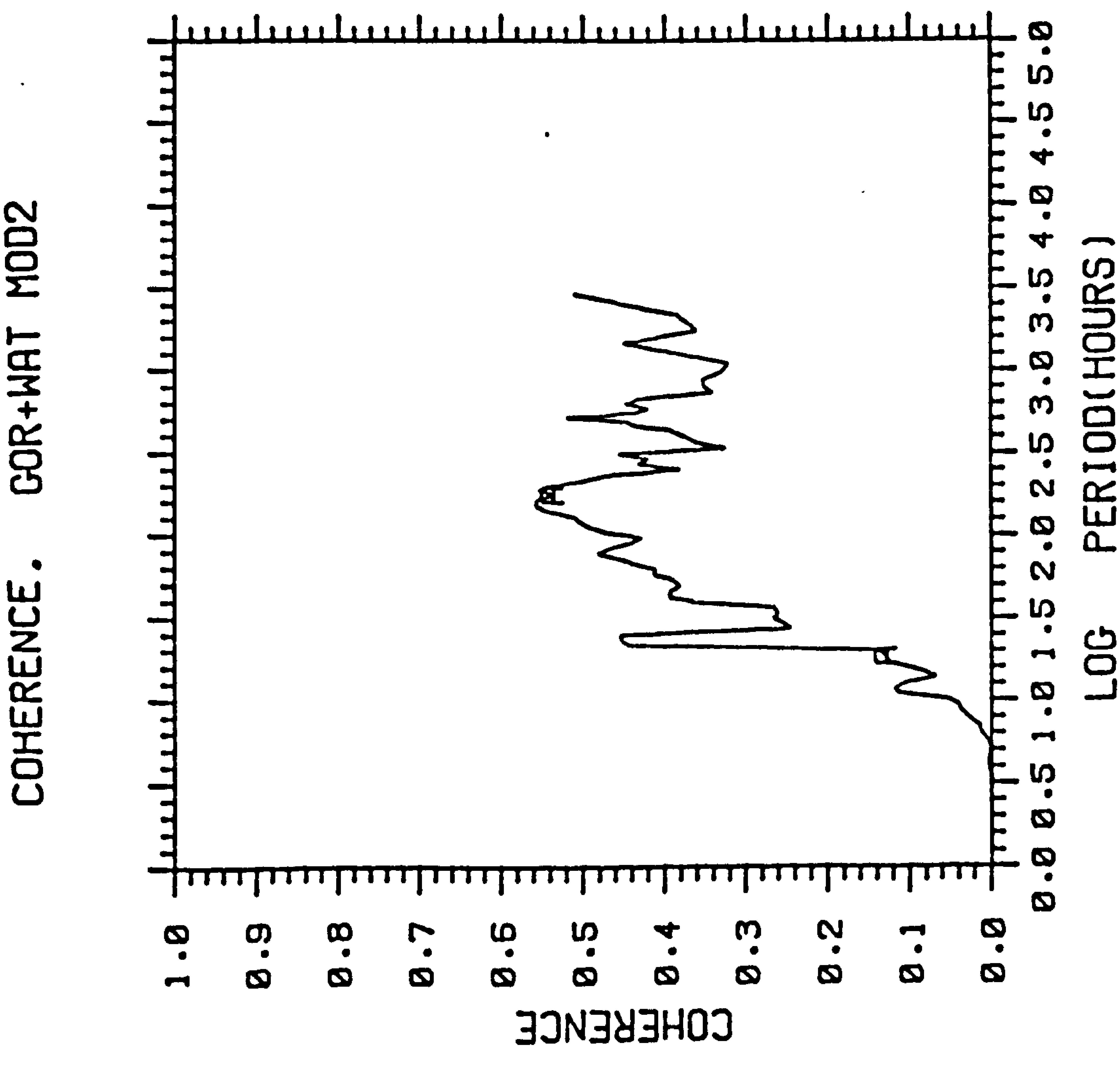


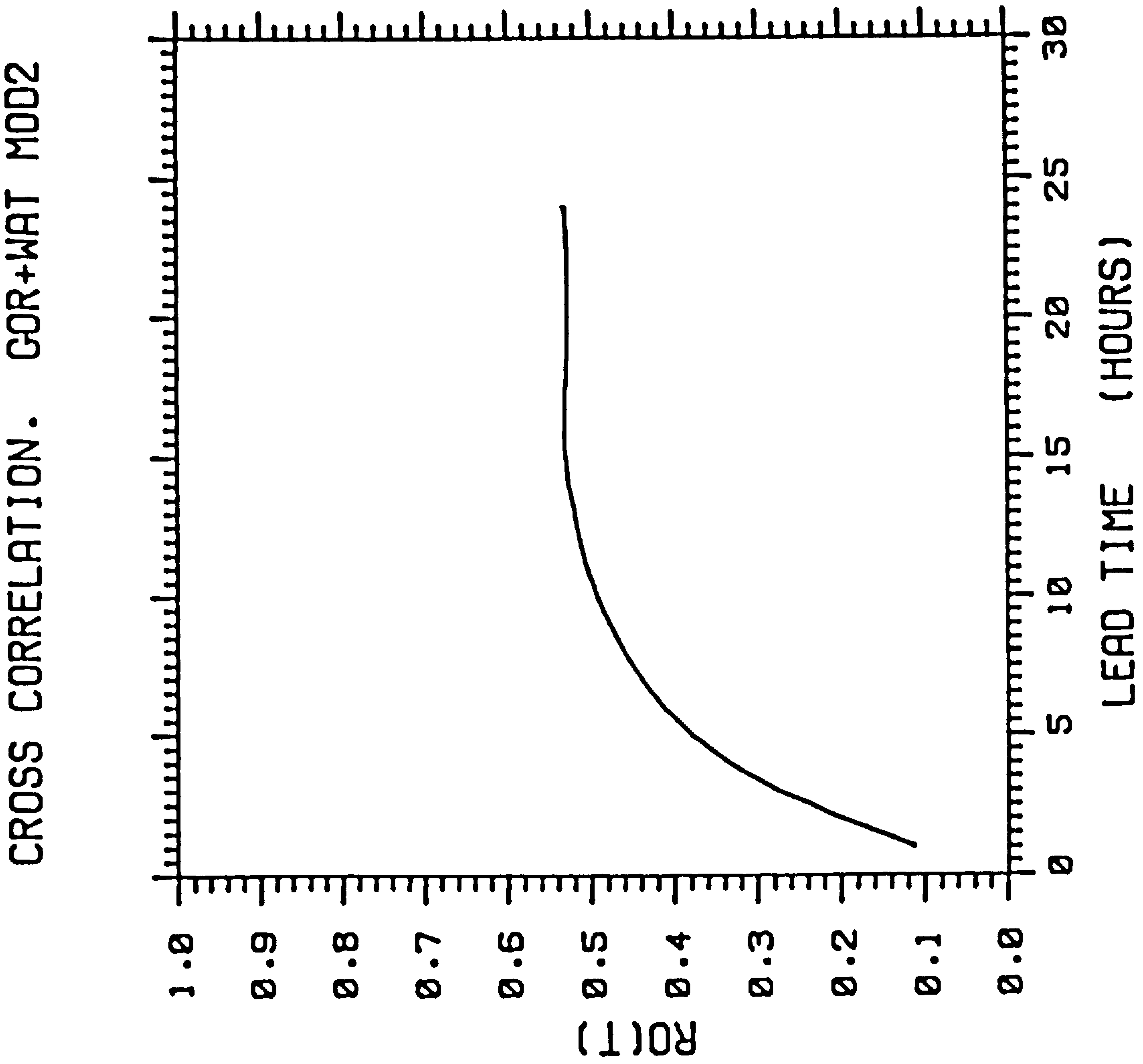
COHERENCE.   STM+WIK MOD2

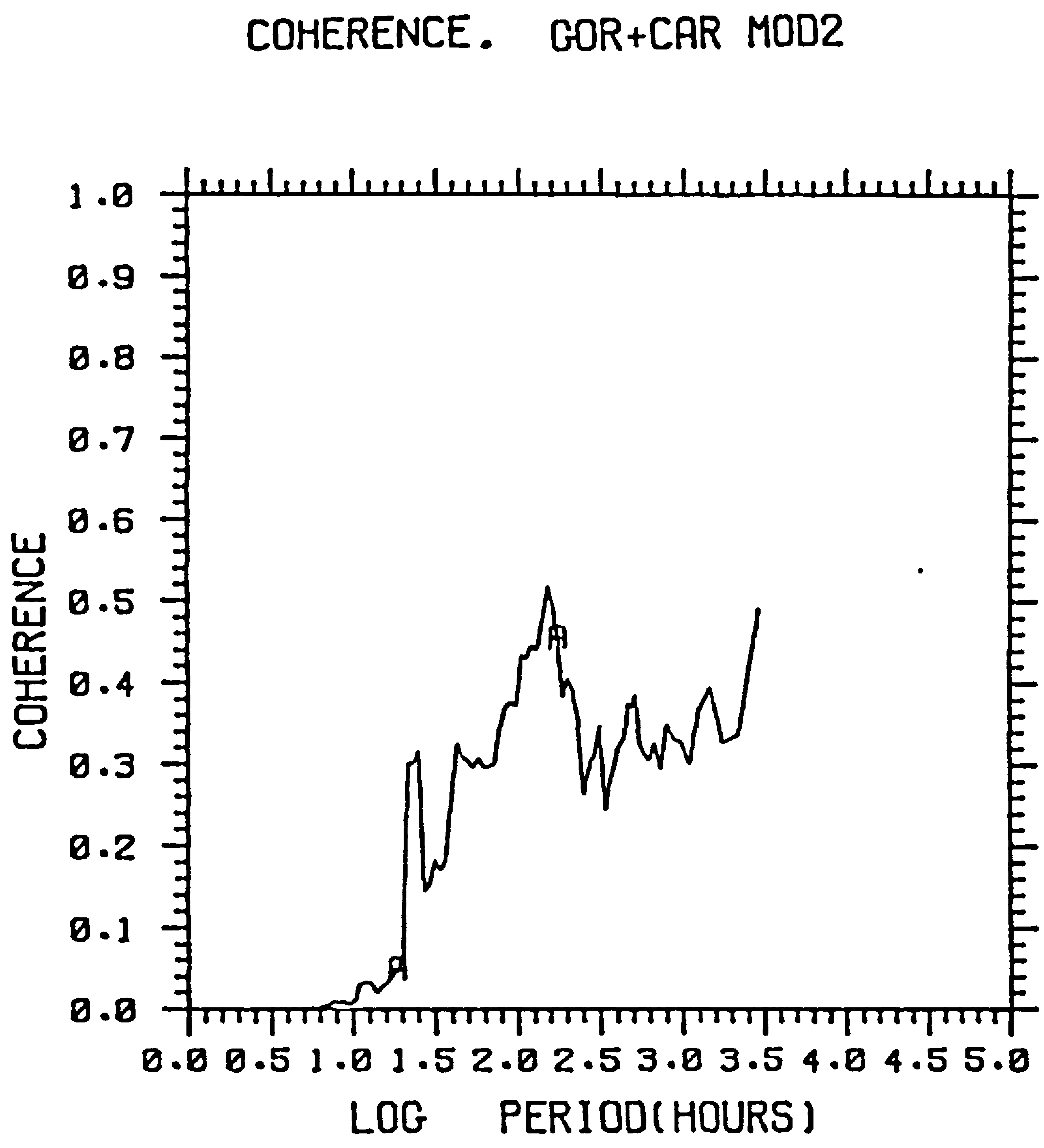


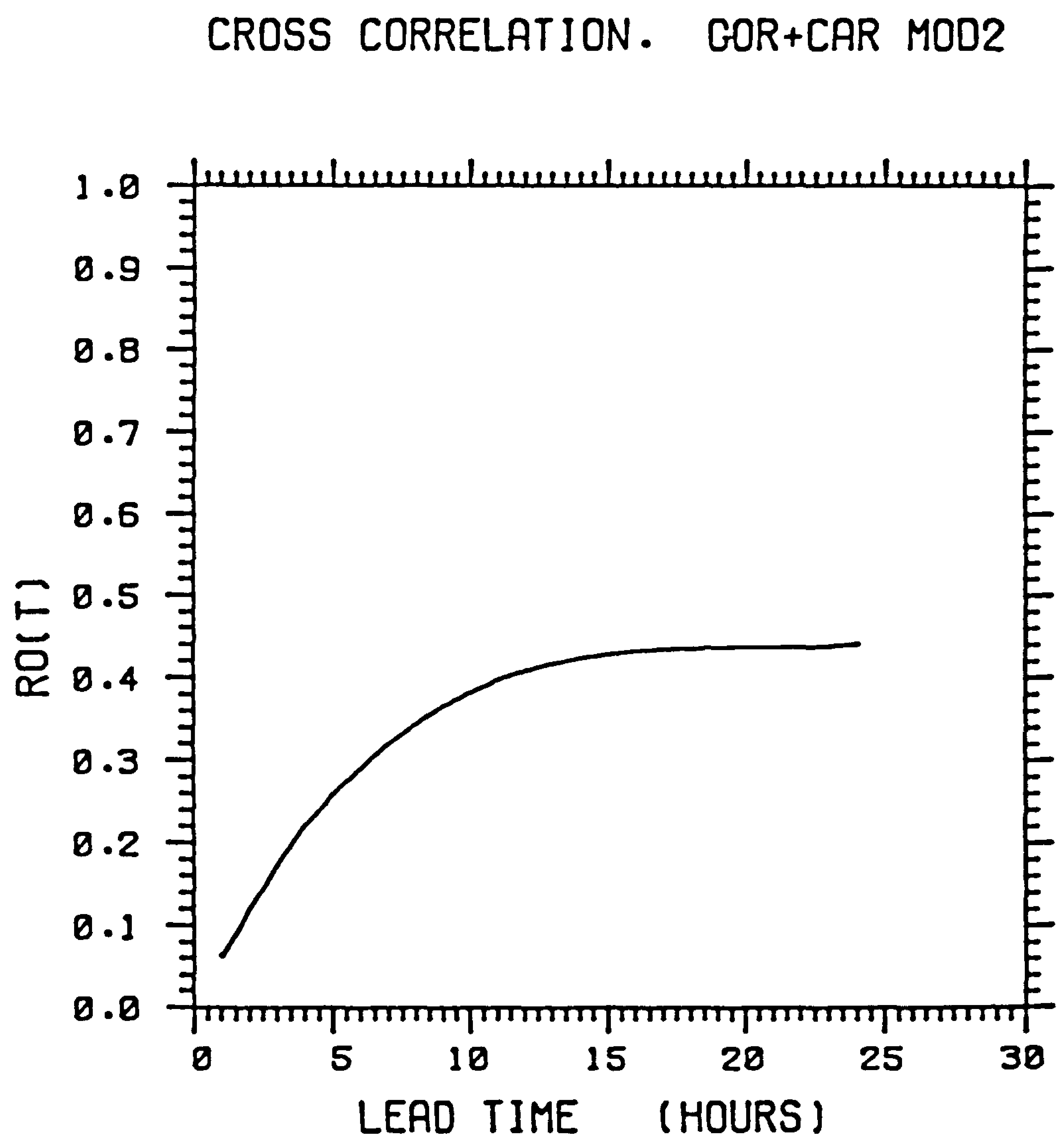
CROSS CORRELATION. STM+WIK MOD2



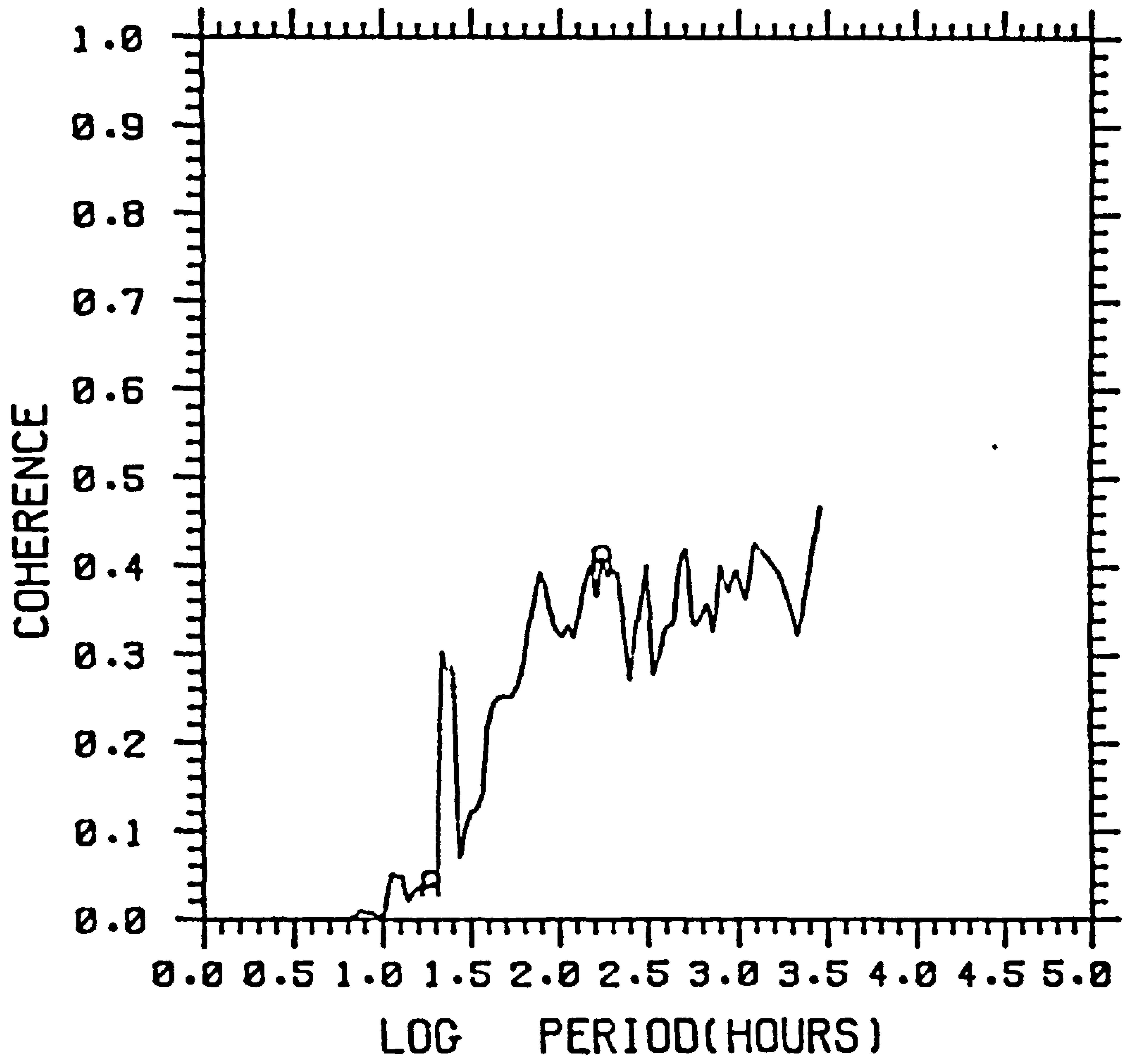

COHERENCE. GOR+WAT MOD2
COHERENCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
LOG PERIOD(HOURS)


CROSS CORRELATION. GOR+WAT MOD2
RO(T)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20
25
30
LEAD TIME (HOURS)

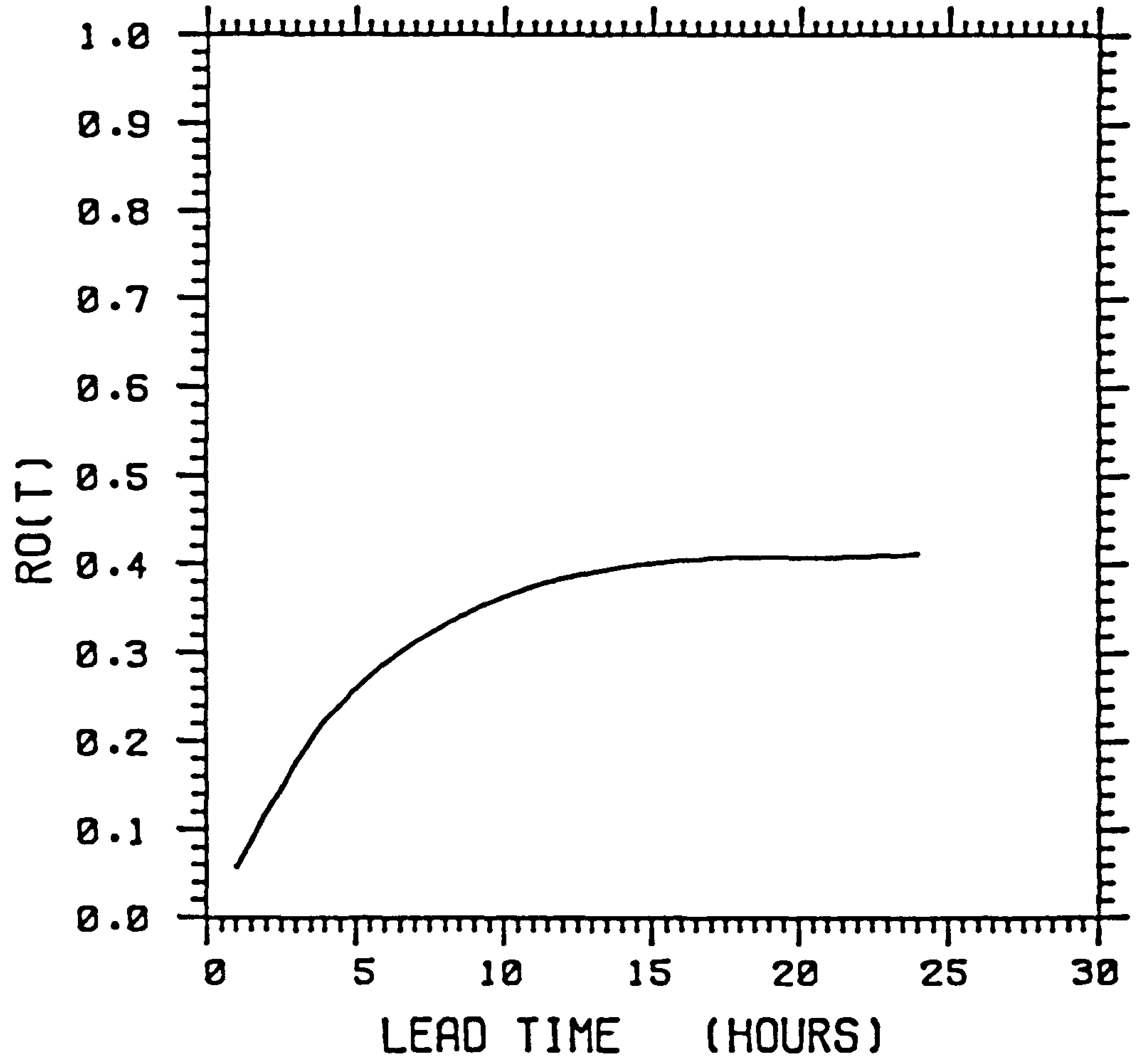

COHERENCE. GOR+CAR MOD2
COHERENCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
LOG PERIOD(HOURS)

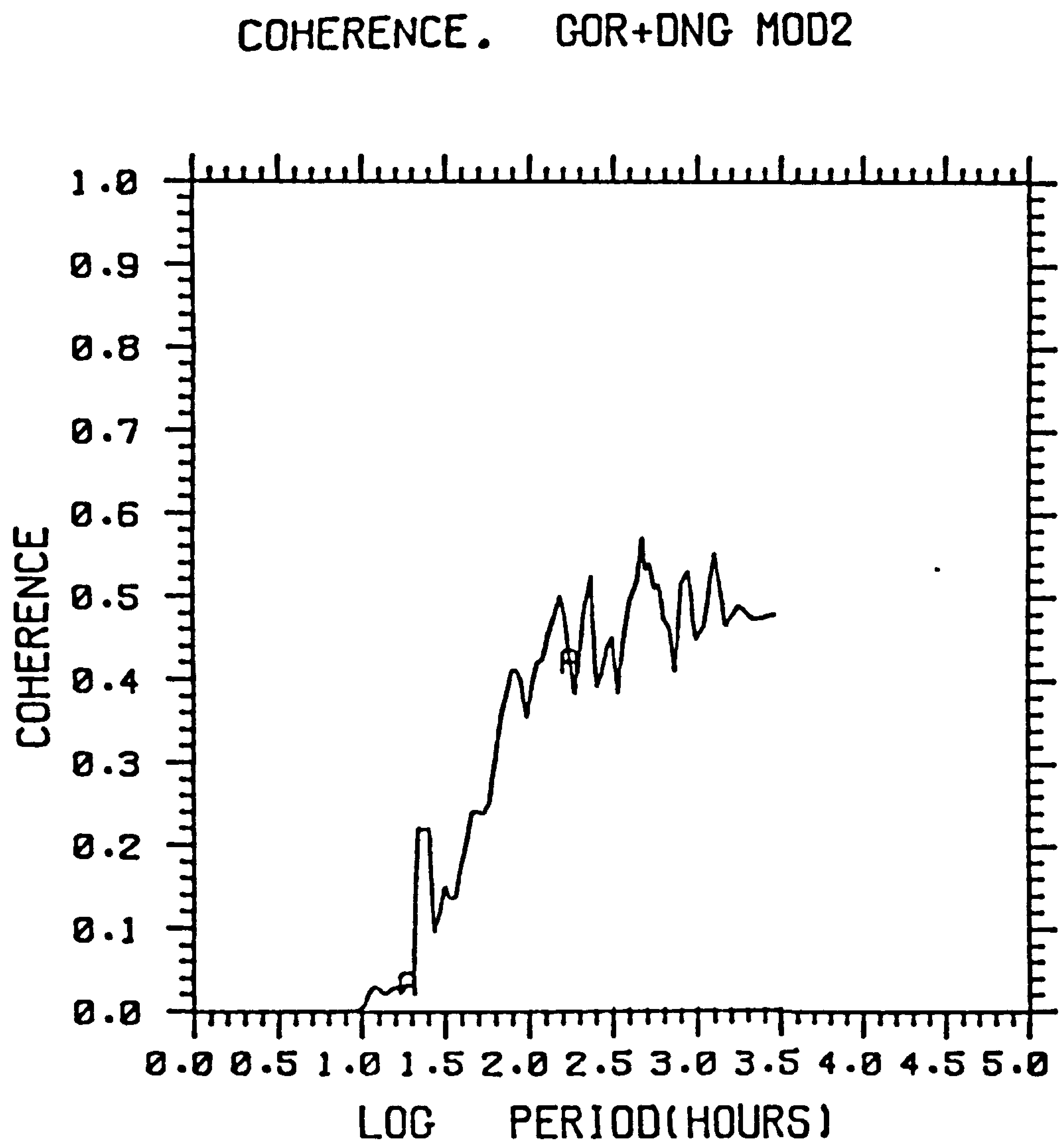

CROSS CORRELATION. GOR+CAR MOD2
RO(T)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 5 10 15 20 25 30
LEAD TIME (HOURS)

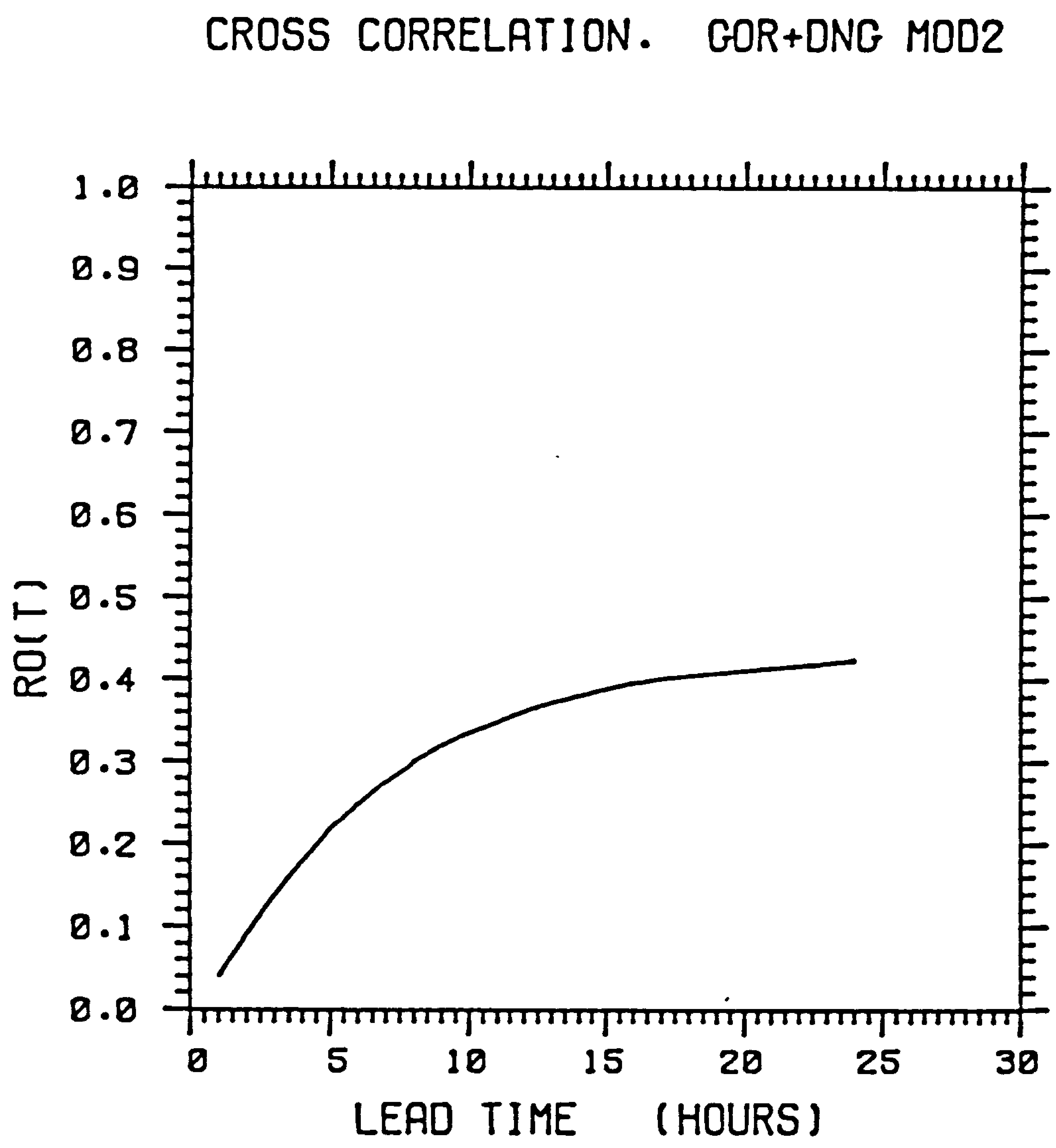
COHERENCE. GOR+GWK MOD2

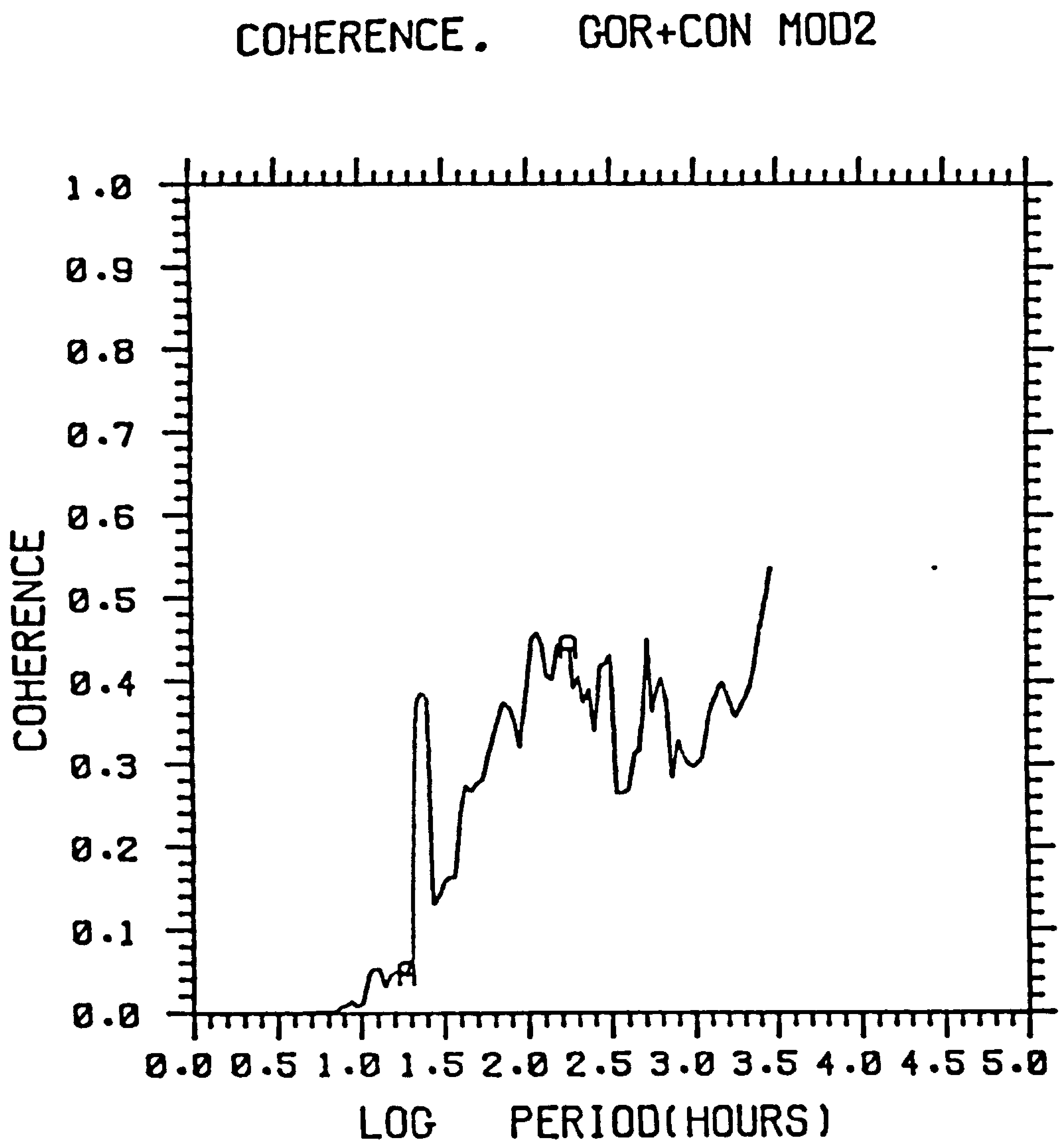


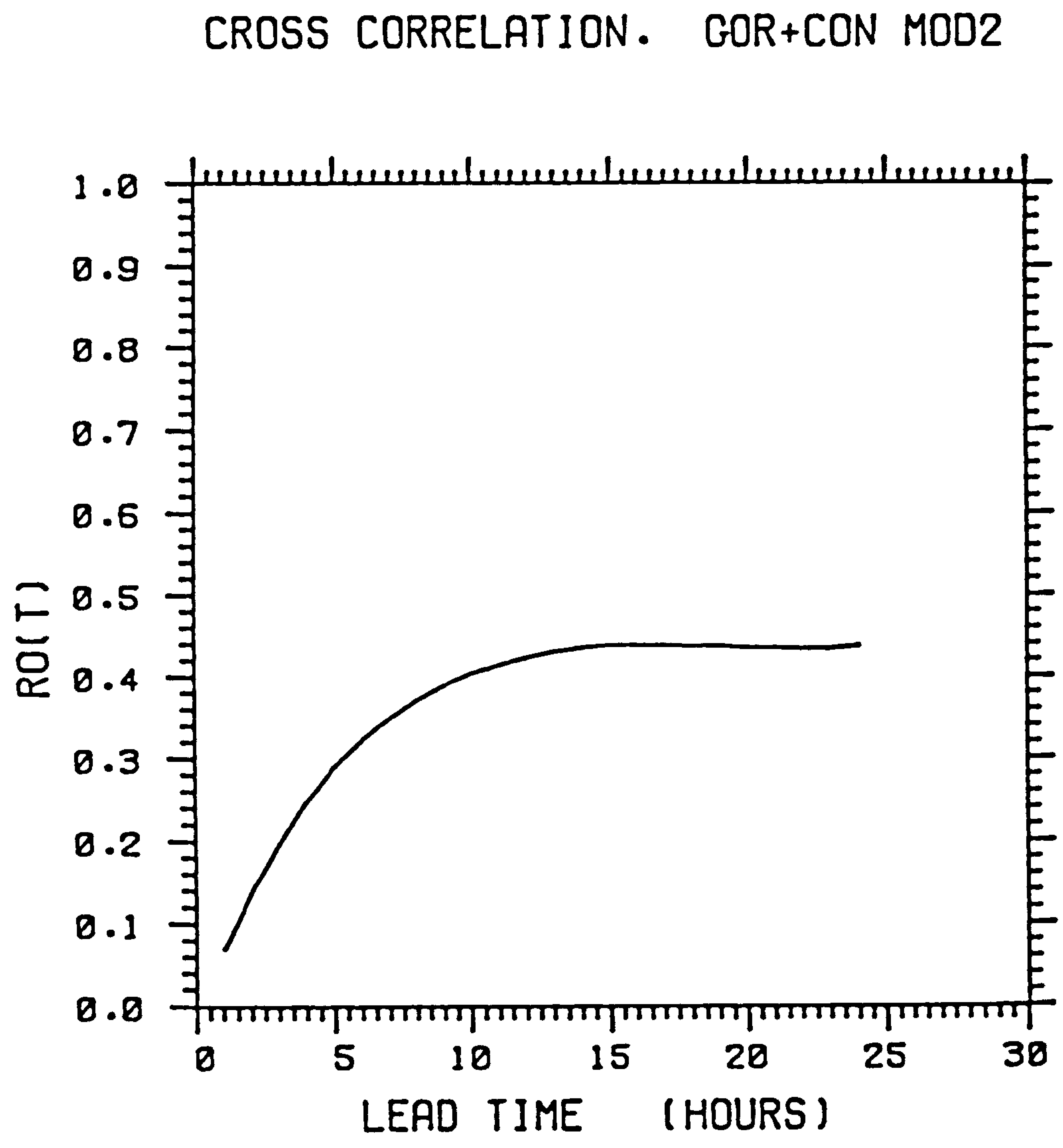
CROSS CORRELATION. GOR+GWK MOD2

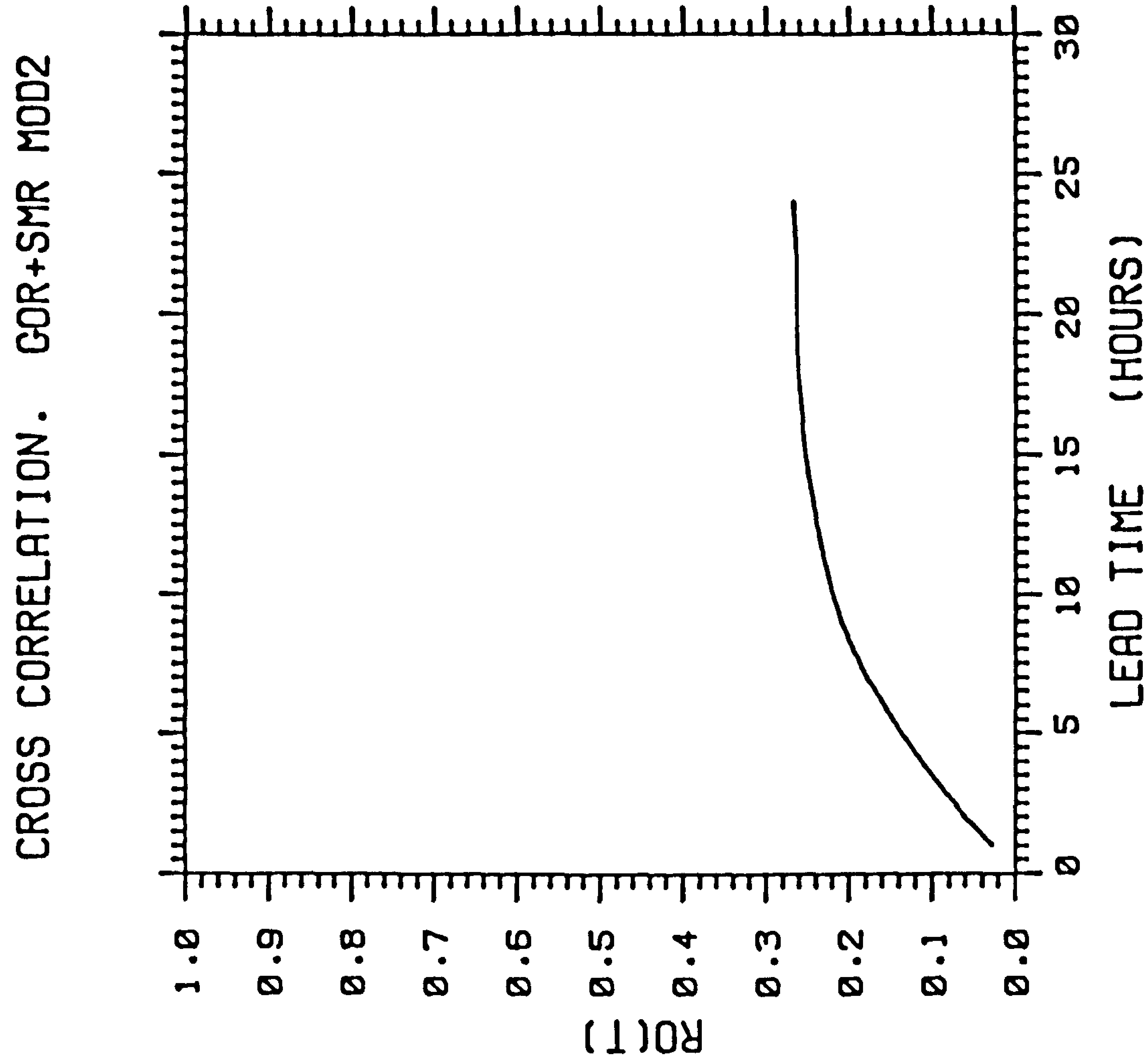



COHERENCE. GOR+DNG MOD2
COHERENCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
LOG PERIOD(HOURS)

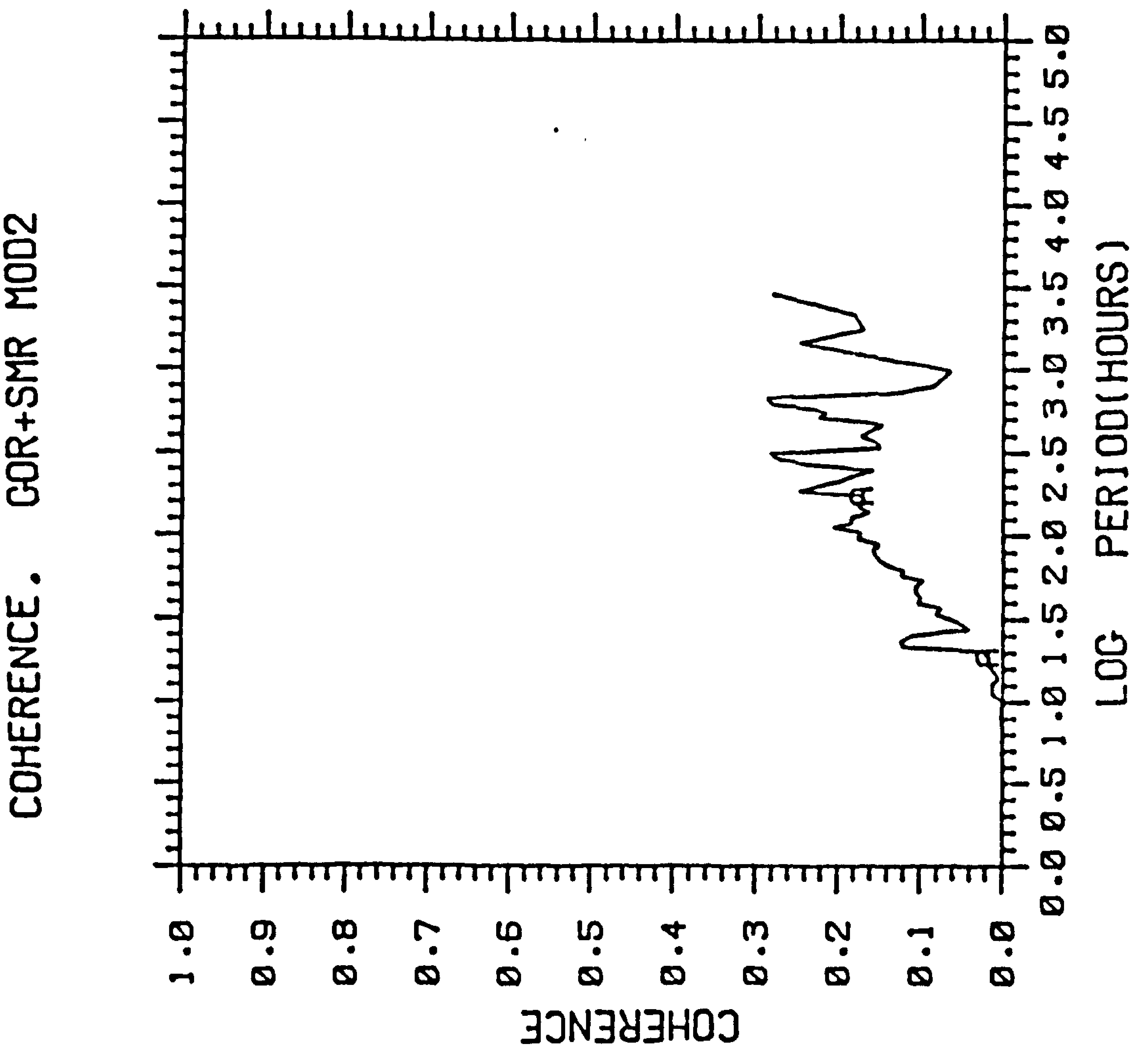

CROSS CORRELATION. GOR+DNG MOD2
RO(T)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 5 10 15 20 25 30
LEAD TIME (HOURS)


COHERENCE. GOR+CON MOD2
COHERENCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
LOG PERIOD(HOURS)


CROSS CORRELATION. GOR+CON MOD2
RO(T)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 5 10 15 20 25 30
LEAD TIME (HOURS)

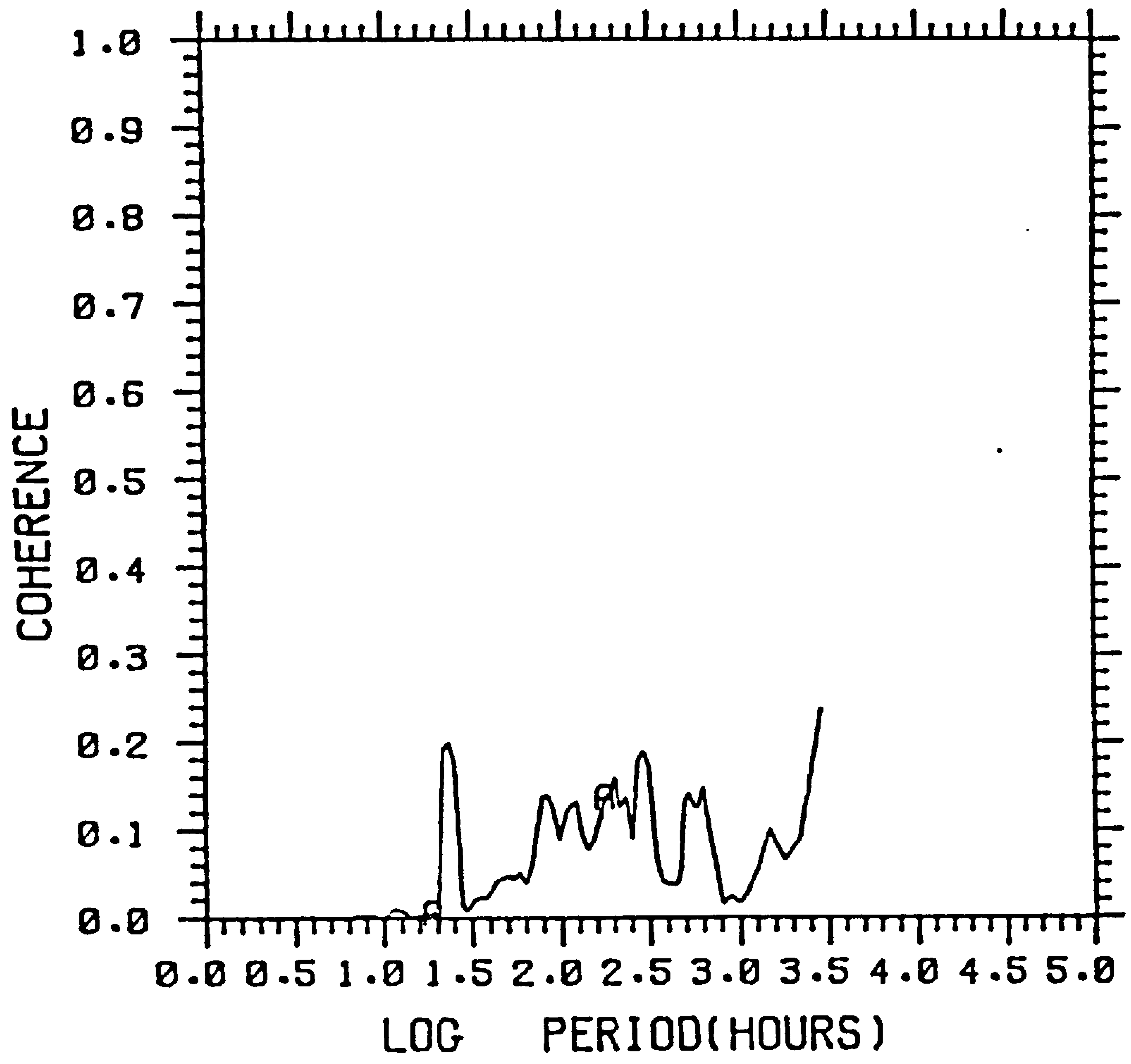


CROSS CORRELATION. GOR+SMR MOD2
RO(T)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20
25
30
LEAD TIME (HOURS)

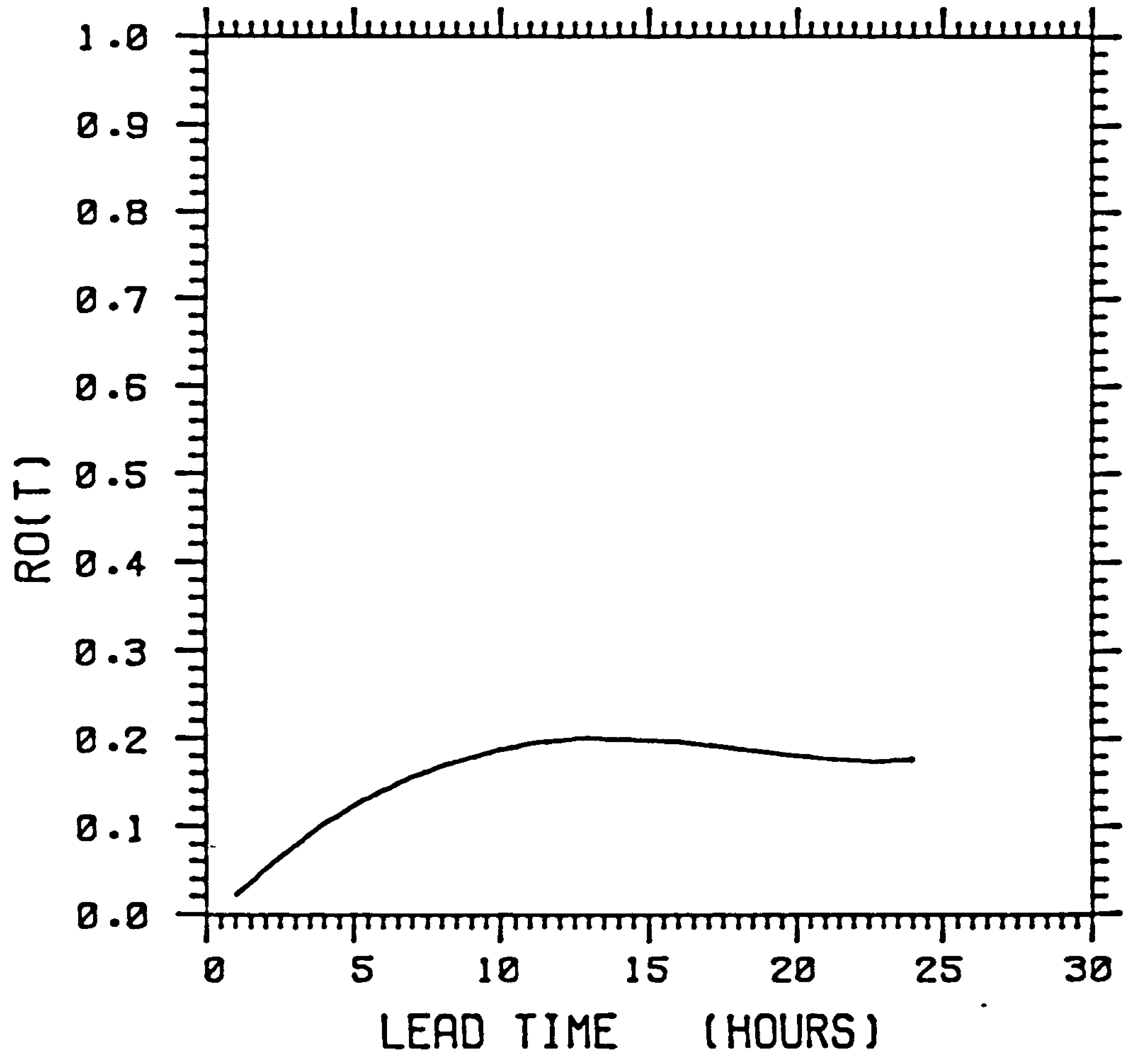


COHERENCE. GOR+SMR MOD2
COHERENCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
LOG PERIOD(HOURS)

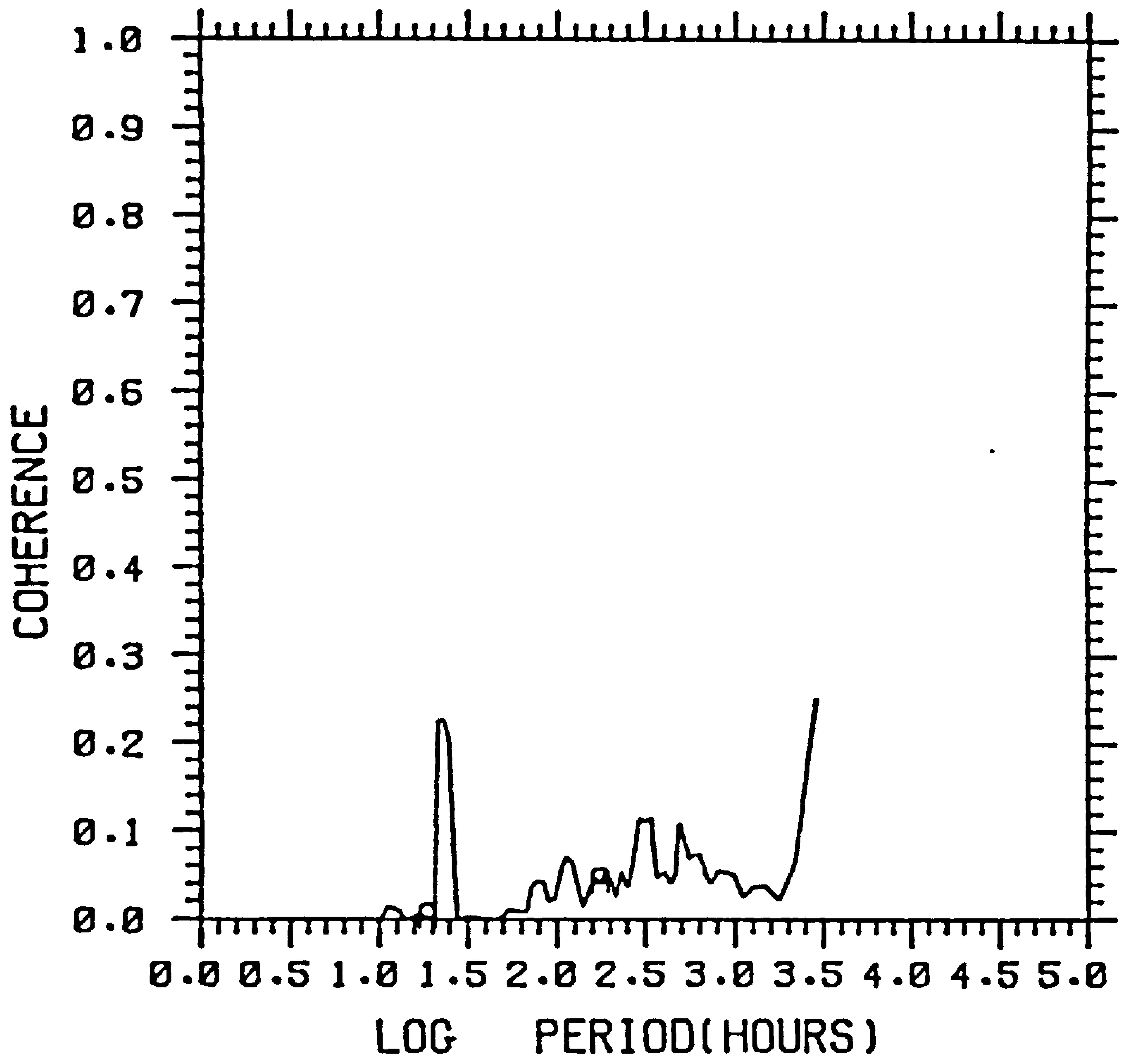
COHERENCE. GOR+DRH MOD2



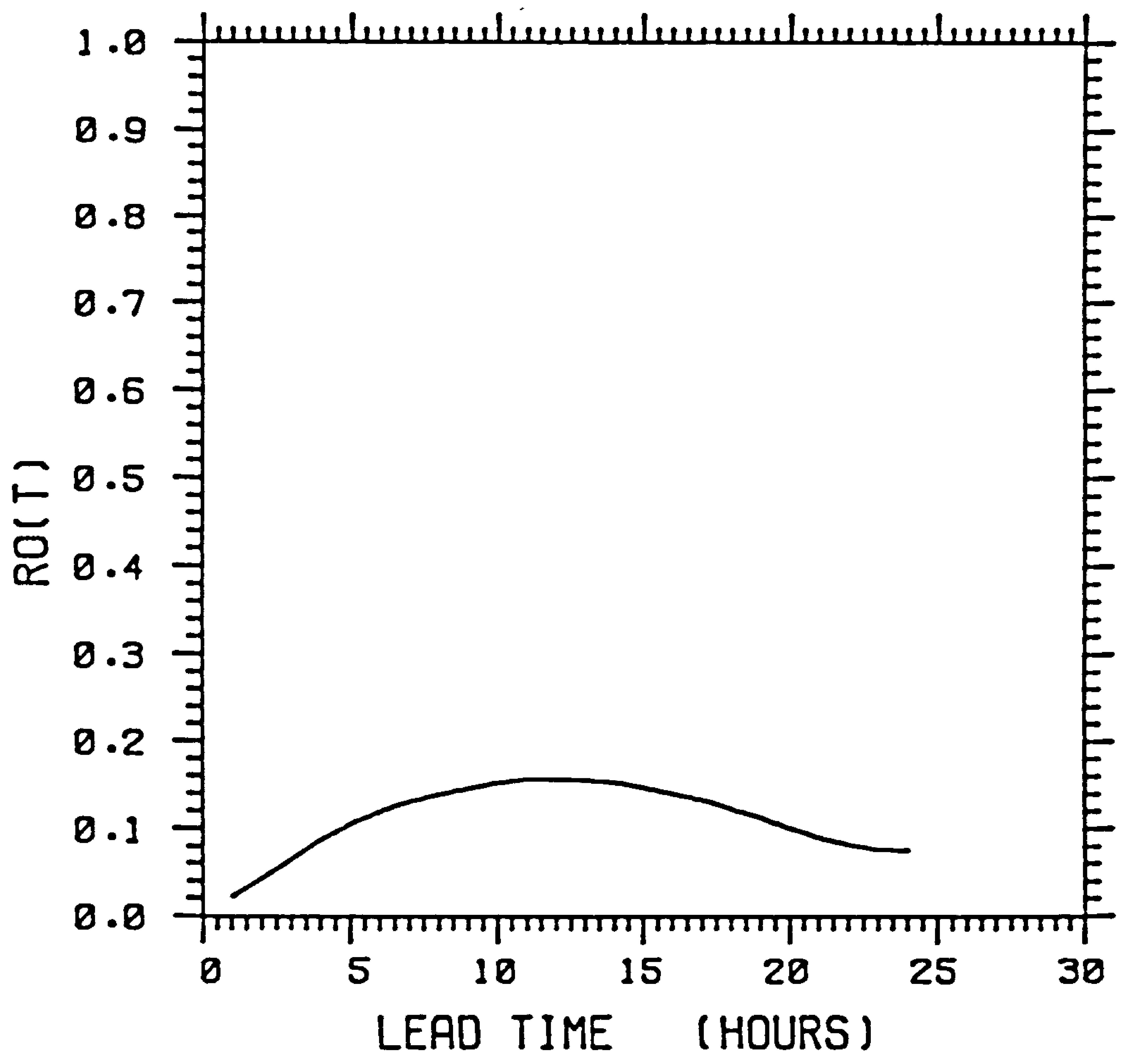
CROSS CORRELATION. GOR+DRH MOD2



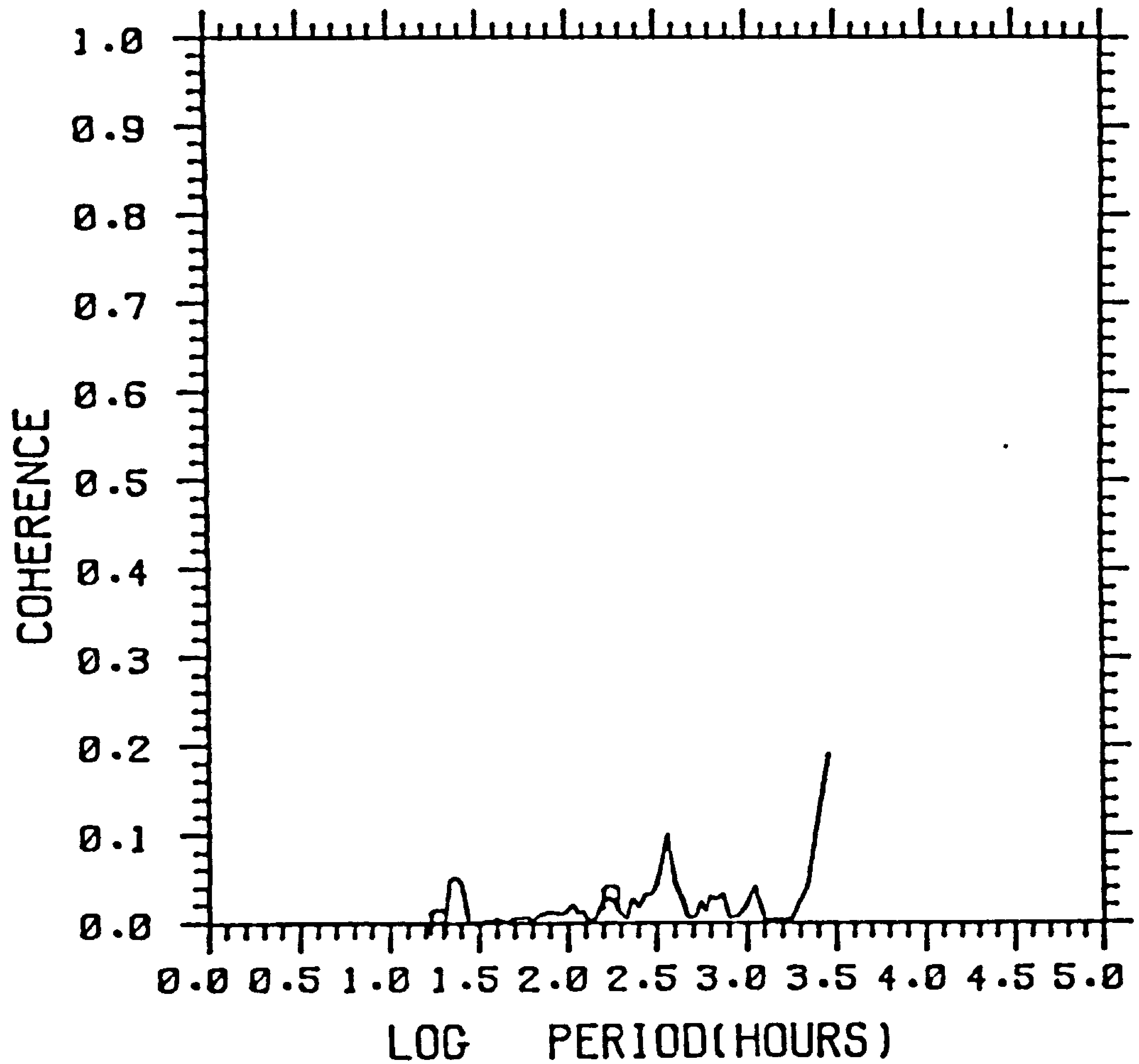
COHERENCE.   GOR+LCH MOD2



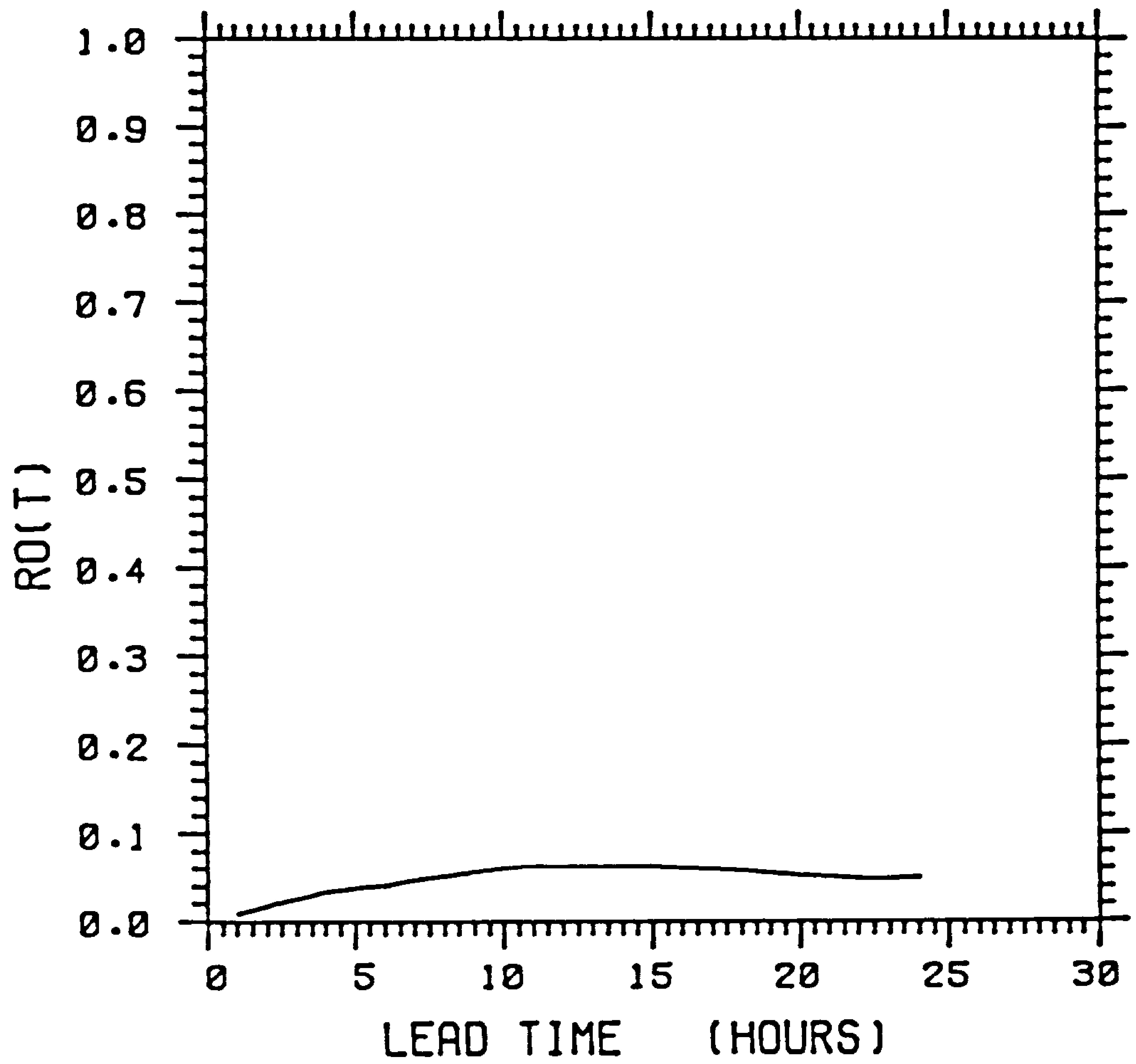
CROSS CORRELATION.   GOR+LCH MOD2

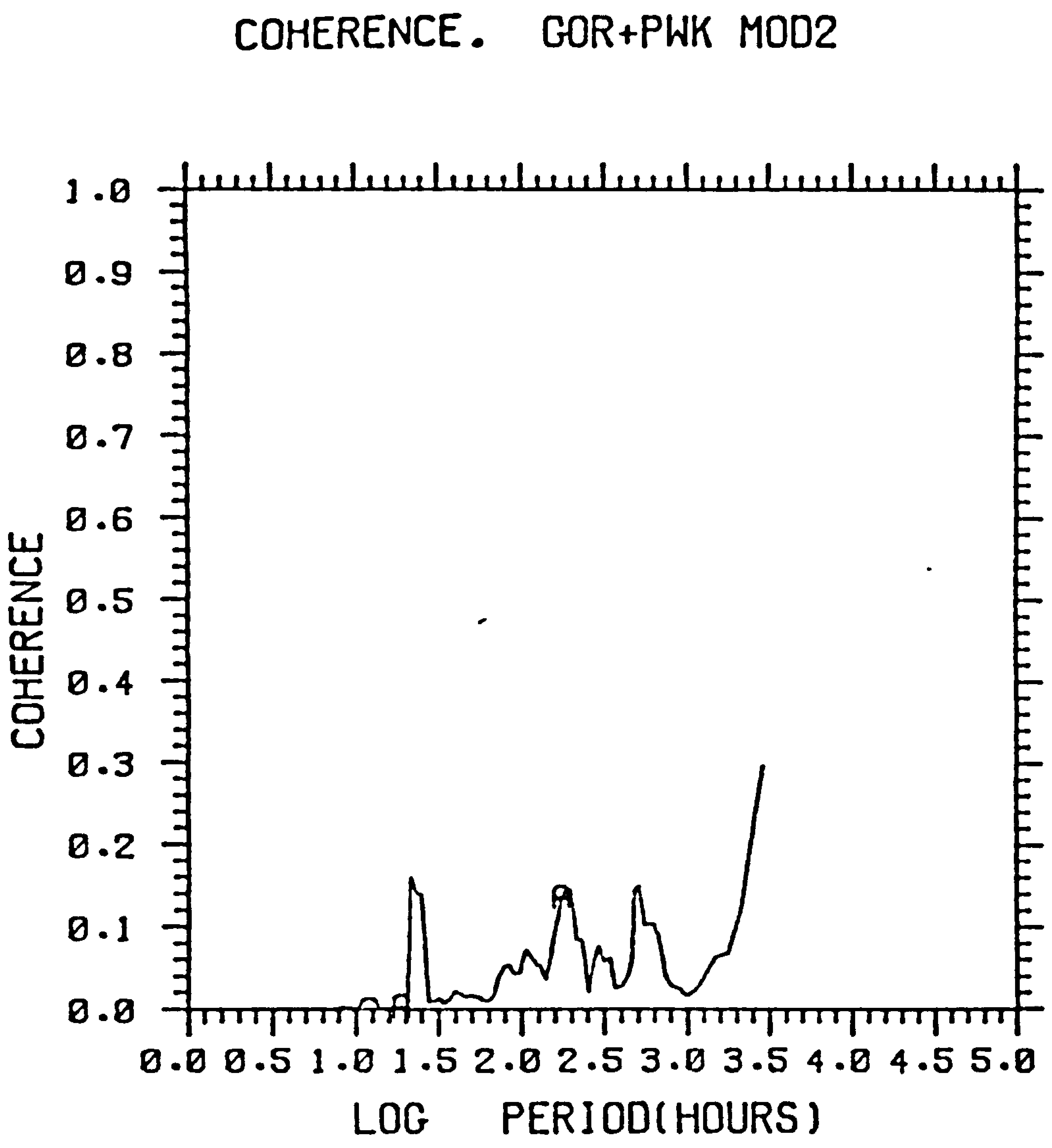


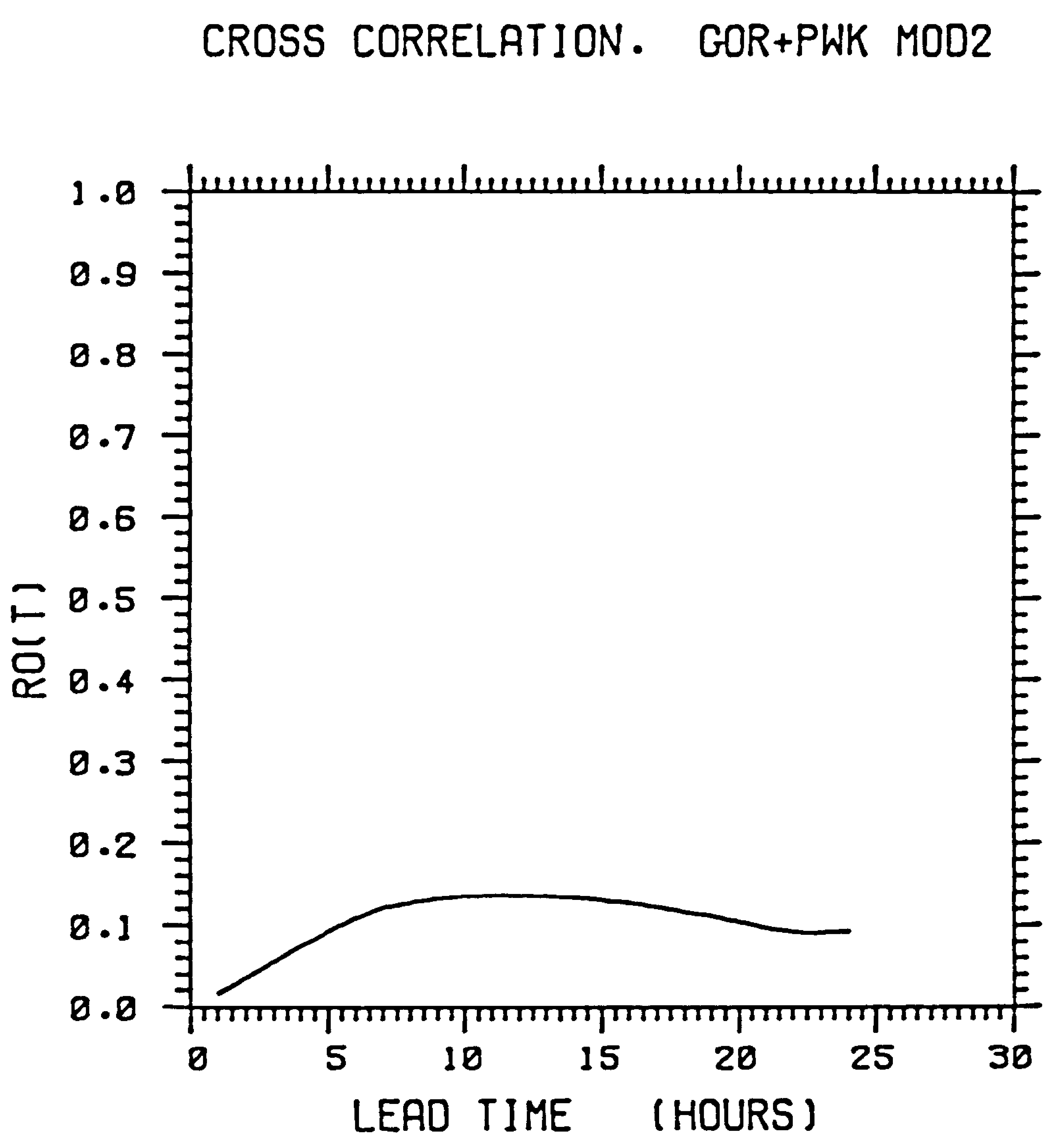
COHERENCE. GOR+FGH MOD2



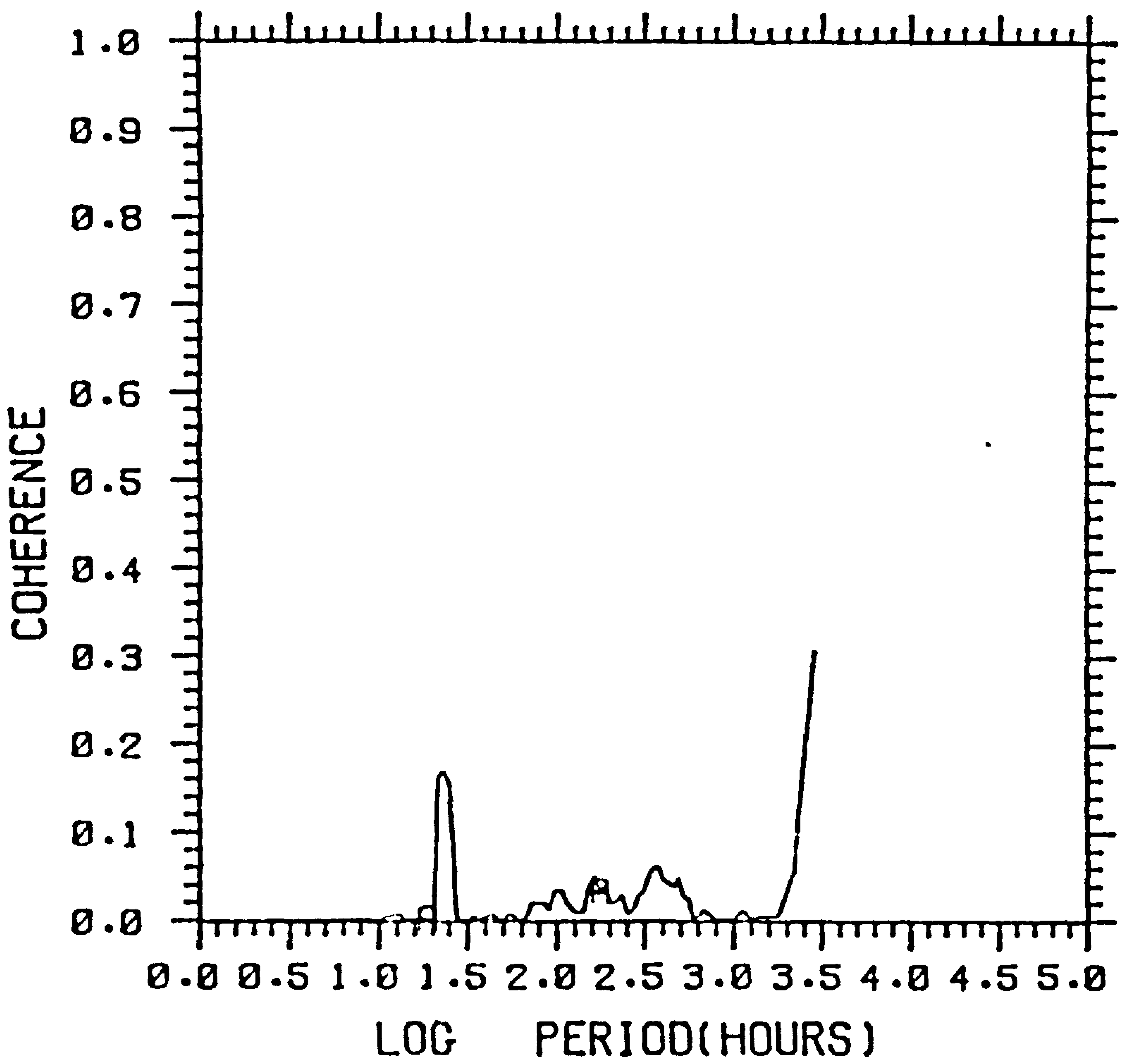
CROSS CORRELATION. GOR+FGH MOD2



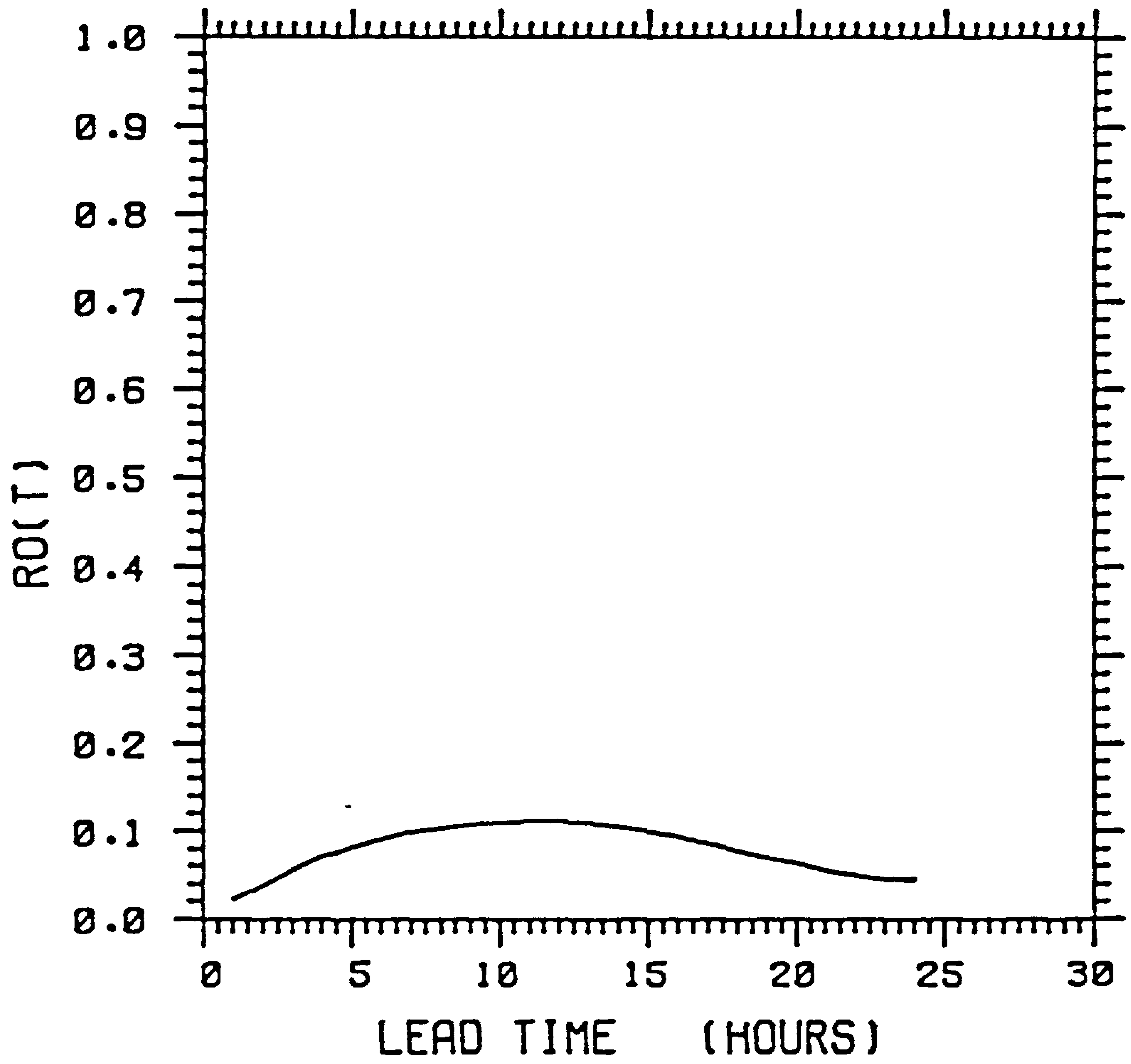

COHERENCE. GOR+PWK MOD2
COHERENCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
LOG PERIOD(HOURS)


CROSS CORRELATION. GOR+PWK MOD2
RO(T)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 5 10 15 20 25 30
LEAD TIME (HOURS)

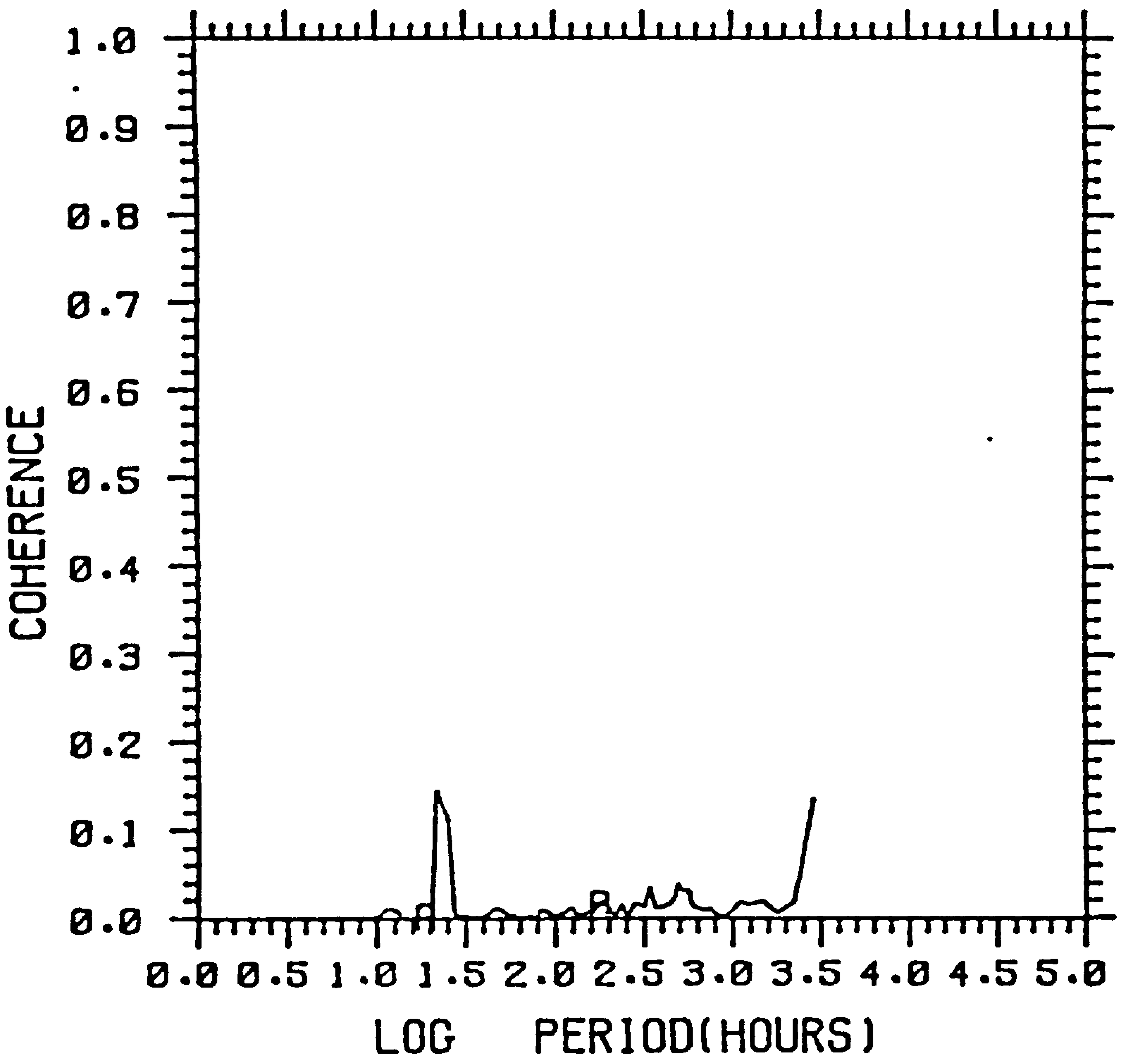
COHERENCE. GOR+WIK MOD2



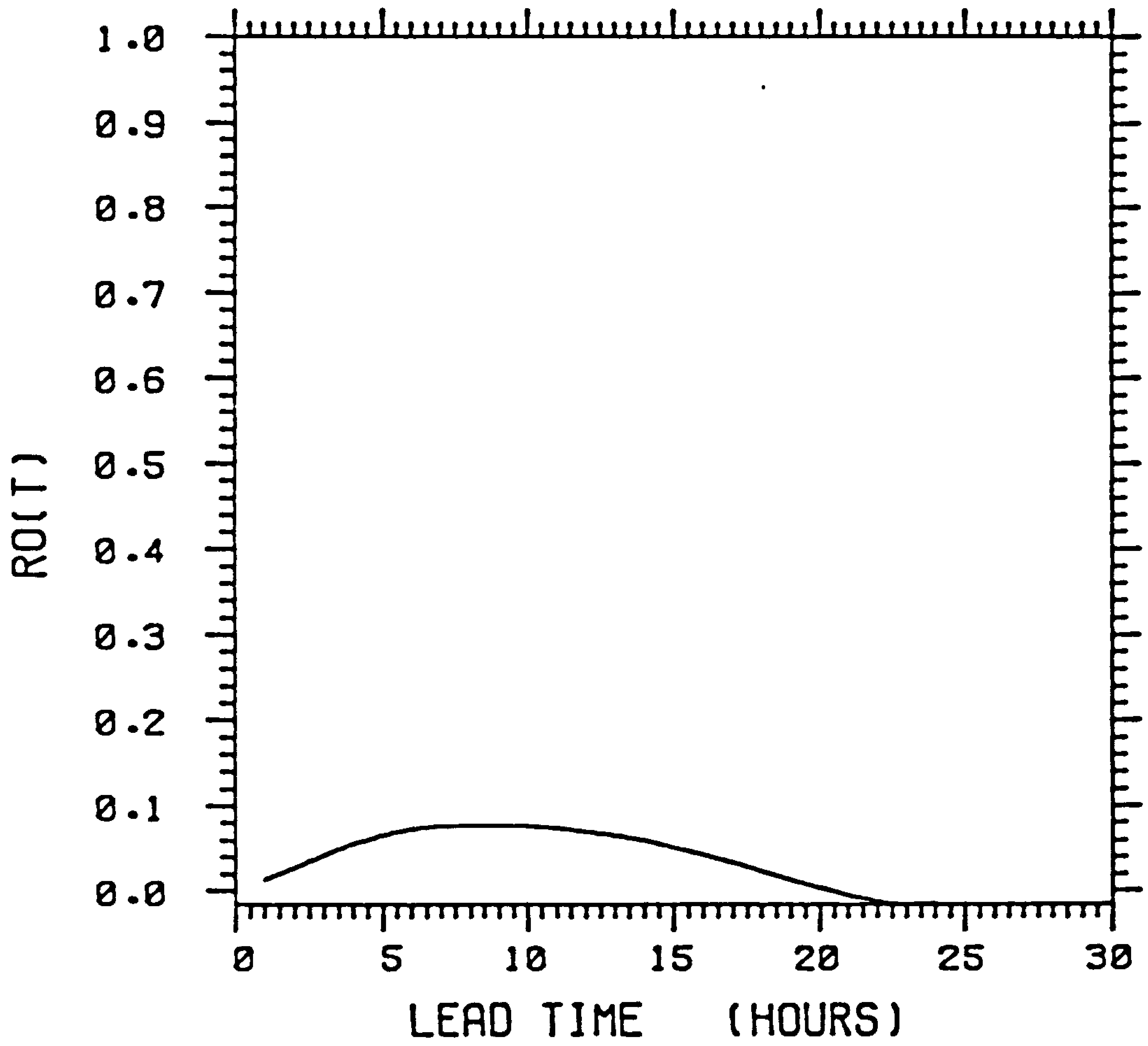
CROSS CORRELATION. GOR+WIK MOD2

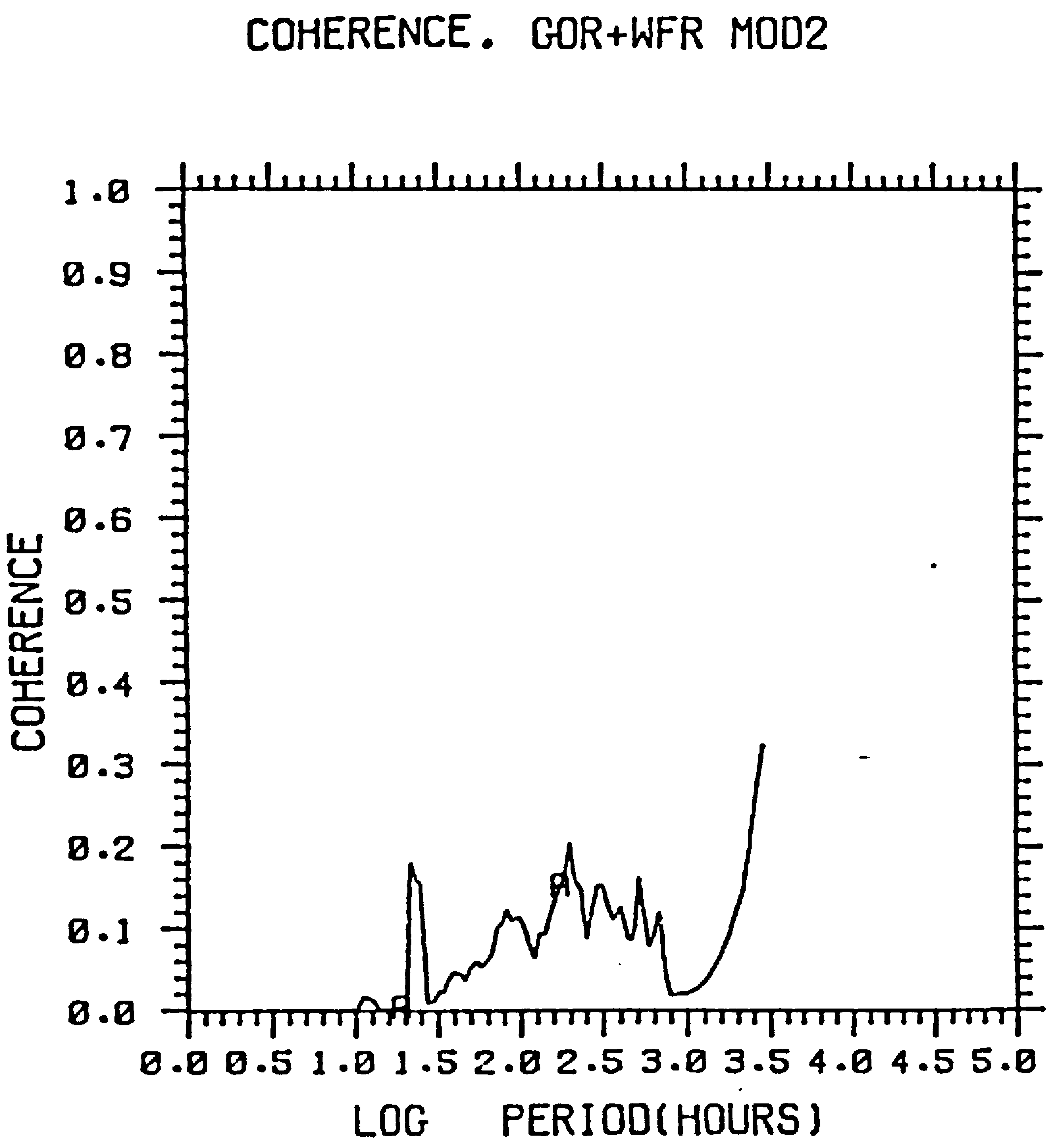


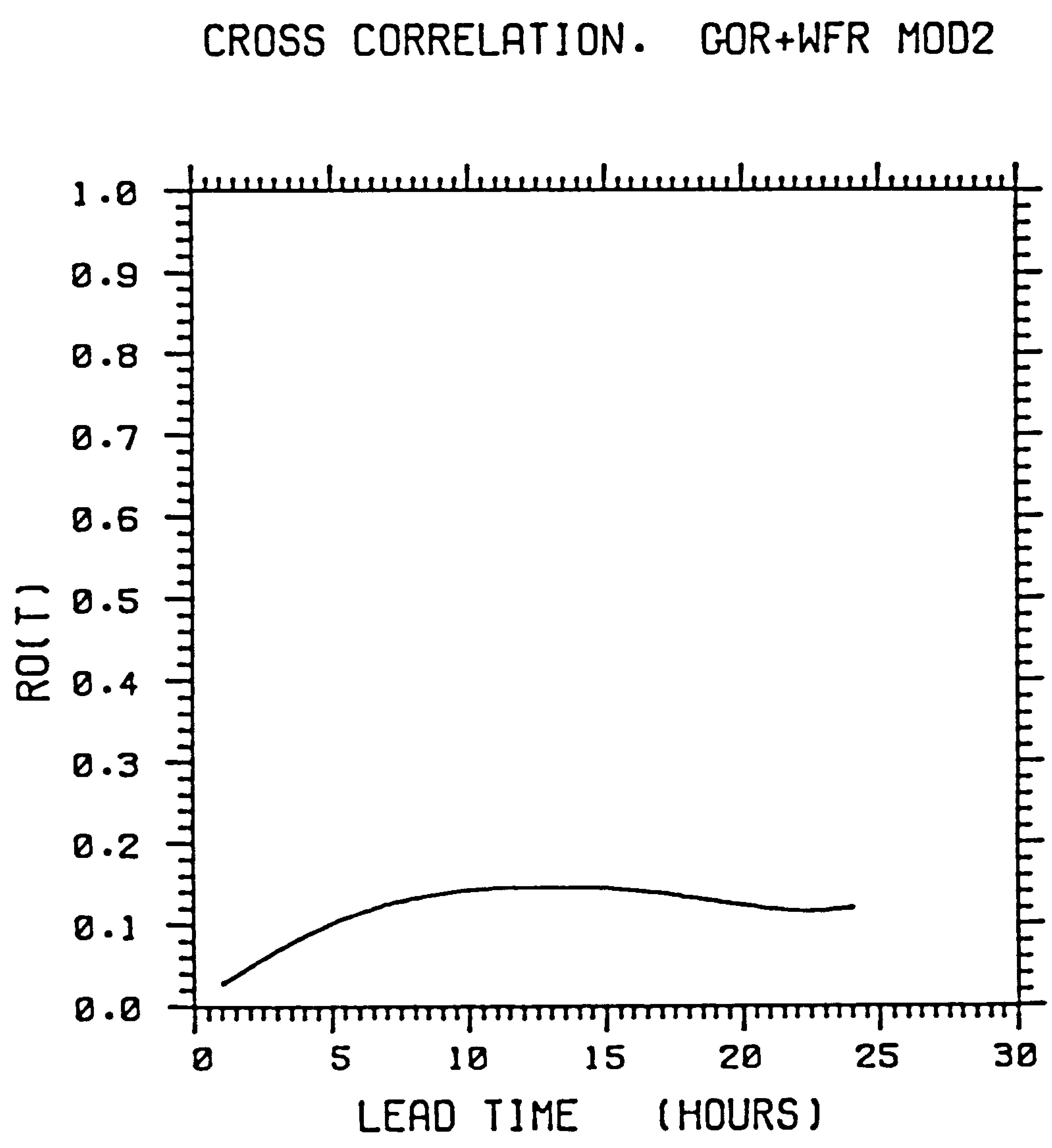
COHERENCE. GOR+STY MOD2



CROSS CORRELATION. GOR+STY MOD2




COHERENCE. GOR+WFR MOD2
COHERENCE
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
LOG PERIOD(HOURS)


CROSS CORRELATION. GOR+WFR MOD2
RO(T)
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0
0 5 10 15 20 25 30
LEAD TIME (HOURS)

APPENDIX 2

This appendix contains listings of the more important and complex of the programs used in completing the work described in this thesis. In addition to the Fortran listings, I have included the job control sequences, written in the system language Pheonix, so that these programs could be run (with a small amount of modification to allow for subsequent changes to Pheonix) on the Cambridge University IBM computer as shown.

F1S1Y9A

The following program generates spectra of wind power in the frequency range 3 yr^{-1} to 0.5 hr^{-1}.

```
REQUEST 'Chain with F1S9Y ?  Y/N. $Y ' CHAIN$Y
REQUEST 'TURNROUND?  Default OVERNIGHT  ' TURNROUND$OVERNIGHT
REQUEST 'PRIORITY?  Default 10 '  PRIORITY$10
REQUEST 'Number of years ?  Default 9  '  YEAR$9
REQUEST 'LIMSTORE  ?  Default 350k for nine years  '  LIMSTORE$350
REQUEST 'TIME ? $90 (9 years)  '     TIME$90
REQUEST 'SITE1?  '     SITE1
REQUEST 'IRUN?  '      IRUN
REQUEST 'TYPE OF MILL?  Default MOD2  '  MILL$MOD2
REQUEST 'NHOUR  ?   Default value 8760  '   NHOUR$8760
REQUEST 'NHOUR2  ?   Default value 4380  '     NHOUR2$4380
REQUEST 'SOURCE or NOSOURCE ? Default NO...  '   SOURCE$NOSOURCE
SUBMIT %H*
JOB RL16 *  F1S1Y9A <MILL>
TURNROUND <TURNROUND>
PRIORITY <PRIORITY>
ROUTE OPENUNIV
LIMSTORE <LIMSTORE>
CHAIN
TIME <TIME>
PLOTTER 200 RECORDS
PRINTER 2K
PAGE 0
SAVE
NOTIFY
TAPE9 RL1602
PHX
TLS WITH %H& TAPE RL1602
```

```
FT .<SITE1>7179R AS &<SITE1>
&
SET CHAR END ;
FORTQCLG(FT01F001=&<SITE1>, ;
FT08F001=RL16.GRAPH<IRUN>/FB/N PROGRAM=%HZ);
 BRIEFC=* SECG=<TIME> LIBRARY=SYS2.CAMLIB+GRAPHICS.NFORTLIB+RL16.LIB ;
PRINTC=* LISTC=<SOURCE> REGG=<LIMSTORE>
      INTEGER*2  IYEAR
      COMPLEX DATA, WORK, A
      DIMENSION IYEAR(8808),S(<NHOUR2>),YPLOT(200,1),XPLOT(200,1),
     +KEYS(6,1),TITLE(1,11),XAX(1,6),YAX(1,5),X(200),Y(200),
     +TITLE1(1,10),KEYS1(1,1),XAX1(1,6),YAX1(1,2),
     +TITLE2(1,10),KEYS2(1,1),XAX2(1,4),YAX2(1,5),
     +XPLOT1(200),YPLOT1(200),XPLOT2(60),YPLOT2(60),
     +DATA(<NHOUR>),WORK(<NHOUR>)
      EQUIVALENCE (IYEAR(1),WORK(1))
C ---------------------------------------------------
C Data input for first graph.
      DATA KEYS1/'    '/
      DATA TITLE1/'EFFE','CT O','F SI','TE D','ISPE','RSAL',
     +'. SI','TE  ','<SITE1> ','<MILL>'/
      DATA XAX1/'LOG ','  FR','EQUE','NCY(','HR-1',')   '/
      DATA YAX1/'N*S(','N)  '/
      ITIT1=10
      KDIM1=1
      LEX1=6
      LEY1=2
C ---------------------------------------------------
C Data input for second graph.
      DATA KEYS2/'    '/
      DATA TITLE2/'EFFE','CT O','F SI','TE D','ISPE','RSAL',
     +'. SI','TE  ','<SITE1>','<MILL>'/
      DATA XAX2/'LOG ','PERI','OD  ','(HR)'/
```

```
      DATA YAX2/'NORM','ALIS','ED V','ARIA','NCE '/
      ITIT2=10
      KDIM2=1
      LEX2=4
      LEY2=5
C ----------------------------------------------------------
      NHOUR=<NHOUR>
      NHOUR2=NHOUR/2
      IRUN=<IRUN>
      YEAR=FLOAT(<YEAR>)
      DO 2 I=1,8808
        IYEAR(I)=0
   2    CONTINUE
      DO 4 J=1,200
        XPLOT1(J)=0.0
        YPLOT1(J)=0.0
        XPLOT(J,1)=0.0
        YPLOT(J,1)=0.0
   4    CONTINUE
      DO 5 J=1,60
        XPLOT2(J)=0.0
        YPLOT2(J)=0.0
   5    CONTINUE
      IDIM=0
      IDUR=NHOUR
      JDIM=200
      DO 6 I=1,NHOUR2
        S(I)=0.0
   6    CONTINUE
      WRITE(6,15)
  15  FORMAT(1X,/,80('-'),/,
     +' SPECTRAL ANALYSIS OF <SITE1>. IRUN=<IRUN>',
     +/,80('-'))
```

```
C ---------------------------------------------------------
C Finds average power at <SITE1>
C ---------------------------------------------------------
      N=0
      AV=0.0
      STOTAL=0.0
      DO 500 JYEAR=1,<YEAR>
        READ(1,90) IYEAR
        DO 100 I=1,NHOUR
          IDUMMY = IYEAR(I)
          IF(IDUMMY.EQ.-32768) GOTO 100
          DUMMY=I<MILL>(IDUMMY)
          AV=AV+DUMMY
          N=N+1
 100      CONTINUE
 500    CONTINUE
      STOTAL = AV/FLOAT(N)
      REWIND 1
      WRITE(6,12) STOTAL, N
  12  FORMAT(1X,/,' Average power  <SITE1>; ',F6.2,4X,' N= ',I5)
C ----------------------------------------------------------
C Start of main DO loop.
C ----------------------------------------------------------
      DO 200 JYEAR=1,<YEAR>
        READ(1,90) IYEAR
  90    FORMAT(100(44A2),4A2)
        DO 10 J=1,NHOUR
          IDUMMY = IYEAR(J)
          DUMMY = I<MILL>(IDUMMY)
          DATA(J) = DUMMY/STOTAL
          IF(IDUMMY.EQ.-32768) DATA(J) = 1.0
  10      CONTINUE
        VAR=0.0
```

```
      AV=0.0
      DO 50 J=1,NHOUR
        AV=AV + (CABS(DATA(J)))
        VAR=VAR + ((CABS(DATA(J)))**2)
 50     CONTINUE
      AV=AV/FLOAT(NHOUR)
      VAR=(VAR/FLOAT(NHOUR)) - (AV**2)
      CALL FFTX(DATA,WORK,NHOUR)
      TVAR=0.0
      DO 55 J=1,NHOUR
        TVAR = TVAR + ((CABS(DATA(J)))**2)
 55     CONTINUE
      WRITE(6,95) VAR, TVAR
 95   FORMAT(1X,' Input variance ',F6.4,'; Output variance ',F6.4)
C-------------------------------------------------------------
      DO 60 J=1,NHOUR2
        A = DATA(J)
        S(J) = S(J) + (((CABS(A))**2)*2.0/YEAR)
 60     CONTINUE
200   CONTINUE
     VARJ1=0.0
     DO 65 J=1,NHOUR2
       VARJ1=VARJ1+S(J)
 65    CONTINUE
     WRITE(6,70) VARJ1
 70  FORMAT(1X,/,50('-'),/,
    +'  Variance (mean over <YEAR> years)  = ',F9.5)
C-------------------------------------------------------------
C End of main DO loop. S contains the spectrum of the power ouput
C at <SITE1>, averaged over <YEAR> years.
C -------------------------------------------------------------
     CALL ALIAS(S,NHOUR2,NHOUR)
     CALL RANWAL(S,NHOUR2,NDIM,YPLOT2,XPLOT2)
```

```
      CALL SMOOT3(XPLOT1,YPLOT1,S,NHOUR2,IDIM)
      WRITE(6,80) (XPLOT2(I), YPLOT2(I), I=1,NDIM)
   80 FORMAT(' ',/,' Output from RANWAL.',/,(4(1X,'+ ',F6.2,F6.2,6X)))
      CALL LOGTRN(YPLOT1,XPLOT1,1.0,NHOUR,IDIM)
      CALL MGRAP2(YPLOT2,1,NDIM,60,XPLOT2,KEYS2,KDIM2,TITLE2,ITIT2,
     +XAX2,LEX2,YAX2,LEY2,1,1,IRUN,4.0,5.0,0.0,0.0)
      CALL MGRAP2(YPLOT1,1,IDIM,JDIM,XPLOT1,KEYS1,KDIM1,TITLE1,
     +ITIT1,XAX1,LEX1,YAX1,LEY1,1,1,
     +IRUN,1.4,0.0,0.0,-5.0)
      STOP
      END
%
C  .MISC:F1S9Y CHAIN=<CHAIN> SITE1=<SITE1>  IRUN=<IRUN> ;
YEAR=9 PERIOD=6 NHOUR=13140 NHOUR2=6570 NHOUR3=6571 MILL=<MILL> ;
TURNROUND=<TURNROUND> LIMSTORE=350K TIME=60 SOURCE=<SOURCE> ;
PRIORITY=<PRIORITY>
*
```

FIS9Y

This program generates spectra of wind power in the frequency range 0.3 yr^{-1} to 1/12 hr^{-1}.

```
IF CHAIN UCEQ N SKIP LABEND
IF CHAIN UCEQ Y SKIP LAB1
REQUEST 'TURNROUND?  Default OVERNIGHT  ' TURNROUND$OVERNIGHT
REQUEST 'PRIORITY?  Default 10 '  PRIORITY$10
REQUEST 'LIMSTORE  ?  Default 350k for nine years  '  LIMSTORE$350
REQUEST 'TIME ? $60 for 9 years.? '        TIME$60
REQUEST 'SITE1?  '     SITE1
REQUEST 'IRUN?  '      IRUN
REQUEST 'Type of mill?  Default MOD2  '  MILL$MOD2
REQUEST 'Number of years ?  $9 '  YEAR$9
REQUEST 'PERIOD ?   Default value 6  '      PERIOD$6
REQUEST 'NHOUR  ?   Default value 13140  '   NHOUR$13140
REQUEST 'NHOUR2  ?   Default value 6570  '     NHOUR2$6570
REQUEST 'NHOUR3  ? Equal to NHOUR2 + 1  '     NHOUR3$6571
REQUEST 'SOURCE or NOSOURCE ? Default NO...  '   SOURCE$NOSOURCE
FILE $ TO .GRAPH<IRUN>/FB/N
*LAB1
SUBMIT %H*
JOB RL16 *  FIS9Y <MILL>
TURNROUND <TURNROUND>
PRIORITY <PRIORITY>
ROUTE OPENUNIV
LIMSTORE <LIMSTORE>
TIME <TIME>
CHAIN
PLOTTER 200 RECORDS
PRINTER 2K
PAGE 0
```

```
SAVE
NOTIFY
TAPE9 RL1602/W
PHX
TLS WITH %H& TAPE RL1602
FT .<SITE1>7179R AS &<SITE1>
&
SET CHAR END ;
FORTQCLG(FT01F001=&<SITE1>,FT03F001=&A/N,;
FT08F001=RL16.GRAPH<IRUN>/FB/MOD PROGRAM=%H%);
 BRIEFC=* SECG=<TIME> LIBRARY=SYS2.CAMLIB+GRAPHICS.NFORTLIB+RL16.LIB ;
PRINTC=* LISTC=<SOURCE> REGG=<LIMSTORE>
      INTEGER*2  IYEAR
      COMPLEX DATA, WORK, A
      DIMENSION IYEAR(8808),S(<NHOUR2>),YPLOT(200,1),XPLOT(200,1),
     +KEYS(6,1),TITLE(1,11),XAX(1,6),YAX(1,6),X(200),Y(200),
     +TITLE1(1,10),KEYS1(1,1),XAX1(1,6),YAX1(1,2),
     +TITLE2(1,10),KEYS2(1,1),XAX2(1,4),YAX2(1,6),
     +XPLOT1(200),YPLOT1(200),XPLOT2(60),YPLOT2(60),
     +DATA(<NHOUR>),WORK(<NHOUR>)
      EQUIVALENCE (DATA(<NHOUR3>), S(1)),  (WORK(1), IYEAR(1))
C -----------------------------------------------------
C Data input for first graph.
      DATA KEYS1/'    '/
      DATA TITLE1/'EFFE','CT O','F SI','TE D','ISPE','RSAL',
     +'. SI','TE  ','<SITE1> ','<MILL>'/
      DATA XAX1/'LOG ','  FR','EQUE','NCY(','HR-1',')   '/
      DATA YAX1/'N*S(','N)  '/
      ITIT1=10
      KDIM1=1
      LEX1=6
      LEY1=2
C -----------------------------------------------------
```

```
C Data input for second graph.
      DATA KEYS2/'    '/
      DATA TITLE2/'EFFE','CT O','F SI','TE D','ISPE','RSAL',
     +'. SI','TE  ','<SITE1> ','<MILL>'/
      DATA XAX2/'LOG ','PERI','OD  ','(HR)'/
      DATA YAX2/'RMS ','STOR','E FL','UCTU','ATIO','N   '/
      ITIT2=10
      KDIM2=1
      LEX2=4
      LEY2=6
C -------------------------------------------------------
      NHOUR=<NHOUR>
      XHOUR=NHOUR
      NHOUR2=<NHOUR2>
      XHOUR2=NHOUR2
      IRUN=<IRUN>
      DO 1 I=1,NHOUR
        DATA(I)=0.
        WORK(I)=0.
    1   CONTINUE
      PERIOD=<PERIOD>
      NYEAR=<YEAR>
      IDIM=0
      IDUR=NHOUR*<PERIOD>
      JDIM=200
      STOTAL=0.0
      WRITE(6,12)
   12 FORMAT(1X,/,80('-'),/,
     +' SPECTRAL ANALYSIS OF <SITE1>. IRUN=<IRUN>',
     +/,80('-'))
C -------------------------------------------------------
C Finds average power at <SITE1>.
C -------------------------------------------------------
```

```
      IMAX=NHOUR*<PERIOD>/NYEAR
      N=0
      AV=0.0
      DO 500 J=1,NYEAR
        READ(1,90) IYEAR
        DO 30 I=1,IMAX
          IDUMMY=IYEAR(I)
          IF(IDUMMY.EQ.-32768) GOTO 30
          DUMMY=IMOD2(IDUMMY)
          AV=AV+DUMMY
          N=N+1
 30       CONTINUE
500     CONTINUE
      STOTAL=AV/FLOAT(N)
      REWIND 1
C -----------------------------------------------------------
C Moving block average, data repair, and normalisation.
C -----------------------------------------------------------
      N=0
      DO 5 I=1,NHOUR
        DATA(I) = (0.0,0.0)
  5     CONTINUE
      DO 200 NY=1,NYEAR
        READ(1,90) IYEAR
 90     FORMAT(100(44A2),4A2)
        DO 10 J=1,IMAX,<PERIOD>
          BLOCK=0.0
          DO 15 I=1,<PERIOD>
            IDUMMY = IYEAR(I+J-1)
            IF(IDUMMY.EQ.-99) WRITE(6,125)
125         FORMAT(1X,'Error in moving block average: n>8760')
            DUMMY = IMOD2(IDUMMY)
            IF(IDUMMY.EQ.-32768) DUMMY = STOTAL
```

```
          BLOCK = BLOCK + DUMMY
  15      CONTINUE
        N=N+1
        DATA(N) = (BLOCK/(PERIOD*STOTAL)) - 1.0
  10    CONTINUE
 200  CONTINUE
C-------------------------------------
      WRITE(6,145) N
 145  FORMAT(1X,'Dimension of DATA:',I8)
      VAR = 0.0
      AV=0.0
      DO 130 I=1,NHOUR
        AV=AV+DATA(I)
        VAR = VAR +  ((CABS(DATA(I))) ** 2)
 130    CONTINUE
      VAR=VAR/XHOUR
      AV=AV/XHOUR
      WRITE(6,140) STOTAL, AV
 140  FORMAT(1X,'Average power for <SITE1> =',F8.2,
     +'; Normalised average power =',F8.6)
C-------------------------------------------------------------
        CALL FOURG(DATA,NHOUR,-1,WORK)
        DO 135 I=1,NHOUR
          DATA(I)=DATA(I)/XHOUR
 135      CONTINUE
C-----------------------------------------------------------
C End of main DO loop.
C-----------------------------------------------------------
      TVAR=0.0
      DO 60 J=1,NHOUR2
        A = DATA(J)
        S(J) = ((CABS(A)) ** 2)*2
        TVAR = TVAR + S(J)
```

```
  60    CONTINUE
      WRITE(6,141) VAR, TVAR
 141  FORMAT(1X,/,' Output variance = ',F9.5,/,
     +' Input  variance = ',F9.5)
C-----------------------------------------------------------
C End of main section. S(J) contains the spectrum of the power output
C from <SITE1> (duration of data, <YEAR> years).
C -----------------------------------------------------------
      CALL STORE(S,NHOUR2,PERIOD,IDUR,NDIM,YPLOT2,XPLOT2)
      CALL SMOOT3(XPLOT1,YPLOT1,S,NHOUR2,IDIM)
      WRITE(6,80) (XPLOT2(I), YPLOT2(I), I=1,NDIM)
  80  FORMAT(' ',/,' Output from STORE.',/,(4(1X,'+ ',F6.1,F6.0,6X)))
      CALL LOGTRN(YPLOT1,XPLOT1,PERIOD,NHOUR,IDIM)
      CALL MGRAP2(YPLOT2,1,NDIM,60,XPLOT2,KEYS2,KDIM2,TITLE2,ITIT2,
     +XAX2,LEX2,YAX2,LEY2,1,1,IRUN,1000.0,5.0,0.0,0.0)
      CALL MGRAP2(YPLOT1,1,IDIM,JDIM,XPLOT1,KEYS1,KDIM1,TITLE1,
     +ITIT1,XAX1,LEX1,YAX1,LEY1,1,1,
     +IRUN,1.4,0.0,0.0,-5.0)
      STOP
      END
%
TLS WITH %H! TAPE RL1602
TT .GRAPH<IRUN> AS .GRAPH<IRUN>
!
*LABEND
```

COHERE

This prgram generates the coherence function of the power output for a pair of wind sites in the frequency range 3 yr^{-1} to 1/2 hr^{-1}.

```
REQUEST 'TURNROUND?  Default OVERNIGHT  ' TURNROUND$OVERNIGHT
REQUEST 'PRIORITY?  Default 10 '  PRIORITY$10
REQUEST 'Number of years ?  Default 9  '  YEAR$9
REQUEST 'LIMSTORE  ?  Default 400k for nine years  '  LIMSTORE$400
REQUEST 'TIME Default 120 '       TIME$120
REQUEST 'SITE1?  '     SITE1
REQUEST 'SITE2?  '     SITE2
REQUEST 'IRUN?  '      IRUN
REQUEST 'TYPE OF MILL?  Default MOD2  '  MILL$MOD2
REQUEST 'NHOUR  ?   Default value 8760  '   NHOUR$8760
REQUEST 'NHOUR2  ?   Default value 4380  '     NHOUR2$4380
REQUEST 'SOURCE or NOSOURCE ? Default NO...  '   SOURCE$NOSOURCE
SUBMIT %H*
JOB RL16 *  COHERE <MILL>
TURNROUND <TURNROUND>
PRIORITY <PRIORITY>
ROUTE OPENUNIV
LIMSTORE <LIMSTORE>
TIME <TIME>
PLOTTER 200 RECORDS
PAGE 0
SAVE
NOTIFY
TAPE9 RL1602/W
PHX
TLS WITH %H& TAPE RL1602
FT .<SITE1>7179R AS &<SITE1>
```

```
FT .<SITE2>7179R AS &<SITE2>
&
SET CHAR END ;
FORTQCLG(FT01F001=&<SITE1>,FT02F001=&<SITE2>,FT03F001=&CO<SITE1><SITE2>/N,;
FT08F001=RL16.PAIR<IRUN>/FB/N PROGRAM=%H%);
 BRIEFC=* SECG=<TIME> LIBRARY=SYS2.CAMLIB+GRAPHICS.NFORTLIB+RL16.LIB ;
PRINTC=* LISTC=<SOURCE> REGG=<LIMSTORE>
      INTEGER*2  IYEAR
      COMPLEX DATA, WORK, STEMP, A, B
      DIMENSION IYEAR(8808), COHERE(200),
     +XYR(<NHOUR2>), XYI(<NHOUR2>),  SX(<NHOUR2>), SY(<NHOUR2>),
     +SXYR(200), SXYI(200),  SXS(200), SYS(200), FREQ(200),
     +KEYS(2,1),TITLE(1,11),XAX(1,5),YAX(1,3),
     +KEYS1(2,1),TITLE1(1,11),XAX1(1,5),YAX1(1,3),
     +X(200),Y(200),S(<NHOUR2>),
     +STOTAL(2),
     +STEMP(<NHOUR2>),DATA(<NHOUR>),WORK(<NHOUR>)
      EQUIVALENCE (IYEAR(1),WORK(1),S(1))
C ------------------------------------------------------
C Data input for first graph.
      DATA KEYS1/'    ','    '/
      DATA TITLE1/'EFFE','CT O','F SI','TE D','ISPE','RSAL',
     +'. SI','TES ','<SITE1>+','<SITE2> ','<MILL>'/
      DATA XAX1/'LOG ','  PE','RIOD','(HOU','RS) '/
      DATA YAX1/'COHE','RENC','E   '/
      ITIT1=11
      KDIM1=2
      LEX1=5
      LEY1=3
C -----------------------------------------------------------
      NHOUR=<NHOUR>
      NHOUR2=NHOUR/2
      IRUN=<IRUN>
```

```
      YEAR=FLOAT(<YEAR>)
      DO 2 I=1,8808
        IYEAR(I)=0
   2  CONTINUE
      IDIM=0
      IDUR=NHOUR
      JDIM=200
      DO 6 I=1,NHOUR2
        XYR(I)=0.0
        XYI(I)=0.0
        SX(I)=0.0
        SY(I)=0.0
        STEMP(I)=(0.0,0.0)
   6  CONTINUE
      WRITE(6,15)
  15  FORMAT(1X,/,80('-'),/,
     +' SPECTRAL ANALYSIS OF <SITE1> AND <SITE2>. IRUN=<IRUN>',
     +/,80('-'))
C -------------------------------------------------------
C Finds average power at both sites.
C -------------------------------------------------------
      DO 400 ISITE=1,2
        N=0
        AV=0.0
        STOTAL(ISITE)=0.0
        DO 500 JYEAR=1,<YEAR>
          READ(ISITE,90) IYEAR
          DO 100 I=1,NHOUR
            IDUMMY = IYEAR(I)
            IF(IDUMMY.EQ.-32768) GOTO 100
            DUMMY=I<MILL>(IDUMMY)
            AV=AV+DUMMY
            N=N+1
```

```
 100        CONTINUE
 500      CONTINUE
          STOTAL(ISITE) = AV/FLOAT(N)
 400    CONTINUE
        REWIND 1
        REWIND 2
        WRITE(6,12) STOTAL(1), STOTAL(2), N
  12    FORMAT(1X,/,' Average power:  <SITE1> ',F6.2,/,
       +'                   <SITE2> ',F6.2,/,'  N= ',I5)
C ------------------------------------------------------------
C Start of main DO loop.
C ------------------------------------------------------------
        DO 200 JYEAR=1,<YEAR>
          DO 300 ISITE=1,2
            READ(ISITE,90) IYEAR
  90        FORMAT(100(44A2),4A2)
            DO 10 J=1,NHOUR
              IDUMMY = IYEAR(J)
              DUMMY = I<MILL>(IDUMMY)
              DATA(J) = DUMMY/STOTAL(ISITE)
              IF(IDUMMY.EQ.-32768) DATA(J) = 1.0
  10        CONTINUE
            CALL FFTX(DATA,WORK,NHOUR)
            IF(ISITE.EQ.2) GOTO 300
              DO 20 J=1,NHOUR2
                STEMP(J) = DATA(J)
  20          CONTINUE
 300      CONTINUE
C---------------------------------------------------------------
          DO 60 J=1,NHOUR2
            A = STEMP(J)
            B = CONJG(DATA(J))
            SX(J) = SX(J) + (((CABS(A))**2)*2.0/YEAR)
```

```
         SY(J) = SY(J) + (((CABS(B))**2)*2.0/YEAR)
         XYR(J) = XYR(J) + (REAL(A*B*2.0/YEAR))
         XYI(J) = XYI(J) + (AIMAG(A*B*2.0/YEAR))
  60    CONTINUE
 200  CONTINUE
C-----------------------------------------------------------------
C End of main DO loop. SX and SY now contain the spectral densities
C of wind power at sites <SITE1> AND <SITE2>. XYR and XYI contain
C the real and imaginary parts of the cross spectral density.
C ----------------------------------------------------------------
      CALL SMOOT3(FREQ(1),SXS(1),SX(1),NHOUR2,IDIM)
      CALL SMOOT3(FREQ(1),SYS(1),SY(1),NHOUR2,IDIM)
      CALL SMOOT3(FREQ(1),SXYR(1),XYR(1),NHOUR2,IDIM)
      CALL SMOOT3(FREQ(1),SXYI(1),XYI(1),NHOUR2,IDIM)
C ----------------------------------------------------------------
      DO 81 I=1,IDIM
        A = CMPLX( SXYR(I), SXYI(I) )
        COHERE(I) = (CABS(A) ** 2)/(SYS(I) * SXS(I))
  81  CONTINUE
      WRITE(6,80) ((FREQ(I), COHERE(I)), I=1,IDIM)
  80  FORMAT(1X,/,80('-'),/,' Coherence function for <SITE1> and <SITE2>.'
     +,/,80('-'),/,(4(1X,F5.0,2X,F6.4,'|')))
C ----------------------------------------------------------------
      DO 130 I=1,IDIM
        SXS(I) = 2.30259*SXS(I)*FREQ(I)
        SYS(I) = 2.30259*SYS(I)*FREQ(I)
        SXYR(I) = 2.30259*SXYR(I)*FREQ(I)
        SXYI(I) = 2.30259*SXYI(I)*FREQ(I)
 130  CONTINUE
      WRITE(3,82) ((FREQ(I),SXS(I),SYS(I),SXYR(I),SXYI(I)), I=1,IDIM)
  82  FORMAT(1X,'Spectral densities and cross spectrum. ',
     +' <SITE1> and <SITE2>',
     +/,(F6.0,4(1X,F10.6)))
```

```
C -----------------------------------------------------------------
      DO 120 I=1,IDIM
        FREQ(I) = ALOG10(NHOUR/FREQ(I))
 120  CONTINUE
      CALL MGRAP2(COHERE,1,IDIM,JDIM,FREQ,KEYS1,KDIM1,TITLE1,
     +ITIT1,XAX1,LEX1,YAX1,LEY1,1,1,
     +IRUN,1.0,5.0,0.0,0.0)
      STOP
      END
%
TLS WITH %H! TAPE RL1602
TT &CO<SITE1><SITE2> AS .CO<SITE1><SITE2>
!
```

WINDCORR

This program generates cross correlation coefficients for changes in the value of wind power at a pair of wind sites, for lead times in the range 1 to 24 hours. In addition the program generates the cross correlation coefficient of wind power at the two sites.

```
REQUEST 'TURNROUND?  Default NOW  ' TURNROUND$NOW
REQUEST 'PRIORITY?  Default 10 '  PRIORITY$10
REQUEST 'SITE1?  '     SITE1
REQUEST 'SITE2?  '     SITE2
REQUEST 'IRUN?  '      IRUN
REQUEST 'TYPE OF MILL?  Default MOD2  '  MILL$MOD2
REQUEST 'SOURCE or NOSOURCE ? Default NO...  '   SOURCE$NOSOURCE
SUBMIT %H*
JOB RL16 *  CORREL <MILL>
TURNROUND <TURNROUND>
PRIORITY <PRIORITY>
ROUTE OPENUNIV
LIMSTORE 250K
TIME 25
PLOTTER 20 RECORDS
PRINTER 2K
PAGE 0
SAVE
NOTIFY
TAPE9 RL1602
PHX
TLS WITH %H& TAPE RL1602
FT .<SITE1>7179R AS &<SITE1>
FT .<SITE2>7179R AS &<SITE2>
&
```

```
SET CHAR END ;
FORTQCLG(FT01F001=&<SITE1>,FT02F001=&<SITE2>,FT08F001=.PAIR<IRUN>/FB/N,;
PROGRAM=%H%);
 BRIEFC=* SECG=25 LIBRARY=SYS2.CAMLIB+GRAPHICS.NFORTLIB+RL16.LIB ;
PRINTC=* LISTC=<SOURCE> REGG=250K
      INTEGER*2 IYEAR
      DIMENSION IYEAR(8808),X(8760),Y(8760),RO(24),T(24),
     +SIGMAX(24),SIGMAY(24),
     +DELX(24),DELY(24),DELXY(24),SQDELX(24),SQDELY(24),
     +KEYS(2,1),TITLE(1,8),XAX(1,5),YAX(1,2)
C -----------------------------------------------------
C Data input for first graph.
      DATA KEYS/'    ','    '/
      DATA TITLE/'CROS','S CO','RREL','ATIO','N.  ',
     +'<SITE1>+','<SITE2> ','<MILL>'/
      DATA XAX/'LEAD',' TIM','E   ','(HOU','RS) '/
      DATA YAX/'RO(T',')   '/
      ITIT=8
      KDIM=2
      LEX=5
      LEY=2
C -----------------------------------------------------
      IRUN=<IRUN>
      WRITE(6,15)
  15  FORMAT(1X,/,80('-'),/,
     +' Correlation coefficient of fluctuations at <SITE1> and <SITE2>',
     +/,' as a function of lead time. IRUN = <IRUN>.'
     +/,80('-'))
C -------------------------------------------------------
      AVX=0.0
      AVY=0.0
      AVXY=0.0
      AVXSQ=0.0
```

```
      AVYSQ=0.0
      DO 160 J=1,24
        DELX(J)=0.0
        DELY(J)=0.0
        DELXY(J)=0.0
        SQDELY(J)=0.0
        SQDELX(J)=0.0
        T(J)=J
 160  CONTINUE
C ---------------------------------------------------------
C Start of main DO loop.
      DO 220 NYEAR=1,9
          CALL WREAD(IYEAR,X,Y)
        DO 180 I=1,8760
          AVX = AVX + X(I)
          AVY = AVY + Y(I)
          AVXY = AVXY + (X(I)*Y(I))
          AVXSQ = AVXSQ + (X(I)*X(I))
          AVYSQ = AVYSQ + (Y(I)*Y(I))
 180    CONTINUE
        DO 200 J=1,24
          IMAX = 8760 - J
          DO 210 I=1,IMAX
            DELX(J)=DELX(J) + (X(I) - X(I+J))
            DELY(J)=DELY(J) + (Y(I) - Y(I+J))
            SQDELX(J) = SQDELX(J) + ((X(I) - X(I+J))**2)
            SQDELY(J) = SQDELY(J) + ((Y(I) - Y(I+J))**2)
            DELXY(J) = DELXY(J) + ((X(I) - X(I+J))*(Y(I) - Y(I+J)))
 210      CONTINUE
 200    CONTINUE
 220  CONTINUE
C ---------------------------------------------------------
      AVX = AVX/(8760*9)
```

```
      AVY = AVY/(8760*9)
      AVXY = AVXY/(8760*9)
      AVXSQ = AVXSQ/(8760*9)
      AVYSQ = AVYSQ/(8760*9)
C ------------------------------------------------------------
      SIGX = SQRT(AVXSQ - (AVX*AVX))
      SIGY = SQRT(AVYSQ - (AVY*AVY))
      ROZERO = (AVXY - (AVX*AVY))/(SIGX*SIGY)
C ------------------------------------------------------------
      DO 240 J=1,24
        DELX(J) = DELX(J)/((8760-J)*9)
        DELY(J) = DELY(J)/((8760-J)*9)
        SQDELX(J) = SQDELX(J)/((8760-J)*9)
        SQDELY(J) = SQDELY(J)/((8760-J)*9)
        DELXY(J) = DELXY(J)/((8760-J)*9)
        SIGMAX(J) = SQRT(SQDELX(J) - (DELX(J)*DELX(J)))
        SIGMAY(J) = SQRT(SQDELY(J) - (DELY(J)*DELY(J)))
        RO(J) = (DELXY(J)-(DELX(J)*DELY(J)))/(SIGMAX(J)*SIGMAY(J))
 240  CONTINUE
C ------------------------------------------------------------
      WRITE(6,105) SIGX,SIGY,ROZERO
 105  FORMAT(1X,/,
     +/,' Total standard deviation at <SITE1> = ',F6.1,
     +/,' Total standard deviation at <SITE2> = ',F6.1,
     +/,' Cross correlation coefficient of power outputs = ',F6.3)
C------------------------------------------------------------
      WRITE(6,106) ((I,RO(I),SIGMAX(I),SIGMAY(I)), I=1,24)
 106  FORMAT(1X,//,80('-'),/,
     +' Lead        Correlation  Sigma        Sigma',/
     +' time(hours) RO(t)        <SITE1>          <SITE2>',/
     +80('-'),/,
     +(I6,6X,F6.3,7X,F6.1,7X,F6.1))
C ------------------------------------------------------------
```

```
      CALL MGRAP2(RO,1,24,24,T,KEYS,KDIM,TITLE,
     +ITIT,XAX,LEX,YAX,LEY,1,1,
     +IRUN,1.0,30.0,0.0,0.0)
      STOP
      END
C -------------------------------------------------------
      SUBROUTINE WREAD(IYEAR,X,Y)
C-----------------------------------------------------------
C Read wind speed data into X and Y.
      INTEGER*2 IYEAR
      DIMENSION IYEAR(8808), X(8760), Y(8760)
      READ(1,90) IYEAR
      DO 180 I=1,8760
        IDUMMY=IYEAR(I)
        X(I)=I<MILL>(IDUMMY)
 180    CONTINUE
      READ(2,90) IYEAR
      DO 175 I=1,8760
        IDUMMY=IYEAR(I)
        Y(I)=I<MILL>(IDUMMY)
 175    CONTINUE
  90  FORMAT(100(44A2),4A2)
C --------------------------------------------------------
      RETURN
      END
%
```

SUBROUTINE LIBRARY

The following contains a listing of the 4 most important subroutines used in the preceeding programs.

```
C ---------------------------------------------------
      SUBROUTINE FFTX(DATA,WORK,NHOUR)
C ---------------------------------------------------
C This subroutine finds the complex fourier transform of
C the array DATA. The result is returned in the same array.
C
      COMPLEX DATA, WORK
      DIMENSION DATA(NHOUR), WORK(NHOUR)
      AV=0.0
      XHOUR=NHOUR
      DO 10 I=1,NHOUR
        AV=CABS(DATA(I))+AV
   10   CONTINUE
      AV=AV/XHOUR
      DO 20 I=1,NHOUR
        DATA(I) = DATA(I) - AV
   20   CONTINUE
      CALL FOURG(DATA,NHOUR,-1,WORK)
      DO 30 I=1,NHOUR
        DATA(I) = DATA(I)/XHOUR
   30   CONTINUE
      RETURN
      END
```

```
C -----------------------------------------------------------
      SUBROUTINE ALIAS(S,NHOUR2,IDUR)
C -----------------------------------------------------------
C The following subroutine corrects the spectrum of wind power output
C for the effect of aliasing. The method is to assume that the true
C logarithmic spectrum,  nS(n)  would have a constant value in the
C spectral gap, equal to half the value of the un-corrected spectrum
C at the nyquist frequency.
C
C
      DIMENSION S(NHOUR2)
      SNY=0.0
      DO 5 I=1,100
        SNY = SNY + S(NHOUR2+1-I)
   5    CONTINUE
      SNY = SNY/100.0
C ----------------------------------------------
      DO 10 N=1,NHOUR2
        FREQ = FLOAT(N)/FLOAT(IDUR)
        SPRIME = 0.0
        PI=3.14159
        X = 1.0-FREQ
        SPRIME = SNY * ((SIN(PI*X)) ** 2) / (16.0 * (X**3))
        S(N) = S(N) - SPRIME
  10    CONTINUE
      RETURN
      END
```

```
C -----------------------------------------------------
      SUBROUTINE RANWAL(S,NHOUR2,NDIM,Y,X)
C -----------------------------------------------------
C The following subroutine integrates S(I) using a weighting
C function
C             FILTER = 4 * ((sin(PI*n*IPER))**2)
C
C The frequency n = I/NHOUR .
C The length of the sampling period of the original data
C is NHOUR. This data will have been block averaged over a
C time interval 1 hour. NHOUR2 is the dimension of the array S.
C The integral is evaluated over a range of values of IPER
C     1 < IPER < NHOUR
C
C The routine estimates the contribution from frequencies above
C the nyquist, on the assumption that nS(n) is constant for n > 1/2
C The integral FINT(IPER) is an estimate of the standard deviation
C of  x(t + IPER) - x(t)  , ie describes the random walk of x(t).
C It is important that the input array only represent spectral
C estimates up to the Nyquist frequency.
C
      DIMENSION S(NHOUR2), Y(60), X(60)
      DO 5 I=1,60
        Y(I)=0.0
        X(I)=0.0
  5     CONTINUE
      PI=3.1415927
      NDIM=0
      NHOUR=NHOUR2*2
      IPER=0
      SNY=0.0
      DO 8 I=1,100
        SNY=SNY+S(NHOUR2-1-I)
```

```
  8    CONTINUE
     SNY=SNY/100.0
C -------------------------------------------------
     DO 10 N=1,60
       IF(IPER.GE.5) GOTO 50
         IPER=IPER+1
         GOTO 60
 50      CONTINUE
         IPER=IPER*1.24
 60      CONTINUE
       IF(IPER.GE.NHOUR) GOTO 10
       NDIM=NDIM+1
       X(N)=ALOG10(FLOAT(IPER))
       FINT=0.0
       IMIN=NHOUR/(20*IPER)
       IF(IMIN.EQ.0) IMIN=1
       DO 20 I=IMIN,NHOUR2
         Z=PI*FLOAT(I)*FLOAT(IPER)/FLOAT(NHOUR)
         FILTER = (SIN(Z))**2
         FINT = FINT + (S(I)*FILTER)
 20      CONTINUE
C -------------------------------------------------
C The next section corrects for frequencies above nyquist.
       I1=NHOUR2+1
       DO 30 I=I1,NHOUR,12
         F=PI*FLOAT(I)/FLOAT(NHOUR)
         Z=PI*FLOAT(I)*FLOAT(IPER)/FLOAT(NHOUR)
         F1=((SIN(F))**2)/(F**3)
         F2 = (SIN(Z))**2
         FINT=FINT + (12.0*SNY*F1*F2/8.0)
  30     CONTINUE
       Y(N) = 4.0 * FINT
 10    CONTINUE
```

```
C ----------------------------------------------------
      RETURN
      END
```

```
C -----------------------------------------------------------
      SUBROUTINE SMOOT3(X,Y,S,ISIZE,IDIM)
C -----------------------------------------------------------
C The following takes raw spectral estimates in S(I), and runs a
C moving block average. The width of the band over which the raw data
C is averaged is DELTA w = 0.24w for high values of w, with a
C lower limit of 5 spectral estimates at the lowest values of w.
C The averaged spectral estimates are returned as Y(I), and the
C central frequency of each block is returned as X(I). The
C number of frequency blocks is returned as IDIM, with a maximum
C value of 200.
C
      DIMENSION X(200), Y(200), S(ISIZE)
      DELK=0.0
      KDEL=0
      K=4
      J=0
      IDIM=0
      DO 5 I=1,200
        X(I)=0.0
        Y(I)=0.0
   5    CONTINUE
C
C The above dimensions the output arrays, X, Y, and the input array S.
C
 100  CONTINUE
        DELK=SQRT(25.0 + 0.0576*(K**2))
        KDEL=(DELK-1)/2
        IF((K+KDEL).GE.(ISIZE)) GOTO 900
        IDIM=IDIM+1
        SAV=0
        IDUMMY=1+(2*KDEL)
        DO 10 I=1,IDUMMY
```

```
  10      SAV=SAV+S(I+K-1-KDEL)
        SAV=SAV/(FLOAT((2*KDEL)+1))
C
C The above averages S(K) over 2*KDEL + 1 values of K. The result is SAV.
C
        J=J+1
        X(J)=FLOAT(K-1)
        Y(J)=SAV
C
C The above puts values of K and SAV into the output arrays.
C
        K = K + (((2*KDEL)+1)/3)
        GOTO 100
 900  CONTINUE
      RETURN
      END
```